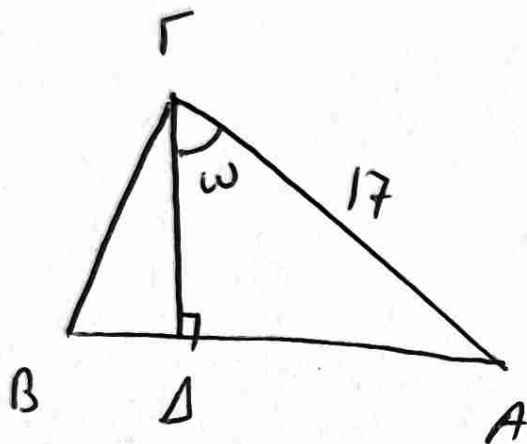


Επαγωγή

Τριγωνομετρία 5

1.



$$\eta\rho\omega = \frac{15}{17}$$



$$\frac{AD}{17} = \frac{15}{17}$$

$$\Rightarrow \underline{\underline{AD=15}}$$

$$AD^2 + CD^2 = AC^2$$

$$15^2 + CD^2 = 17^2$$

$$225 + CD^2 = 289$$

$$CD^2 = 64$$

$$\underline{\underline{CD=8}}$$

$$6^2 + 8^2 = BC^2$$

$$36 + 64 = BC^2$$

$$\underline{\underline{BC=10}}$$

$$\epsilon\varphi B = \frac{4}{3}$$

$$\frac{CD}{BD} = \frac{4}{3}$$

$$\frac{8}{BD} = \frac{4}{3}$$

$$\underline{\underline{BD=6}}$$

$$2. \quad A = \sigma w^2 \frac{61n}{3} + \sigma w^2 \frac{73n}{6} - \sigma y \frac{9n}{4}$$

$$\bullet \sigma w \frac{61n}{3} = \sigma w \frac{60n + n}{3} = \sigma w (20n + \frac{n}{3}) = \sigma w \frac{n}{3} = \frac{1}{2}$$

$$\bullet \sigma w \frac{73n}{6} = \sigma w (\frac{72n}{6} + \frac{n}{6}) = \sigma w (12n + \frac{n}{6}) = \sigma w \frac{n}{6} = \frac{\sqrt{3}}{2}$$

$$\bullet \sigma y \frac{9n}{4} = \sigma y (\frac{8n}{4} + \frac{n}{4}) = \sigma y (2n + \frac{n}{4}) = \sigma y \frac{n}{4} = 1$$

$$A = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$A = \frac{1}{4} + \frac{3}{4} - 1$$

$$A = 0.$$

$$3. \quad 3\sqrt{5} \eta^2 x - 2\eta x - \sqrt{5} = 0$$

$$\pi < x < \frac{3\pi}{2}$$

$$\underline{\underline{\eta x = t}}$$

$$3\sqrt{5} t^2 - 2t - \sqrt{5} = 0$$

$$\Delta = 4 + 4 \cdot 3\sqrt{5} \cdot \sqrt{5}$$

$$\Delta = 4 + 12 \cdot 5$$

$$\Delta = 64$$

$$t = \frac{2 \pm 8}{6\sqrt{5}} = \frac{1 \pm 4}{3\sqrt{5}}$$

$$\begin{array}{l} \nearrow \frac{5}{3\sqrt{5}} \\ \searrow \frac{-3}{3\sqrt{5}} \end{array}$$

$$\eta x = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\eta x = -\frac{\sqrt{5}}{5}$$

$$\eta^2 x + 5w^2 x = 1$$

$$\left(-\frac{\sqrt{5}}{5}\right)^2 + 5w^2 x = 1$$

$$\frac{5}{25} + 5w^2 x = 1$$

$$5w^2 x = \frac{20}{25}$$

$$5w x = \frac{2\sqrt{5}}{5}$$

$$5w x = -\frac{2\sqrt{5}}{5}$$

$$E\psi_X = \frac{\psi_X}{\sigma\psi_X} = \frac{\frac{-\sqrt{5}}{5}}{\frac{-2\sqrt{5}}{5}} = \frac{1}{2}$$

$$c\psi_X = \frac{1}{2}$$

$$\sigma\psi_X = 2$$

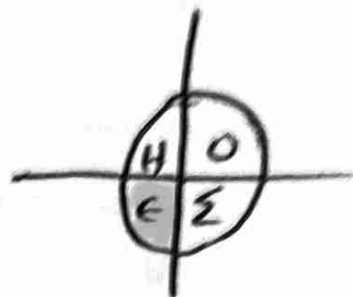
$$A = \frac{3 \cdot \left(-\frac{\sqrt{5}}{5}\right)^2 + \left(\frac{\sqrt{5}}{5}\right)\left(-\frac{2\sqrt{5}}{5}\right)}{2 - 2 \cdot \frac{1}{2}}$$

$$A = 3 \cdot \frac{5}{25} + \frac{5 \cdot (-2)}{5 \cdot 5}$$

$$A = \frac{3}{5} - \frac{2}{5} = \frac{1}{5}$$

4. $\epsilon\psi^2x - \frac{2}{\sigma\psi x} = 8$

$$\pi < x < \frac{3\pi}{2}$$



$$\boxed{\begin{aligned}\epsilon\psi x \cdot \sigma\psi x &= 1 \\ \sigma\psi x &= \frac{1}{\epsilon\psi x}\end{aligned}}$$

$$\epsilon\psi^2x - \frac{2}{\frac{1}{\epsilon\psi x}} = 8$$

$$\epsilon\psi^2x - 2\epsilon\psi x - 8 = 0$$

$$\underline{\underline{\epsilon\psi x = t}}$$

$$t^2 - 2t - 8 = 0$$

$$t = 4$$

$$t = -2$$

$$\epsilon\psi x = 4$$

~~$$\epsilon\psi x = -2$$~~

$$\sigma\psi x = \frac{1}{4}$$

$$\sigma_w^2 x = \frac{1}{1 + cy^2x} = \frac{1}{1 + 16} = \frac{1}{17}$$

$$\sigma_w x = \frac{1}{\sqrt{17}}$$

$$\Rightarrow \sigma_w x = -\frac{1}{\sqrt{17}}$$

$$E y x = \frac{w x}{\sigma_w x}$$

$$4 = \frac{w x}{-\frac{1}{\sqrt{17}}}$$

$$\Rightarrow w x = -\frac{4}{\sqrt{17}}$$

5. (a) $(\eta p x - \sigma w x)^2 + (\eta p x + \sigma w x)^2 = 2$

$$\eta p^2 x - 2 \cancel{\eta p x \sigma w x} + \sigma w^2 x + \eta p^2 x + 2 \cancel{\eta p x \sigma w x} + \sigma w^2 x = 2$$

$$2\eta p^2 x + 2\sigma w^2 x = 2$$

$$\eta p^2 x + \sigma w^2 x = 1 \quad \checkmark$$

(b) $\frac{1 + \eta p^2 x}{\sigma w^2 x} - 2 \eta p^2 x = 1$

$$\frac{1 + \eta p^2 x}{\sigma w^2 x} - 2 \frac{\eta p^2 x}{\sigma w^2 x} = 1$$

$$1 + \eta p^2 x - 2 \eta p^2 x = \sigma w^2 x$$

$$1 - \eta p^2 x = \sigma w^2 x$$

$$1 = \sigma w^2 x + \eta p^2 x$$

$$1 = 1 \quad \checkmark$$

①

$$\frac{1}{nr^x} - \frac{nr^x}{1 + \sigma w^x} = \sigma w^x$$

$$\cancel{nr^x} (1 + \sigma w^x) \frac{1}{\cancel{nr^x}} - \cancel{nr^x} (1 + \sigma w^x) \frac{nr^x}{\cancel{1 + \sigma w^x}} = \cancel{nr^x} (1 + \sigma w^x) \frac{\sigma w^x}{\cancel{nr^x}}$$

$$1 + \sigma w^x - nr^{2x} = (1 + \sigma w^x) \sigma w^x$$

$$\cancel{1 + \sigma w^x} - nr^{2x} = \cancel{\sigma w^x} + \sigma w^2 x$$

$$\sigma w^2 x = \sigma w^2 x$$

6.

$$\frac{n_p(\frac{41n}{2} - \theta) \cdot \sigma_w(\frac{35n}{2} + \theta) \varepsilon_y(21n - \theta)}{\sigma_w(\frac{25n}{2} + \theta) \sigma_y(\frac{5n}{2} - \theta) \sigma_w(33n + \theta)},$$

$$\cdot n_p(\frac{41n}{2} - \theta) = n_p(\cancel{\frac{40n}{2}} + \frac{n}{2} - \theta) = \sigma_w \theta$$

$$\cdot \sigma_w(\frac{35n}{2} + \theta) = \sigma_w(\frac{34n}{2} + \frac{n}{2} + \theta) = \sigma_w(17n + \frac{n}{2} + \theta)$$

$$= \sigma_w(\cancel{17n} + n + \frac{n}{2} + \theta) = \sigma_w(n + \frac{n}{2} + \theta)$$

$$= -\sigma_w(\frac{n}{2} + \theta) = -\sigma_w(\frac{n}{2} - (-\theta)) =$$

$$= -n_p(-\theta) = \sigma_w \theta$$

$$\cdot \varepsilon_y(21n - \theta) = \varepsilon_y(\cancel{20n} + n - \theta) = \varepsilon_y(n - \theta)$$

$$= -\sigma_y \theta$$

$$\bullet \sigma_w \left(\frac{25n}{2} + \theta \right) = \sigma_w \left(\frac{24n}{2} + \frac{n}{2} + \theta \right) = \sigma_w \left(12n + \frac{n}{2} + \theta \right)$$

$$= \sigma_w \left(\frac{n}{2} + \theta \right) = \sigma_w \left(\frac{n}{2} - (-\theta) \right) = \eta \mu (-\theta)$$

$$= -\eta \mu \theta$$

$$\bullet \sigma_\varphi \left(\frac{5n}{2} - \theta \right) = \sigma_\varphi \left(\frac{4n}{2} + \frac{n}{2} - \theta \right) = \varepsilon \varphi \theta$$

$$\bullet \sigma_w \left(33n + \theta \right) = \sigma_w \left(32n + n + \theta \right)$$

$$= \sigma_w (n + \theta) = -\sigma_w \theta$$

$$= \frac{\cancel{\sigma_w \theta} \cancel{\eta \mu \theta} (-\cancel{\varepsilon \varphi \theta})}{-\cancel{\eta \mu \theta} \cancel{\varepsilon \varphi \theta} (-\cancel{\sigma_w \theta})} = -1$$

7. a) $f(x) = 2\omega^3 x + 2$

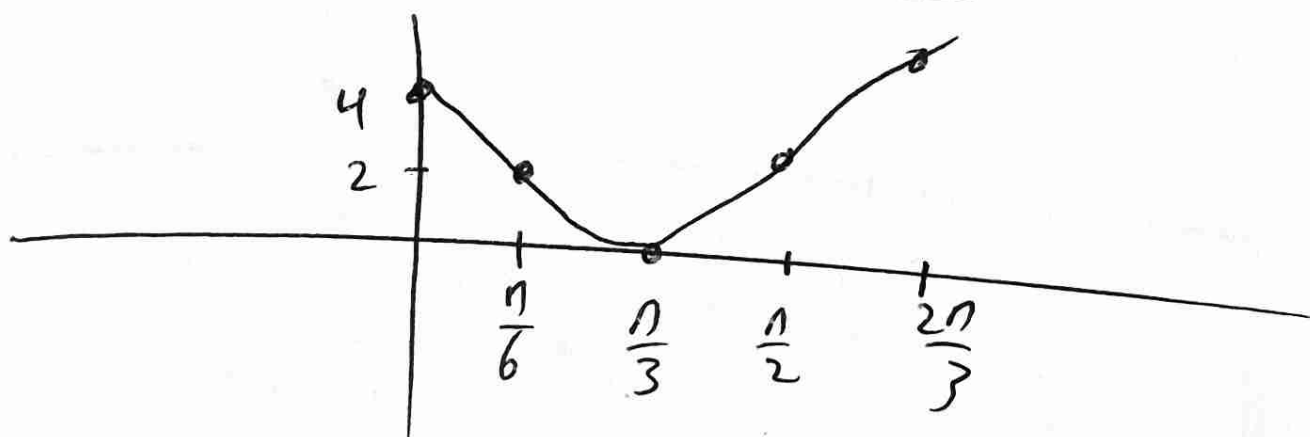
$$\boxed{\omega = 3 \quad p = 2 \quad c = 2}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$$

$$M = |p| + c = |2| + 2 = 4$$

$$m = -|p| + c = -|2| + 2 = 0$$

$3x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$\omega^3 x$	1	0	-1	0	1
$2\omega^3 x$	2	0	-2	0	2
$f(x)$	4	2	0	2	4



(B) $f(x) = 2\sqrt{2}x - 3$

$$T = \frac{2\pi}{2} = \pi$$

$$M = |2| - 3 = -1$$

$$m = -2 - 3 = -5$$

$2x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\sqrt{2}x$	0	1	0	-1	0
$2\sqrt{2}x$	0	2	0	-2	0
$f(x)$	-3	-1	-3	-5	-3

