Exercita 24

1. $f(x) = x^7 + 5x - 6$

(01) $f'(x) = 7x^6 + 5 > 0 \quad f \nearrow$

(B) $3 < \pi \Rightarrow f(3) < f(\pi)$

(8) i) Av $x \in (1,2)$ vo $f(x) > 0$

$1 < x < 2$

$f \nearrow$

$f(1) < f(x) < f(2)$

$0 < f(x) < \infty$

ii) Av $\lambda > \mu$ vo $\dfrac{\lambda^7 - \mu^7}{\lambda - \mu} > -5$

$\lambda > \mu \Rightarrow f(\lambda) > f(\mu)$

$\lambda^7 + 5\lambda - 6 > \mu^7 + 5\mu - 6$

$\lambda^7 - \mu^7 > 5\mu - 5\lambda$

$\lambda^7 - \mu^7 > -5(\lambda - \mu)$

$\dfrac{\lambda^7 - \mu^7}{\lambda - \mu} > -5$ ✓

11) $f(x^7+x-3) + x^7 = -2x-3$

$f(x^7+x-3) = -x^7-2x-3$

$f(x^7+x-3) = f(-x)$.

$f : 1-1$

$x^7+x-3 = -x$

$x^7+2x-3 = 0$

$f(x) = 0$

$f(x) = f(1)$

$x = 1$

11) $e^{7x} + 2e^x > 3$

$e^{7x} + 2e^x - 3 > 0$

$f(e^x) > 0$

$f(e^x) > f(1)$

$f \nearrow$

$e^x > 1$

$e^x > e^0$

$x > 0$

O i) $f(e^x - 1) - 2x = x^7 - 3$

$f(e^x - 1) = x^7 + 2x - 3$

$f(e^x - 1) = f(x)$

$f \exists 1-1$

$e^x - 1 = x$

• $e^x \geq x + 1$ To "=" $x = 0$

(δ) $\lim_{x \to 1^+} \frac{1}{f(x)} = +\infty$.

(+)

$x > 1 \Rightarrow f(x) > f(1) \Rightarrow f(x) > 0$

2. $f(x) = x^7 + 2x - 3$ $\quad f'(x) = 7x^6 + 2 > 0$

$f \nearrow$

(α) $x^7 + 2x = 3$

$x^7 + 2x - 3 = 0$

$f(x) = 0$

$f(x) = f(1)$

f 1-1

$x = 1$

(β) i) $x < \frac{3}{x^6 + 2}$ $\Rightarrow x^7 + 2x < 3$

$x^7 + 2x - 3 < 0$

$f(x) < 0$

$f(x) < f(1)$

$f \nearrow$

$x < 1$

⑧ $\lim_{x \to 1} \frac{1}{f(x)}$ δω υπερχ-1

x		1	
f(x)	↗	− 0	+ ↗

$$\lim_{x \to 1^-} \frac{1}{f(x)} = -\infty$$

$$\lim_{x \to 1^+} \frac{1}{f(x)} = +\infty$$

3. (a) $f(x) = x - \ln(x^2+1)$, $x \in \mathbb{R}$.

$$f'(x) = 1 - \frac{2x}{x^2+1} = \frac{x^2+1-2x}{x^2+1}$$

$$f'(x) = \frac{(x-1)^2}{x^2+1} \geq 0$$

$f \nearrow$

(B) i) $x = \ln(x^2+1)$

$x - \ln(x^2+1) = 0$

$f(x) = 0$

$f(x) = f(0)$

$x = 0$

ii). $f(x^2+1) > f(e^x)$

$f \uparrow$

$x^2+1 > e^x$

$\ln(x^2+1) > x$

$0 > x - \ln(x^2+1)$

$0 > f(x)$

$f(0) > f(x)$

$f \uparrow$

$x < 0$

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iii) $e^{f(x)} - 1 = f^2(x) \Rightarrow e^{f(x)} = f^2(x) + 1$

$f(x) = \ln(f^2(x)+1)$

$f(x) - \ln(f^2(x)+1) = 0$

$f(f(x)) = 0$

$f(f(x)) = f(0)$

$f(x) = 0$
$f(x) = f(0)$

$x = 0$

(8) Να δ. $\ln(n^2+1) + f(\ln n) < n$

$$f(\ln n) < n - \ln(n^2+1)$$

$$f(\ln n) < f(n)$$

$$f \nearrow$$

$$\ln n < n$$

✓

• $\ln x \le x - 1$

$$\ln n \le n - 1$$

$$\ln n - n \le -1$$

$$\ln n - n < 0$$

$$\ln n < n$$

(δ) $\lim_{x \to 0} \frac{\ln x}{f(3x) - f(2x)} = \lim_{x \to 0^+} \ln x \, \frac{1}{f(3x) - f(2x)}$

$= -\infty \cdot (+\infty) = -\infty.$

$\longrightarrow 2x < 3x \Rightarrow f(2x) < f(3x)$

$\boxed{f(3x) - f(2x) > 0}$

(ε) Αν $f(a) < 0$ τότε $e^a < 1$

$f(a) < 0$

$f(a) < f(0)$

$f \nearrow$

$a < 0$

$e^a < e^0$

$e^a < 1$ ✓

4. $f(x) = x^2 + \ln x - 1 \quad , x > 0$

(a) Pilul $f(x)$

$f'(x) = 2x + \frac{1}{x} > 0$

$f \nearrow$

$f(x) = 0$

$f(x) = f(1)$

$f \overset{?}{=} 1-1$

$x = 1$

x	1
$f(x)$	$-\infty \quad 0 \quad +\nearrow$

(B) $(x^2+1)^2 + \ln(x^2+1) = 1$

$f(x^2+1) = 0$

$f(x^2+1) = f(1)$

$f \overset{?}{=} 1-1$

$x^2+1 = 1$

$x^2 = 0$

$x = 0$

⑧ $xe^{x^2} > e$ $(0, +\infty)$.

$\ln xe^{x^2} > \ln e$

$\ln x + \ln e^{x^2} > 1$

$\ln x + x^2 > 1$

$x^2 + \ln x - 1 > 0$

$f(x) > 0$

$f(x) > f(1)$

$f \nearrow$

$x > 1$

⑧ $\ln \frac{x^2+1}{x+1} > (x+1)^2 - (x^2+1)^2$

$\ln(x^2+1) - \ln|x+1| > (x+1)^2 - (x^2+1)^2$

$\ln(x^2+1) + (x^2+1)^2 - 1 > (x+1)^2 + \ln|x+1| - 1$

$f(x^2+1) > f(x+1) \longrightarrow$ $x+1>0$

$f \nearrow$ $x>-1$

$x^2+1 > x+1$

$x^2 - x > 0$

x		0		
x^2-x	$+$	$-$	$/$	$+$

$x \in (-\infty, 0) \cup (1, +\infty)$

$x \in (-1, 0) \cup (1, +\infty)$

29. $f:\mathbb{R}\to\mathbb{R}$ δυο φορές παρ/μη

$f''(x)<0 \quad \forall x\in\mathbb{R}$

$f'(1)=0$

(α) Μονοτονία $f(x)$

x		1	
f''		−	−
f'	↘	+ 0	− ↘
f		↗	↘

$x<1 \Rightarrow f'(x)>f'(1)$

$f'(x)>0$

(β) i) Γνησίως $f(2-x)=f(3) \qquad (-\infty,0)$

$f(2-x)=f(3)$

• $3>1$

• $x<0 \Rightarrow -x>0 \Rightarrow 2-x>2$

$\forall x>1$ η f 1-1

$2-x=3 \qquad x=-1$

11). $f(e^{2-x}) = f\left(\frac{1}{x-1}\right)$ $(1,2]$

- $1 < x \leq 2$

$-1 > -x \geq -2$

$1 > 2-x \geq 0$

$\boxed{e > e^{2-x} \geq 1}$

- $1 < x \leq 2$

$0 < x-1 \leq 1$

$\boxed{\frac{1}{x-1} \geq 1}$

f 1-1 $\forall x > 1$

$$e^{2-x} = \frac{1}{x-1}$$

$$e^{2-x} - \frac{1}{x-1} = 0$$

$$\frac{1}{e^{x-2}} - \frac{1}{x-1} = 0$$

$$\frac{x-1-e^{x-2}}{(x-1)e^{x-2}} = 0$$

$$x - 1 - e^{x-2} = 0$$

$e^x \geq x + 1 \longrightarrow x = 0$

$e^{x-2} \geq x - 2 + 1 \longrightarrow x - 2 = 0$

$x = 2$

$e^{x-2} \geq x - 1$

$0 \geq x - 1 - e^{x-2}$

⑧ $f(e^{x-2}) = f\left(\frac{1}{x}\right)$ $(1,2)$

μονοτονία

- $1 < x < 2$

$-1 < x-2 < 0$

$e^{-1} < e^{x-2} < 1$

- $1 < x < 2$

$1 > \frac{1}{x} > \frac{1}{2}$

$e^{x-2} = \frac{1}{x}$

$e^{x-2} - \frac{1}{x} = 0$

$f(x)$ $f'(x) = e^{x-2} + \frac{1}{x^2} > 0$

$f(1) = e^{-1} - 1 = \frac{1}{e} - 1 < 0$ $f \nearrow$

$f(2) = 1 - \frac{1}{2} > 0$

$f(1)f(2) < 0$ Bolzano $\exists \xi \in (1,2)$ τ.ω $f(\xi) = 0$

Rolle

f συνεχώς $[a,\beta]$
f παραγωγίσιμη (a,β)
$f(a)=f(\beta)$
} $\exists \xi \in (a,\beta)$ τ.ω $f'(\xi)=0$.

Βοήθεια 20

2. Ⓑ $f(x)=\frac{e^x}{x}$ $\Delta=[\ln 2, \ln 4]$

f συνεχώς $[\ln 2, \ln 4]$ ως π.σ.σ

f παραγωγίσιμη $(\ln 2, \ln 4)$ ως π.π.σ

$$f(\ln 2)=\frac{e^{\ln 2}}{\ln 2}=\frac{2}{\ln 2}$$

$$f(\ln 4)=\frac{e^{\ln 4}}{\ln 4}=\frac{4}{\ln 4}=\frac{4}{\ln 2^2}=\frac{4}{2\ln 2}=\frac{2}{\ln 2}$$

$f(\ln 2)=f(\ln 4)$

Rolle $\exists \xi \in (\ln 2, \ln 4)$ τ.ω $f'(\xi)=0$

$$f'(x) = \frac{e^x x - e^x}{x^2}$$

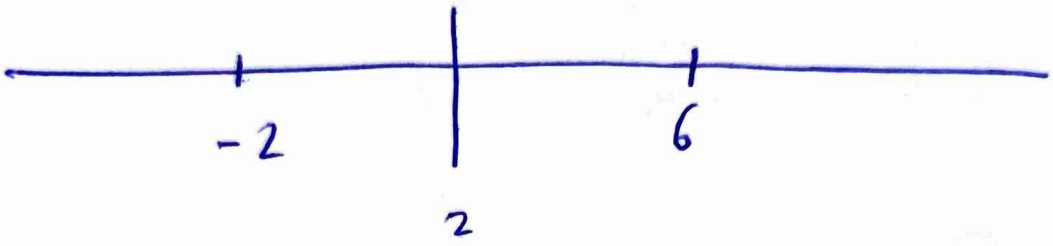
$$f'(1) = 0 \Rightarrow \frac{e^1 1 - e^1}{1^2} = 0$$

$$e^1(1-1) = 0$$

$$\underline{\underline{1 = 1}}$$

16. ⓐ $f(x) = \begin{cases} x^2 - 6x + 3, & x > 2 \\ -2x - 1, & x \le 2 \end{cases}$

Ισχύει το Rolle στο $[-2, 6]$



$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} -2x - 1 = -5$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 - 6x + 3) = -5$$

$$\left.\begin{array}{l} \lim\limits_{x \to 2} f(x) = -5 \\ f(2) = -5 \end{array}\right\} \text{συνεχή στο } 2$$

Η f συνεχή στο $[-2, 2) \cup (2, 6]$ αλλά η Σ.Θ.Μ

συνεχή $[-2, 6]$

$$\lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2^-} \frac{-2x-1-(-5)}{x-2} =$$

$$= \lim_{x\to 2^-} \frac{-2x-1+5}{x-2} =$$

$$= \lim_{x\to 2^-} \frac{-2x+4}{x-2} = \lim_{x\to 2^-} \frac{-2(x-2)}{x-2}$$

$$= -2$$

$$\lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2^+} \frac{x^2-6x+3+5}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-6x+8}{x-2} = \lim_{x\to 2^+} \frac{2x-6}{1}$$

$$= -2$$

$f'(2) = -2$

Η f παραγωγίσιμη στο 2.

Η f παρ/μη $(-2, 2)$ και $(2, 6)$ με π.π.σ

Άρα η f παρ/μη $(-2, 6)$ ✓

$f(-2) = 3$
$f(6) = 3$ ✓

Rolle ✓

$x \leq 2$

$f'(x) = -2$

$x > 2$

$f'(x) = 2x - 6$

$2x - 6 = 0$

$x = 3$