Ενότητα 13

2. $f: \mathbb{R} \to \mathbb{R}$ συνεχής
$f(1) = 2$
$f(x) \neq 0 \quad \forall x \in \mathbb{R}.$

Αφού f συνεχής και $f(x) \neq 0$
$\Rightarrow f(x) > 0$ ή $f(x) < 0$ $\forall x \in \mathbb{R}$

Αφού $f(1) = 2$
$\Rightarrow f(x) > 0 \quad \forall x \in \mathbb{R}$

$$g(x) = \frac{1}{\sqrt{f(x)}}$$

πρέπει $f(x) > 0$ που είναι!

$A_f = \mathbb{R},$

5. $f: \mathbb{R} \to \mathbb{R}$, συνεχής, $f(x) \neq 0$ και $f(1) = -2$

$\Rightarrow$ Αφού f συνεχής και $f(x) \neq 0$
$\Rightarrow f(x) > 0$ ή $f(x) < 0$ $\forall x \in \mathbb{R}$
και αφού $f(1) = -2 \Rightarrow f(x) < 0$

(α) $$\lim_{x \to 1} \frac{|f(x)| - 2}{f^2(x) + 2f(x)} \underset{\substack{x \to 1 \\ t \to -2}}{\overset{f(x) = t}{=}} \lim_{t \to -2} \frac{\overset{\ominus}{|t|} - 2}{t^2 + 2t}$$

$$= \lim_{t \to -2} \frac{-t-2}{t(t+2)} = \lim_{t \to -2} \frac{-\cancel{(t+2)}}{t\cancel{(t+2)}} = \frac{1}{2},$$

(B) $\lim_{x \to -\infty} (f(2)-1)x^3+5x-1 =$

$= \lim_{x \to \infty} (f(2)-1)x^3 = \underset{\ominus}{(f(2)-1)}(-\infty) = +\infty$,

$H \quad f(x) < 0 \quad \forall x \in \mathbb{R}$.

$\Rightarrow f(2) < 0$

$f(2)-1 < -1$

$\underline{\underline{f(2)-1 < 0}}$

7. $f: \mathbb{R} \to \mathbb{R}$ συνεχής

$f(x) \neq 0 \quad \forall x \in \mathbb{R}$

Ν.δ.ο. η $x f(x) = x^2 - 4$ έχει ρίζα $(-2, 2)$

$$\underbrace{x f(x) - x^2 + 4}_{g(x)} = 0$$

Η $g(x)$ συνεχής $[-2, 2]$ ως π.σ.σ.

$g(-2) = -2f(-2) - 4 + 4 = -2f(-2)$

$g(2) = 2f(2)$

$g(-2)\,g(2) = -4 \underbrace{f(-2) f(2)}_{+} < 0$ Bolzano $\exists \xi \in (-2, 2)$ τ.ω $g(\xi) = 0$

Αφού $f(x) \neq 0$ και συνεχής

$\Rightarrow f(x) > 0$ ή $f(x) < 0 \quad \forall x \in \mathbb{R}$.

Σε κάθε περίπτωση $f(-2), f(2)$ ομόσημοι

$\Rightarrow f(-2) f(2) > 0$

9. $f: \mathbb{R} \to \mathbb{R}$ συνεχής

$f(2) = 1$

x		1		4	
$f(x)$	;	0	$+$	0	;

$$\lim_{x\to-\infty}\left(f(3)x^3-2x+3\right) = \lim_{x\to-\infty}\left(f(3)x^3\right) =$$

$$= \underset{+}{f(3)}\,(-\infty) = -\infty$$

11. (a) $f(x) = 3x^3 - 2x - 1$

$\rightarrow f(x) = 0 \Rightarrow 3x^3 - 2x - 1 = 0$

3	0	-2	-1	1
$\downarrow$	3	3	1	
3	3	1	0	

$$(x-1)(3x^2+3x+1) = 0$$

$\Delta < 0$

$x = 1$

x		1	
$f(x)$	$-$	0	$+$

$f(0) = -1$ $\quad f(2) = 19$

ή

x		1	
$x-1$	$-$	0	$+$
$3x^2+3x+1$	$+$		$+$
$f(x)$	$-$		$+$

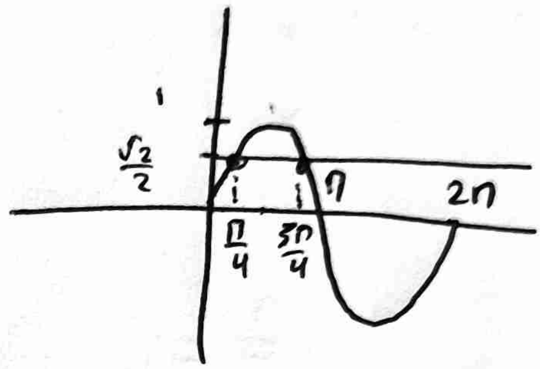
12. ⓐ $f(x)=\sqrt{2}\,\eta\mu x-1 \quad , \; x\in[0,\pi]$

$f(x)=0 \Rightarrow \sqrt{2}\,\eta\mu x-1=0$

$\sqrt{2}\,\eta\mu x=1$

$\eta\mu x=\dfrac{1}{\sqrt{2}} \Rightarrow \eta\mu x=\dfrac{\sqrt{2}}{2}$

x	0		$\frac{\pi}{4}$		$\frac{3\pi}{4}$		π
$f(x)$		$-$	0	$+$	0	$-$	



$f(0)=-1$

$f(\pi)=-1$

$f(\frac{\pi}{2})=\sqrt{2}-1$

B' τρόπος

$\eta\mu x=\dfrac{\sqrt{2}}{2}$

$\eta\mu x=\eta\mu\dfrac{\pi}{4}$

$\boxed{x=2\kappa\pi+\frac{\pi}{4}}$ ή $x=2\kappa\pi+\pi-\frac{\pi}{4}$

$\boxed{x=2\kappa\pi+\frac{3\pi}{4}}$

$$0 \le x \le \pi$$

$$0 \le 2k\pi + \frac{\pi}{4} \le \pi$$

$$0 \le 2k + \frac{1}{4} \le 1$$

$$0 \le 8k + 1 \le 4$$

$$-1 \le 8k \le 3$$

$$-\frac{1}{8} \le k \le \frac{3}{8}$$

$$k \in \mathbb{Z}$$

$$k = 0$$

$$x = \frac{\pi}{4}$$

$$0 \le x \le \pi$$

$$0 \le 2k\pi + \frac{3\pi}{4} \le \pi$$

$$0 \le 2k + \frac{3}{4} \le 1$$

$$0 \le 8k + 3 \le 4$$

$$-3 \le 8k \le 1$$

$$-\frac{3}{8} \le k \le \frac{1}{8}$$

$$k \in \mathbb{Z}$$

$$k = 0$$

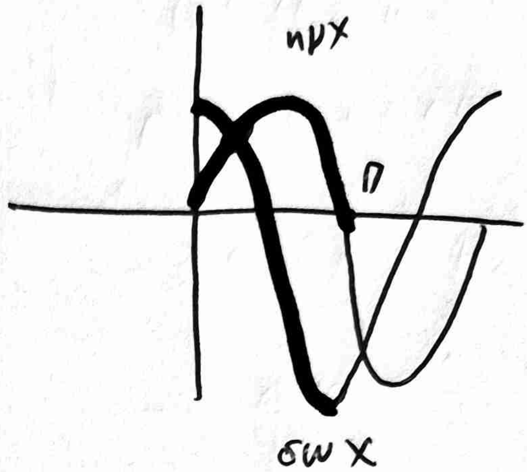
$$x = \frac{3\pi}{4}$$

Γ) $f(x) = \eta\mu x - \sigma\upsilon\nu x \qquad x \in [0, \pi]$

$f(x) = 0 \Rightarrow \eta\mu x - \sigma\upsilon\nu x = 0$

$\eta\mu x = \sigma\upsilon\nu x$

$x = \frac{\pi}{4}$



x	0	$\pi/4$	π
$f(x)$	$-$	0	$+$

$f(0) = -1$

$f(\pi) = 1$

f συνεχής

$f(0) = 2$

14. (B) $|f(x)| = e^x + 1$

Ρίζα $f(x)$

$f(x) = 0$

$|f(x)| = 0$

$e^x + 1 = 0$

Άτοπο!

$f(x) \neq 0$ και συνεχής

$\Rightarrow f(x) > 0$ ή $f(x) < 0$

$f(0) = 2 \Rightarrow f(x) > 0$

$|\overset{+}{f(x)}| = e^x + 1$

$f(x) = e^x + 1$

δ) $f^2(x) = 2 - ημx$ $\qquad$ $f(0) = \sqrt{2}$

$$f^2(x) = \left(\sqrt{2-ημx}\right)^2$$

$$|f(x)| = \left|\sqrt{2-ημx}\right|$$

$$|f(x)| = \sqrt{2-ημx} \Rightarrow f(x) = \sqrt{2-ημx}$$

Ρίζες $f(x)$

$f(x)=0$

$|f(x)|=0$

$\sqrt{2-ημx}=0$

$2-ημx=0$

$2=ημx$

Άτοπο!

$f(x) \neq 0$ και συνεχής

$\Rightarrow f(x)>0$ ή $f(x)<0$

$\forall x \in \mathbb{R}$.

$f(0)=\sqrt{2} \Rightarrow \underline{f(x)>0}$

15. (B) $f^2(x) + 4ημx = ημ^2x + 4$ $\qquad f(0)=2$

$$f^2(x) = ημ^2x - 4ημx + 4$$

$$f^2(x) = (ημx - 2)^2$$

$$|f(x)| = |ημx - 2| \qquad ημx \leq 1$$

$$ημx < 2$$

$$ημx - 2 < 0$$

$$|f(x)| = 2 - ημx$$

$$\boxed{f(x) = 2 - ημx}$$

<u>Ρίζα f(x)</u>

$f(x) = 0$

$|f(x)| = 0$

$2 - ημx = 0$

Άτοπο

$f(x) \neq 0$ και συνεχής

$f(x) > 0$ ή $f(x) < 0$

$f(0) = 2$

$\Rightarrow f(x) > 0$

17. (B) $f^2(x) = 1-2xf(x)$ $\qquad f(0)=-1$

$$f^2(x)+2xf(x) = 1$$

$$f^2(x)+2xf(x)+x^2 = x^2+1$$

$$\left(f(x)+x\right)^2 = x^2+1$$

$$\left(f(x)+x\right)^2 = \sqrt{x^2+1}^{\,2}$$

$$|f(x)+x| = |\overset{+}{\sqrt{x^2+1}}|$$

$$|\underbrace{f(x)+x}_{h(x)}| = \sqrt{x^2+1}$$

$$|\overset{-}{h(x)}| = \sqrt{x^2+1} \Rightarrow -h(x) = \sqrt{x^2+1}$$

$$-(f(x)+x) = \sqrt{x^2+1}$$

$$-f(x)-x = \sqrt{x^2+1}$$

$$-f(x) = x+\sqrt{x^2+1}$$

Ρίζα $h(x)$

$h(x)=0$

$|h(x)|=0$

$\sqrt{x^2+1}=0$

Άτοπο.

$h(x)\neq 0$ και συνεχής

$h(x)>0$ ή $h(x)<0$

$h(0)=f(0)+0 = -1+0=-1$

$h(x)<0$

$$f(x) = -x-\sqrt{x^2+1}$$

17. (B) $f^2(x) + x^4 = 1 + 2x^2 f(x)$ $\quad f(0)=1$

$f^2(x) - 2x^2 f(x) + x^4 = 1$

$(f(x) - x^2)^2 = 1^2$

$|f(x) - x^2| = |1|$ $\oplus$

$\oplus$

$|h(x)| = 1 \Rightarrow h(x) = 1 \Rightarrow f(x) - x^2 = 1$

$f(x) = x^2 + 1$

Ρ.Τ.Ι $h(x)$

$h(x) = 0$

$|h(x)| = 0$

$1 = 0$

Άτοπο!

$h(x) \neq 0$ και συνεχής

$h(x) > 0$ ή $h(x) < 0$

$h(0) = f(0) - 0 = 1$

$h(x) > 0$

18. $e^{f(x)}\left(e^{f(x)}-2x\right)=1 \quad \forall x\in\mathbb{R}.$

$$e^{2f(x)}-2xe^{f(x)}=1 \longrightarrow \underline{x=0}: \quad e^{2f(0)}=1,\quad 2f(0)=0,\quad \underline{\underline{f(0)=0}}$$

$$e^{2f(x)}-2xe^{f(x)}+x^2=x^2+1$$

$$\left(e^{f(x)}-x\right)^2=x^2+1$$

$$\left(e^{f(x)}-x\right)^2=\sqrt{x^2+1}^{\,2}$$

$$\left|e^{f(x)}-x\right|=\left|\overset{\oplus}{\sqrt{x^2+1}}\right|$$

$$\left|\overset{\oplus}{h(x)}\right|=\sqrt{x^2+1} \Rightarrow h(x)=\sqrt{x^2+1}$$

$$e^{f(x)}-x=\sqrt{x^2+1}$$

$$e^{f(x)}=\sqrt{x^2+1}+x$$

$$\underline{\underline{f(x)=\ln\left(\sqrt{x^2+1}+x\right)}}$$

<u>Ρ.Τ.Δ $h(x)$:</u>

$h(x)=0$

$|h(x)|=0$

$\sqrt{x^2+1}=0$

Άτοπο!

$h(x)\neq 0$ και συνεχής

$h(x)>0$ ή $h(x)<0 \ \forall x\in\mathbb{R}$

$h(0)=e^{f(0)}-0=e^0=1$

$h(0)=1$

$\underline{\underline{h(x)>0}}$

19. (B) $f^2(x) + 2e^x = e^{2x} + 1$ $\qquad$ $\underline{\underline{x \geq 0}}$

$f(\ln 2) = -1$

$f^2(x) = e^{2x} - 2e^x + 1$

$f^2(x) = (e^x - 1)^2$

$|\overset{\ominus}{f(x)}| = |\overset{\oplus}{e^x - 1}|$

Ρίζα $f(x)$

$f(x) = 0$

$|f(x)| = 0$

$|e^x - 1| = 0$

$e^x - 1 = 0$

$e^x = 1$

$\underline{\underline{x = 0}}$

x		0	
$f(x)$		0	$-$

$f(\ln 2) = -1$

Αφού $x \geq 0$

$e^x \geq e^0$

$e^x \geq 1$

$e^x - 1 \geq 0$

$-f(x) = e^x - 1$

$f(x) = 1 - e^x$

20. B $f^2(x) = 4 - x^2$ $\quad x \in [-2, 2]$

$f(0) = 2$

$$f^2(x) = \sqrt{4-x^2}^{\,2}$$

$$|f(x)| = |\sqrt{4-x^2}|$$

$$|f(x)| = \sqrt{4-x^2}$$

$$f(x) = \sqrt{4-x^2}$$

Find $f(x)$

$$f(x) = 0$$

$$|f(x)| = 0$$

$$\sqrt{4-x^2} = 0$$

$$4 - x^2 = 0$$

$$x = \pm 2$$

x		-2		2	
$f(x)$			+		

$f(0) = 2$

23. (B)

$$f^2(x) = e^{2x} - 2e^x + 1$$

$$f^2(x) = (e^x - 1)^2$$

$$|f(x)| = |e^x - 1| .$$

- $x > 0$
  $e^x > e^0$
  $e^x > 1$
  $e^x - 1 > 0$
- $x < 0$
  $e^x - 1 < 0$

P,TJ $f(x)$

$f(x) = 0$

$|f(x)| = 0$

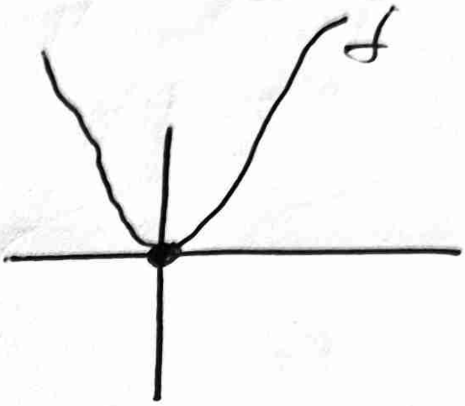
$|e^x - 1| = 0$

$e^x - 1 = 0$

$e^x = 1$

$x = 0$.

1.



x		0	
$f(x)$	;		;

$x < 0$

$|f(x)| = |e^x - 1|$

$f(x) = 1 - e^x$

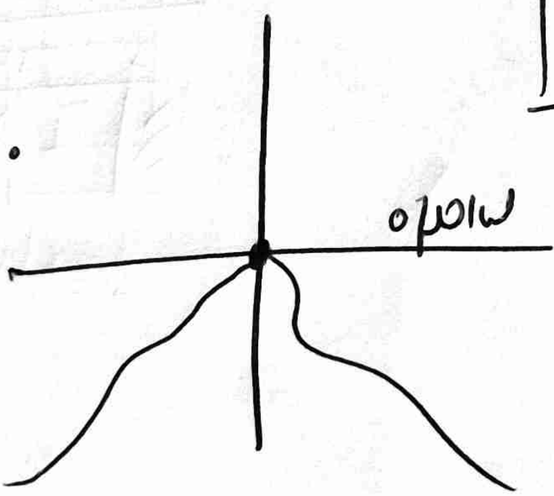
$x \geq 0$

$|f(x)| = |e^x - 1|$

$f(x) = e^x - 1$

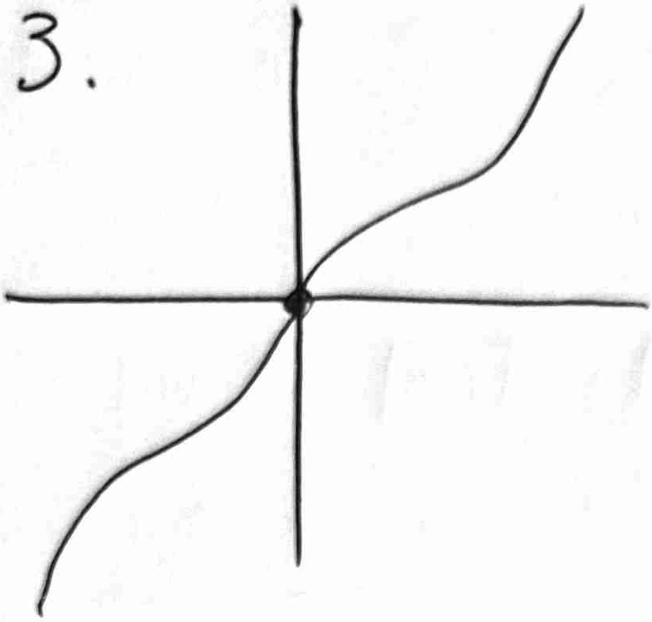
$$f(x) = \begin{cases} 1 - e^x , & x < 0 \\ e^x - 1 , & x \geq 0 \end{cases}$$

2.



$$f(x) = \begin{cases} e^x - 1 , & x < 0 \\ 1 - e^x , & x \geq 0 \end{cases}$$

3.



$x < 0$

$|f(x)| = |e^x - 1|$

$-f(x) = 1 - e^x$

$f(x) = e^x - 1$

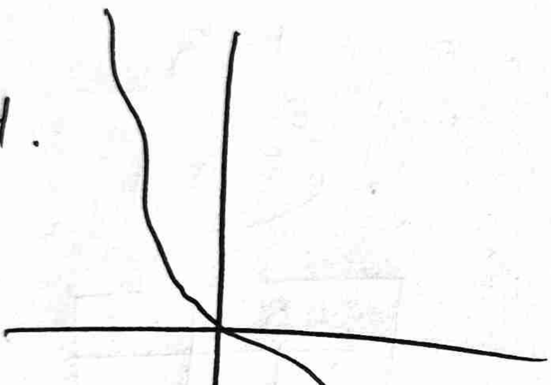
$x \geq 0$

$|f(x)| = |e^x - 1|$

$f(x) = e^x - 1$

$f(x) = e^x - 1$

4.



opow

$f(x) = 1 - e^x$

27. ⓐ $\underbrace{e^x + x - 1}_{\varphi(x)} = 0 \Rightarrow \varphi(x) = 0$

$\varphi \nearrow$

$\varphi(x) = \varphi(0)$

p8|-|

$\underline{\underline{x = 0}}$

ⓑ $f^2(x) = (e^x + x - 1)^2$

$|f(x)| = |e^x + x - 1|$

$x < 0 \Rightarrow \varphi(x) < \varphi(0)$

$e^x + x - 1 < 0$

$x > 0 \Rightarrow e^x + x - 1 > 0$

גרף $f(x)$

$f(x) = 0$

$|f(x)| = 0$

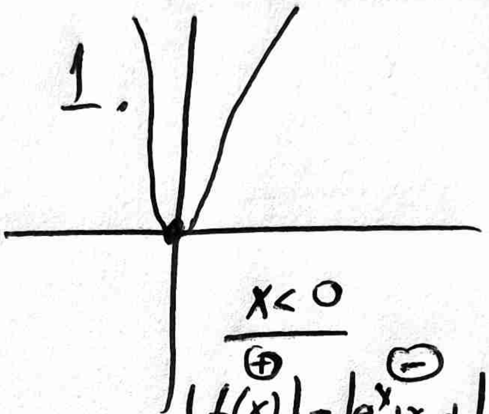
$|e^x + x - 1| = 0$

$e^x + x - 1 = 0$

$x = 0$

x		0	
$f(x)$	$\nearrow$	0	$\nearrow$

1.



$x < 0$

$|\overset{\oplus}{f(x)}| = |\overset{\ominus}{e^x + x - 1}|$

$\underline{\underline{f(x) = -e^x - x + 1}}$

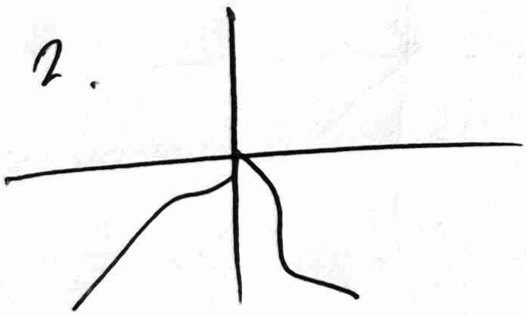
$x \geq 0$

$|\overset{\oplus}{f(x)}| = |\overset{\oplus}{e^x + x - 1}|$

$\underline{\underline{f(x) = e^x + x - 1}}$

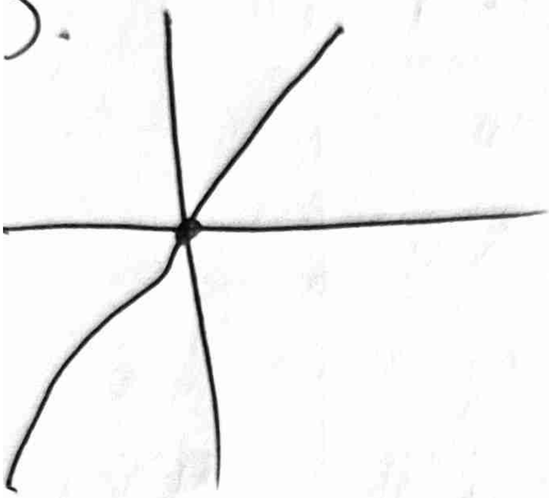
$$f(x) = \begin{cases} -e^x - x + 1, & x < 0 \\ e^x + x - 1, & x \geq 0 \end{cases}$$

2.



$$f(x) = \begin{cases} e^x + x - 1, & x < 0 \\ -e^x - x + 1, & x \geq 0. \end{cases}$$

3.



$x < 0$

$|f(x)| = |e^x + x - 1|$

$-f(x) = -e^x - x + 1$

$f(x) = e^x + x + 1$

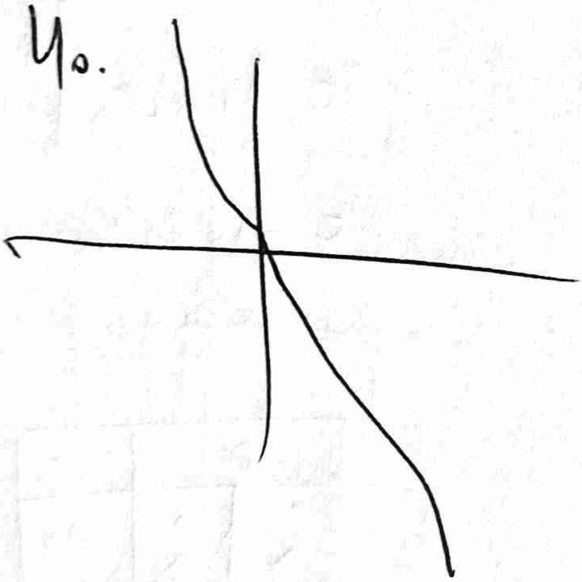
$x \geq 0$

$|f(x)| = |e^x + x - 1|$

$f(x) = e^x + x - 1$

$f(x) = e^x + x - 1$

4.



$f(x) = -e^x - x + 1$

Ενοτητα 14

14. $f: [0,2] \to \mathbb{R}$ συνεχης.

Ν.δ.ο. $\exists x_0 \in [0,2]$ τ.ω. $f(x_0) = \dfrac{f(0)+5f(1)+4f(2)}{10}$

---

f συνεχης στο $[0,2]$

$\Rightarrow$ ΘΜΕΤ $\quad m \le f(x) \le M \quad \forall x \in [0,2]$

$$\left.\begin{aligned} m \le f(0) \le M \quad &\Rightarrow \quad m \le f(0) \le M \\ m \le f(1) \le M \quad &\Rightarrow \quad 5m \le 5f(1) \le 5M \\ m \le f(2) \le M \quad &\Rightarrow \quad 4m \le 4f(2) \le 4M \end{aligned}\right\} +$$

$$10m \le f(0)+5f(1)+4f(2) \le 10M$$

$$m \le \frac{f(0)+5f(1)+4f(2)}{10} \le M.$$

O αριθμοί $\frac{f(0)+5f(1)+4f(2)}{10}$

$\in [m, M]$

δηλαδή ανήκει στο $ΣΤ_f$.

άρα $\exists x_0 \in [0,2]$ τω

$$f(x_0) = \frac{f(0)+5f(1)+4f(2)}{10}.$$

8. $f:[a,\beta]\to\mathbb{R}$ συνεχω $f\nearrow$

Νδο $\exists!\,\xi\in(a,\beta)$ τ.ω

$$f(\xi)=\frac{f(a)+f(\beta)+f\left(\frac{a+\beta}{2}\right)}{3}$$

Αφου f συνεχης $[a,\beta]$

απο ΘΜΕΤ η $m\le f(x)\le M$

$\forall x\in[a,\beta]$

$$\left.\begin{array}{l} m\le f(a)\le M \\ m\le f(\beta)\le M \\ m\le f\left(\frac{a+\beta}{2}\right)\le M \end{array}\right\}\oplus \quad 3m\le f(a)+f(\beta)+f\left(\frac{a+\beta}{2}\right)\le 3M$$

$$m\le\frac{f(a)+f(\beta)+f\left(\frac{a+\beta}{2}\right)}{3}\le M$$

ο αριθμός $\dfrac{f(\alpha)+f(\beta)+f\left(\frac{\alpha+\beta}{2}\right)}{3}$

$\in \Sigma T f$ άρα $\exists!\ \xi \in [\alpha,\beta]$

τω $f(\xi) = \dfrac{f(\alpha)+f(\beta)+f\left(\frac{\alpha+\beta}{2}\right)}{3}$

Η μοναδικότητα προκύπτει

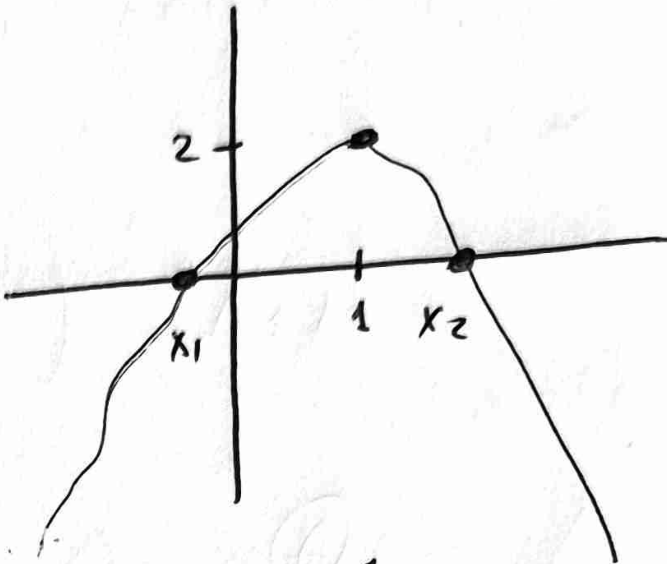
αφού $f \nearrow$

19.

x	$-\infty$	2	$+\infty$
$f(x)$	$-\infty$ ↗	1	↘ $-\infty$

(α) ΣΤ$_f = (-\infty, 2]$

(β)



$x \leq 1$

- f συνεχής
- f ↗
- ΣΤ$_f = (-\infty, 2]$

το $0 \in$ ΣΤ$_f$

$\exists!\, x_1 < 1$

τ.ω $f(x_1) = 0$

$x > 1$

- f συνεχής
- f ↘
- ΣΤ$_f = (-\infty, 2]$

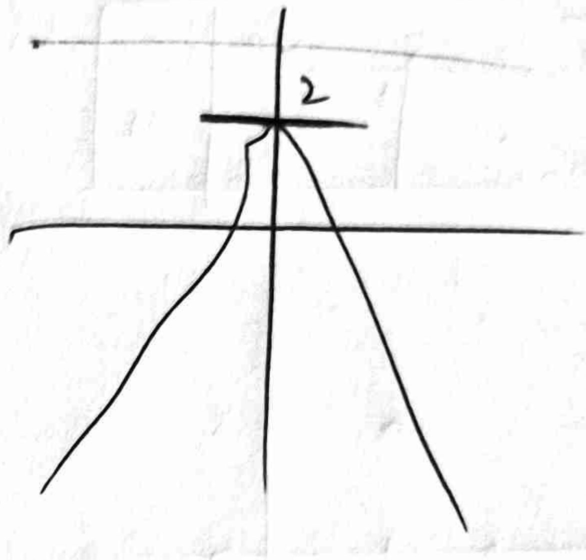
∴ το $0 \in$ ΣΤ$_f$

άρα $\exists!\, x_2$

τ.ω

$f(x_2) = 0$

① i) $f(x) = \alpha$



Αν $\alpha > 2$ 0 ρίζα

Αν $\alpha = 2$ 1 ρίζα

Αν $\alpha < 2$ 2 ρίζα.

ii) $f(x) = \frac{1}{\alpha}$ $\alpha > 1$

Αν $\alpha > 1$ προφανώς $0 < \frac{1}{\alpha} < 1$

2 ρίζες

18. $f(x) = \ln x - \frac{1}{x} + 1, \quad x > 0$

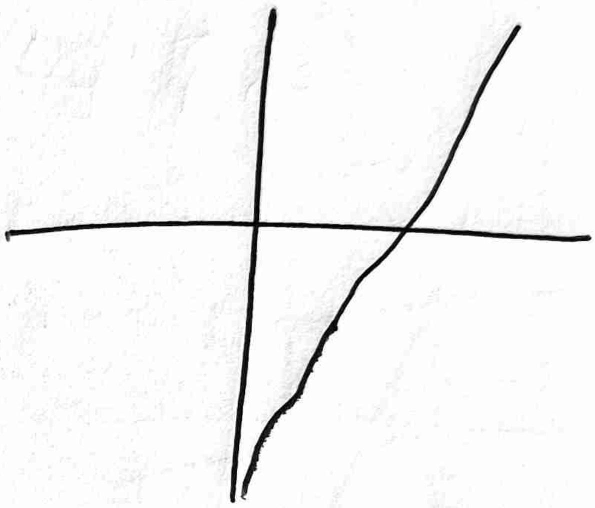
(α) Νδο η f αντιστρ. και να βρω $D_{f^{-1}}$

- $x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2$ (+) $f \nearrow$
- $x_1 < x_2 \Rightarrow \frac{1}{x_1} > \frac{1}{x_2} \Rightarrow -\frac{1}{x_1} < -\frac{1}{x_2} \Rightarrow f$ 1-1

$\Rightarrow f$ αντιστρ.

$D_{f^{-1}} = ΣΤ_f$

x	0	$+\infty$
$f(x)$	$-\infty$ $\nearrow$	$+\infty$



$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(\ln x - \frac{1}{x} + 1 \right)$

$= -\infty - \infty = -\infty$

$\lim_{x \to +\infty} f(x) = +\infty$

$ΣΤ_f = \mathbb{R}$.

$D_{f^{-1}} = \mathbb{R}$.

(B) Νδο η $f^{-1}\left(\ln x - \frac{1}{x}\right) = 1$

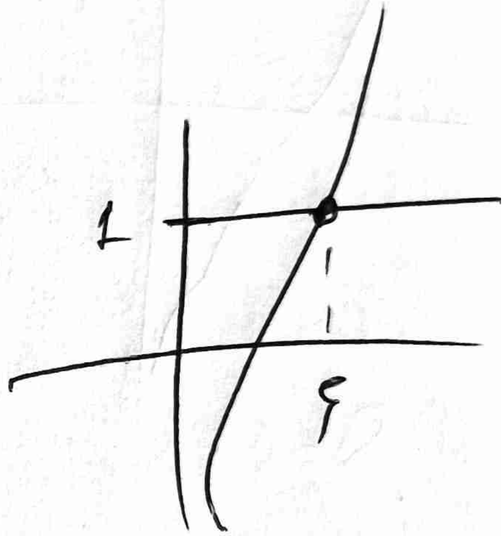
έχει μοναδική ρίζα στο $(0,+\infty)$.

$$f\left(f^{-1}\left(\ln x - \frac{1}{x}\right)\right) = f(1)$$

$$\ln x - \frac{1}{x} = 0$$

$$\ln x - \frac{1}{x} + 1 = 1$$

$$f(x) = 1.$$



- f συνεχής
- $f \nearrow$
- $\Sigma T_f = \mathbb{R}$.

Το $1 \in \Sigma T_f$ άρα $\exists!\ \xi > 0$ τ.ω

$$f(\xi) = 1$$

17. $f(x) = e^{-x} - \ln x - 1$

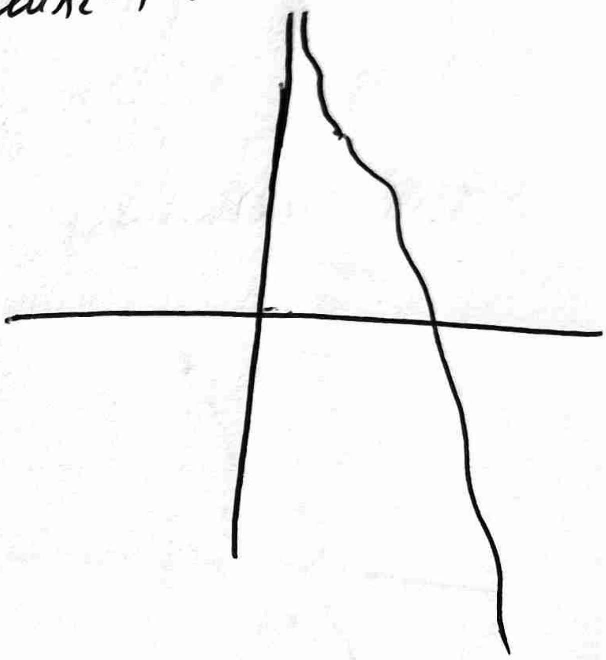
ⓐ Σύνολο Τιμών.

Πεδίο ορισμού

$A_f = (0,+\infty)$

Μονοτονία

$$\left.\begin{array}{l} \bullet\ x_1 < x_2 \Rightarrow e^{-x_1} > e^{-x_2} \\ \bullet\ x_1 < x_2 \Rightarrow -\ln x_1 - 1 > -\ln x_2 - 1 \end{array}\right\} \oplus \quad f \downarrow$$



x	0	+∞
f(x)	+∞ ↘	−∞

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} e^{-x} - \ln x - 1$$

$$= 1 - \ln 0 - 1$$

$$= -(-\infty) = +\infty$$

$$\Sigma T_f = \mathbb{R}.$$

$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} e^{-x} - \ln x - 1$$

$$= e^{-\infty} - \ln(+\infty) - 1$$

$$0 - \infty = -\infty$$

(B) Νδο $\exists!\, x_0 > 0$ τ.ω $e^{x_0} \ln x_0 + e^{x_0} = 1$.

$$e^{x} \ln x + e^{x} = 1$$

$$\ln x + 1 = \frac{1}{e^{x}}$$

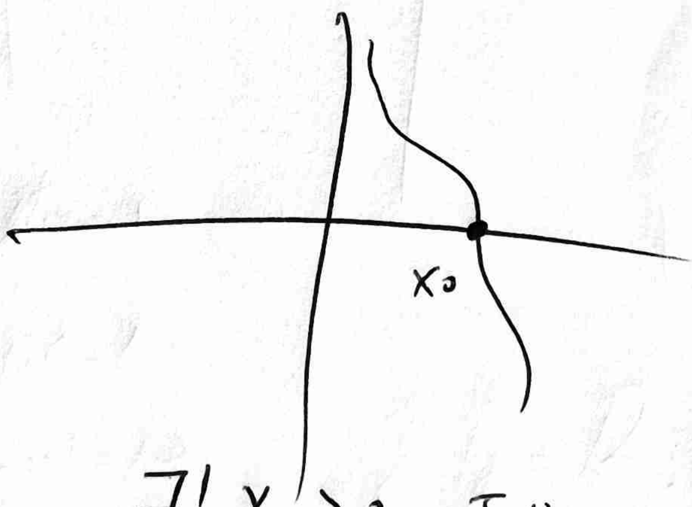
$$\ln x + 1 = e^{-x}$$

$$0 = e^{-x} - \ln x - 1$$

$$0 = f(x)$$

Αρκεί νδο η f τέμνει τον $x'x$ μοναδική φορά

- f συνεχής.
- $f \downarrow$
- $\Sigma T_f = \mathbb{R}$.

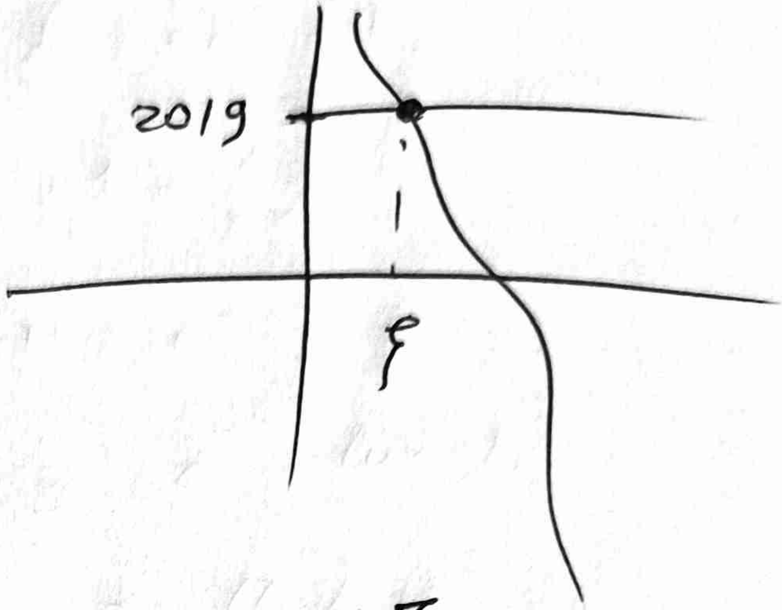


Το $0 \in \Sigma T_f$ άρα $\exists!\, x_0 > 0$ τ.ω

$$f(x_0) = 0,$$

① νδο η εξισωση $f(x)=2019$ εχει

μοναδικη ριζα



- f συνεχης
- $f \downarrow$
- $ΣΤ_f = \mathbb{R}$.

Το $2019 \in ΣΤ_f$ αρα $\exists!\ ξ > 0$

τ.ω $f(ξ) = 2019$.

Επόμενο Μάθημα

13.

13

14 α γ

15 α

17 α γ

19 α

20 α

21

22

23 α

25

26

14

6

15 α

16 α.