

Esercizio 6

$$25. \quad (a) \quad \frac{x^2+2x}{3x+6} = \frac{x(\cancel{x+2})}{3(\cancel{x+2})} = \frac{x}{3}.$$

$$(b) \quad \frac{3x^2-x}{6x-2} = \frac{x(\cancel{3x-1})}{2(\cancel{3x-1})} = \frac{x}{2}$$

$$(c) \quad \frac{x^2-1}{x^2+x} = \frac{(x-1)(\cancel{x+1})}{x(\cancel{x+1})} = \frac{x-1}{x}$$

$$(d) \quad \frac{3x-x^2}{x^2-9} = \frac{x(\cancel{3-x})}{(x-3)(x+3)} = \frac{-x(\cancel{x-3})}{(\cancel{x-3})(x+3)}$$
$$= \frac{-x}{x+3}$$

$$(e) \quad \frac{x^2-25}{10x-2x^2} = \frac{(x-5)(\cancel{x+5})}{2x(5-x)} = - \frac{(\cancel{x-5})(x+5)}{2x(\cancel{x-5})}$$
$$= \frac{-(x+5)}{2x}.$$

$$\textcircled{22} \quad \frac{x^2 - 2x + 1}{x^3 - 3x^2 + 3x - 1} = \frac{(x-1)^2}{(x-1)^3} = \frac{1}{x-1}$$

$$27. \textcircled{a} \quad \frac{x^3 - x}{x^2 - x} = \frac{\cancel{x}(x^2 - 1)}{\cancel{x}(x-1)}$$

$$= \frac{(\cancel{x-1})(x+1)}{\cancel{x-1}} = x+1$$

$$\textcircled{b} \quad \frac{x^3 - 2x^2 + x}{x^2 - 1} = \frac{x(x^2 - 2x + 1)}{(x-1)(x+1)} =$$

$$= \frac{x(x-1)\cancel{1}}{(\cancel{x-1})(x+1)} = \frac{x(x-1)}{x+1}$$

$$\textcircled{c} \quad \frac{x^4 - x}{x^2 - x} = \frac{\cancel{x}(x^3 - 1)}{\cancel{x}(x-1)} = \frac{(\cancel{x-1})(x^2 + x + 1)}{\cancel{x-1}}$$

$$= x^2 + x + 1$$

(52)

$$\frac{1-9x^2}{6x^2-2x} = \frac{(1-3x)(1+3x)}{2x(3x-1)}$$

$$= - \frac{(\cancel{3x-1})(1+3x)}{2x(\cancel{3x-1})} = - \frac{1+3x}{2x}$$

26.

$$\textcircled{a} \quad \frac{x^2-4x+4}{x^2-2x} = \frac{(x-2)^2}{x(\cancel{x-2})} = \frac{x-2}{x}$$

$$\textcircled{b} \quad \frac{x^2-9}{x^2-6x+9} = \frac{(\cancel{x-3})(x+3)}{(x-3)^2} = \frac{x+3}{x-3}$$

$$\textcircled{c} \quad \frac{9x^2-6x+1}{9x^2-1} = \frac{(\cancel{3x-1})^2}{(\cancel{3x-1})(3x+1)} = \frac{3x-1}{3x+1}$$

$$\textcircled{d} \quad \frac{x^3+1}{x^2+x} = \frac{(\cancel{x+1})(x^2-x+1)}{x(\cancel{x+1})} = \frac{x^2-x+1}{x}$$

$$\textcircled{e} \quad \frac{x^4-1}{x^3-1} = \frac{(x^2-1)(x^2+1)}{(x-1)(x^2+x+1)} = \frac{(\cancel{x-1})(x+1)(x^2+1)}{(\cancel{x-1})(x^2+x+1)}$$

$$\textcircled{8} \quad \frac{8x^3 - 2x}{2x^2 + x} = \frac{\cancel{2x}(4x^2 - 1)}{\cancel{x}(2x+1)} = \frac{2(2x-1)\cancel{(2x+1)}}{\cancel{2x+1}}$$

$$= 2 \cdot (2x-1).$$

$$\textcircled{9} \quad \frac{x^5 - 16x}{x^3 + 4x} = \frac{\cancel{x}(x^4 - 16)}{\cancel{x}(x^2 + 4)} = \frac{(x^2 - 4)(x^2 + 4)}{x^2 + 4}$$

$$= \frac{(x-2)(x+2)\cancel{(x^2 + 4)}}{\cancel{x^2 + 4}} = (x-2)(x+2).$$

$$\textcircled{10} \quad \frac{(x+1)^3 - 1}{x} = \frac{(x+1-1)((x+1)^2 + x+1 + 1^2)}{x}$$

$$= \frac{\cancel{x}(x^2 + 2x + 1 + x + 2)}{\cancel{x}} = x^2 + 3x + 3.$$

$$\textcircled{7} \left(x - \frac{1}{x}\right)^2 \cdot \frac{x^3 - x^2}{(x-1)^3} =$$

$$= \left(\frac{x}{1} - \frac{1}{x}\right)^2 \cdot \frac{x^2 \cancel{(x-1)}}{(x-1)^3} =$$

$$= \left(\frac{x^2}{x} - \frac{1}{x}\right)^2 \cdot \frac{x^2}{(x-1)^2}$$

$$= \left(\frac{x^2 - 1}{x}\right)^2 \cdot \frac{x^2}{(x-1)^2}$$

$$= \left(\frac{(x-1)(x+1)}{x}\right)^2 \cdot \frac{x^2}{(x-1)^2}$$

$$\frac{\cancel{(x-1)}^2 (x+1)^2}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{\cancel{(x-1)}^2}$$

$$= (x+1)^2$$

$$\textcircled{n} . \frac{x-1}{x^2-x+1} \cdot \frac{x^3+1}{x^2-1} =$$

$$= \frac{\cancel{x-1}}{\cancel{x^2-x+1}} \cdot \frac{\cancel{(x+1)}(\cancel{x^2-x+1})}{\cancel{(x-1)}(\cancel{x+1})} = 1$$

Ενότητα 7

1. (B) $A \vee \alpha > 2$ και $1 < B$

$$\rightarrow \alpha > 2 \Rightarrow \underline{\alpha - 2 > 0}$$

$$\rightarrow B > 1 \Rightarrow \underline{B - 1 > 0}$$

$$\rightarrow \underline{\frac{\alpha - 2}{B - 1} > 0}$$

2. (B) $A \vee 2 < x < 3$

$$\rightarrow 2 < x < 3$$

$$2 - 2 < x - 2 < 3 - 2$$

$$\underline{0 < x - 2 < 1}$$

$$\rightarrow 2 < x < 3$$

$$2 - 3 < x - 3 < 3 - 3$$

$$\underline{-1 < x - 3 < 0}$$

$$\underline{\begin{matrix} (x-2) & (x-3) \\ \oplus & \ominus \end{matrix}} < 0$$

$$3. \textcircled{1} \text{ Nдо } a(a+2) \geq -1.$$

$$a^2 + 2a \geq -1$$

$$a^2 + 2a + 1 \geq 0$$

$$(a+1)^2 \geq 0 \text{ по } 1 \text{ ст. } x \text{ и } 6.$$

$$\textcircled{02} 2(a^2+9) \geq (a+3)^2$$

$$2a^2 + 18 \geq a^2 + 6a + 9.$$

$$a^2 - 6a + 9 \geq 0,$$

$$(a-3)^2 \geq 0$$



$$4. \text{ Nдо } \frac{\overset{+}{x^2}}{\underset{-}{x+1}} < 0 \quad \text{or } x < -1$$

$$\bullet x < -1 \Rightarrow x+1 < 0$$

5. (B) No $x^2 + y^2 - 4x + 6y \geq -13$.

$$x^2 + y^2 - 4x + 6x + 13 \geq 0.$$

$$x^2 - 4x + 4 + y^2 + 6x + 9 \geq 0.$$

$$(x-2)^2 + (y+3)^2 \geq 0$$

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26. (B) No $x^2y^2 + 3x^2 + y^2 + 3 \geq 2xy^2 + 6x$

$$x^2y^2 + 3x^2 + y^2 + 3 - 2xy^2 - 6x \geq 0.$$

$$x^2(y^2 + 3) + y^2 + 3 - 2x(y^2 + 3) \geq 0$$

$$(y^2 + 3)(x^2 + 1 - 2x) \geq 0$$

$$(y^2 + 3)(x-1)^2 \geq 0$$

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Επορω Μαθημ

7

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