

Ασκήσεις ΣΟ Σ Απόλυτα

1. Να βρεθεί το x σε κάθε περίπτωση

α) $|2x-5|=7$

β) $d(x, -3) = d(x, 7)$

γ) $d(x, -1) \geq 2$

δ) $|2x+8| < 12$

ε) $|x-3|=6$

ζ) $|3x-1| = 2|2x+1|$

Προσοχή: $d(a, b) = |a-b|$.

2. $A = |3x-1| - 2|2x-5| - |-x|$

Εάν $1 < x < 2$

Να γραφεί χωρίς απόλυτα.

3. $A = |x-3| - 2x$

Γραψε χωρίς αποδοτα.

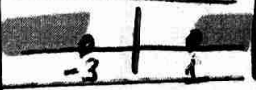
4. $A = \frac{|2x-1|}{|1-2x|} - \frac{|x-2|}{|2-x|}$

Γραψε χωρίς αποδοτα.

5. κανε αποδοποιωση.

$$\frac{x^2 - 2|x| + 1}{x^2 - |x|}$$

6.

$d(x, 1) < 2$		
	$ x+2 \geq 4$	-1
$d(x, -1) > 2$	$ x+1 > 2$	

Ασκσεις απολυτων

1. (a) $|2x-5|=7$

$$2x-5=7 \quad \vee \quad 2x-5=-7$$

$$2x=7+5$$

$$2x=12$$

$$\boxed{x=6}$$

$$2x=-7+5$$

$$2x=-2$$

$$\boxed{x=-1}$$

(b) $d(x, -3) = d(x, 7)$

$$|x+3| = |x-7|$$

$$\cancel{x}+3 = \cancel{x}-7 \quad \vee \quad x+3 = -x+7$$

$$3 = -7$$

αδυναται

$$2x = 7-3$$

$$2x = 4$$

$$x = 2$$

(γ) $d(x, -1) \geq 2$

$$|x+1| \geq 2$$

$$\Rightarrow x+1 \geq 2 \quad \vee \quad x+1 \leq -2$$

$$\underline{\underline{x \geq 1}}$$

$$\underline{\underline{x \leq -3}}$$

$$2. \quad A = |3x-1|^{\oplus} - 2|2x-5|^{\ominus} - |-x|^{\ominus}.$$

$$\bullet \quad 1 < x < 2 \quad \Rightarrow \quad 3 < 3x < 6 \quad \Rightarrow \quad 2 < 3x-1 < 5$$

$$\bullet \quad 1 < x < 2 \quad \Rightarrow \quad 2 < 2x < 4 \quad \Rightarrow \quad -3 < 2x-5 < -1$$

$$\bullet \quad 1 < x < 2 \quad \Rightarrow \quad -1 > -x > -2$$

$$A = 3x-1 - 2(-2x+5) - (-x)$$

$$A = 3x-1 + 4x-10 - (-x)$$

$$\underline{\underline{A = 6x - 11}}$$

$$\textcircled{8} \quad |2x+8| < 12$$

$$-12 < 2x+8 < 12$$

$$-12-8 < 2x < 12-8$$

$$-20 < 2x < 4$$

$$\underline{-10 < x < 2}$$

$$\textcircled{9} \quad |x-3| = 6$$

$$x-3=6$$

∨

$$x-3 = -6$$

$$\textcircled{x=9}$$

$$\textcircled{x=-3}$$

$$\textcircled{10} \quad |3x-1| = 2|2x+1|$$

$$|3x-1| = |4x+2|$$

$$3x-1 = 4x+2$$

$$\vee \quad 3x-1 = -4x-2$$

$$3x-4x = 2+1$$

$$3x+4x = 1-2$$

$$-x = 3$$

$$\textcircled{x=-3}$$

$$7x = -1$$

$$\textcircled{x = -\frac{1}{7}}$$

$$3. A = |x-3| - 2x$$

$$1. \text{ Av } x-3 \geq 0 \text{ Toz } A = |x-3| - 2x$$

$$A = x-3 - 2x$$

$$\underline{A = -3 - x}$$

$$2. \text{ Av } x-3 < 0 \text{ Toz } A = -x+3 - 2x$$

$$\underline{A = -3x+3}$$

$$4. A = \frac{|2x-1|}{|1-2x|} - \frac{|x-2|}{|2-x|} =$$

$$A = \frac{|2x-1|}{|2x-1|} - \frac{|x-2|}{|x-2|} = 1 - 1 = 0.$$

$$5. \frac{x^2 - 2|x| + 1}{x^2 - |x|} = \frac{|x|^2 - 2|x| + 1}{|x|^2 - |x|} = \frac{(|x|-1)^2}{|x|(|x|-1)}$$

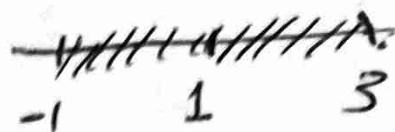
$$= \frac{|x|-1}{|x|}.$$

6.

$$d(x, 1) < 2$$

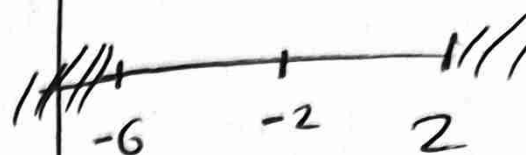
$$|x-1| < 2$$

$$x \in (-1, 3)$$



$$d(x, -2) \geq 4$$

$$|x+2| \geq 4$$



$$x \in (-\infty, -6] \cup [2, \infty)$$

Ασκήσεις 505

Π.7.1

1. Νδδ $\sqrt{3} \cdot \sqrt{2\sqrt{2}-\sqrt{5}} \cdot \sqrt{2\sqrt{2}+\sqrt{5}} = 3$

2.
$$\frac{\sqrt{20} - 2\sqrt{8} + 3\sqrt{12}}{\sqrt{45} - 2\sqrt{18} + 3\sqrt{27}}$$

3. Ρητοποίηση $\frac{3}{\sqrt{8}}$ και $\frac{\sqrt{2}}{\sqrt{50}-\sqrt{12}}$

4. Νδδ $\frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 4.$

5. Υπολογίσε $\sqrt{5 \sqrt[3]{5 \sqrt[4]{5 \sqrt{25}}}}$

6. Αν $A = \sqrt{x^2+2x+1} - \sqrt{1-4x+4x^2}$

και $d(3x, -1) \leq 2$ ανάλωσε

Άσκηση 5 πίκας

$$1. \sqrt{3(2\sqrt{2}-\sqrt{5})(2\sqrt{2}+\sqrt{5})} = 3$$

$$\sqrt{3[(2\sqrt{2})^2 - (\sqrt{5})^2]} = 3$$

$$\sqrt{3 \cdot (8 - 5)} = 3$$

$$\sqrt{9} = 3$$

$$2. \frac{\sqrt{20} - 2\sqrt{8} + 3\sqrt{12}}{\sqrt{45} - 2\sqrt{18} + 3\sqrt{27}} =$$

$$= \frac{2\sqrt{5} - 4\sqrt{2} + 6\sqrt{3}}{3\sqrt{5} - 6\sqrt{2} + 9\sqrt{3}} = \frac{2(\sqrt{5} - 2\sqrt{2} + 3\sqrt{3})}{3(\sqrt{5} - 2\sqrt{2} + 3\sqrt{3})}$$

$$= \frac{2}{3}$$

$$3. \quad \frac{3}{\sqrt{8}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\begin{aligned} \frac{\sqrt{2}}{\sqrt{50}-\sqrt{12}} &= \frac{\sqrt{2}}{5\sqrt{2}-2\sqrt{3}} = \frac{\sqrt{2}(5\sqrt{2}+2\sqrt{3})}{(5\sqrt{2})^2-(2\sqrt{3})^2} \\ &= \frac{5 \cdot 2 + 2\sqrt{6}}{25 \cdot 2 - 12} = \frac{10+2\sqrt{6}}{38} \end{aligned}$$

$$4. \quad \frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 4$$

$$\frac{\sqrt{5}(\sqrt{5}+\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2} - \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2} = 4$$

$$\frac{\sqrt{25}+\sqrt{15}}{2} - \frac{\sqrt{15}-3}{2} = 4$$

$$\frac{5+\sqrt{15}-\sqrt{15}+3}{2} = 4$$

$$\frac{8}{2} = 4 \quad \checkmark$$

$$5. \sqrt{5 \sqrt[3]{5 \sqrt[4]{25}}}$$

$$= \sqrt{5 \sqrt[3]{5 \cdot 25^{\frac{1}{4}}}}$$

$$= \sqrt{5 \sqrt[3]{5 \cdot (5^2)^{\frac{1}{4}}}}$$

$$= \sqrt{5 \sqrt[3]{5 \cdot 5^{\frac{2}{4}}}}$$

$$= \sqrt{5 \sqrt[3]{5^1 \cdot 5^{\frac{1}{2}}}}$$

$$= \sqrt{5 \sqrt[3]{5^{\frac{3}{2}}}}$$

$$= \sqrt{5 \cdot 5^{\frac{3}{2}}}$$

$$= \sqrt{5^1 \cdot 5^{\frac{3}{2}}}$$

$$= \sqrt{5^1 \cdot 5^{\frac{1}{2}}}$$

$$= \sqrt{5^{\frac{3}{2}}}$$

$$= 5^{\frac{3}{2} \cdot \frac{1}{2}} = 5^{\frac{3}{4}} = \sqrt[4]{5^3}$$

$$6. d(3x, -1) < 2$$

$$|3x+1| < 2$$

$$-2 < 3x+1 < 2$$

$$-3 < 3x < 1$$

$$\boxed{-1 < x < \frac{1}{3}}$$

$$A = \sqrt{x^2 + 2x + 1} - \sqrt{1 - 4x + 4x^2}$$

$$A = \sqrt{(x+1)^2} - \sqrt{(2x-1)^2}$$

$$A = |x+1|^{\oplus} - |2x-1|^{\ominus}$$

$$\bullet -1 < x < \frac{1}{3} \Rightarrow 0 < x+1 < \frac{4}{3}$$

$$\bullet -1 < x < \frac{1}{3} \Rightarrow -2 < 2x < \frac{2}{3} \Rightarrow -3 < 2x-1 < -\frac{1}{3}$$

$$A = x+1 - (-2x+1) = \underline{\underline{3x}}$$