Ενοτητα 24

61. $f(x) = \begin{cases} \alpha - \dfrac{\eta\mu x}{x}, & -\pi < x < 0 \\ 0, & x = 0 \\ -2e^{1-x} - \dfrac{x^3}{3} - x + \beta, & x > 0 \end{cases}$ Συνεχής!

(α) $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \left(\alpha - \dfrac{\eta\mu x}{x}\right) = \alpha - 1$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \left(-2e^{1-x} - \dfrac{x^3}{3} - x + \beta\right) =$

$= -2e + \beta$

Γνωρίζω ότι $f(0) = 0$

$\Rightarrow \alpha - 1 = 0$ και $-2e + \beta = 0$

$\boxed{\alpha = 1}$ $\boxed{\beta = 2e.}$

$f(x) = \begin{cases} 1 - \dfrac{\eta\mu x}{x}, & -\pi < x < 0 \\ 0, & x = 0 \\ -2e^{1-x} - \dfrac{x^3}{3} - x + 2e, & x > 0. \end{cases}$

Β

$-\pi < x < 0$

$$f_1(x) = 1 - \frac{\eta\mu x}{x}$$

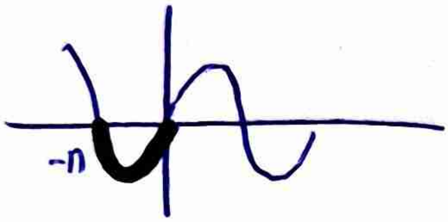
$$f_1'(x) = -\frac{x\sigma\upsilon\nu x - \eta\mu x}{x^2}$$

$$\boxed{\varphi(x) = x\sigma\upsilon\nu x - \eta\mu x} \quad \varphi(0) = 0$$

$$\varphi'(x) = \sigma\upsilon\nu x - x\eta\mu x - \sigma\upsilon\nu x$$

$$\varphi'(x) = -x\eta\mu x < 0$$

⊖ ⊖



x	$-\pi$	0
φ'	$-$	
φ	↘ $+$	
$-x^2$	$-$	
f_1'	$-$	
f_1	↘	

$x < 0$

$\varphi(x) > \varphi(0)$

$\varphi(x) > 0$

$x > 0$

$$f_2(x) = -2e^{1-x} - \frac{x^3}{3} - x + 2e$$

$$f_2'(x) = 2e^{1-x} - x^2 - 1$$

$$f_2'(1) = 0$$

$$f_2''(x) = -2e^{1-x} - 2x < 0$$

x	0	1
f_2''	$-$	$-$
f_2'	↘ $+$	0 ↘ $-$
f_2	↗	↘

x	$-\pi$	0	1
$f(x)$	↘	↗	↘

γ $\quad f(e^{-x^2}-3) = f(-2)$

• $x^2 \geq 0 \Rightarrow -x^2 \leq 0 \Rightarrow e^{-x^2} \leq e^0 \Rightarrow e^{-x^2} \leq 1$

$\Rightarrow e^{-x^2} - 3 \leq -2$

$\forall x < 0$

$f d$

$f \frac{3}{3}|-1$

$$e^{-x^2} - 3 = -2$$

$$e^{-x^2} = 1$$

$$x^2 = 0 \qquad \underline{x = 0}$$

62. $f: \mathbb{R} \to \mathbb{R}$ παρ/μη και f' συνεχ/ς.

$f'(x) \neq 0$

$f'(0) < 0$

(α) Αφού $f'(x) \neq 0$ και συνεχ/ς

$\Rightarrow f'(x) < 0$ ή $f'(x) > 0$

$f'(0) < 0 \Rightarrow f'(x) < 0 \Rightarrow f \downarrow$

(β) Ν.δ.ο. $f(2e^x) < f(1-x^2)$

$f \downarrow$

$2e^x > 1 - x^2$

$\underbrace{2e^x - 1 + x^2}_{g(x)} > 0$

ορατα Αναλυση, δεν ειδα τιποτα!

$g'(x) = 2e^x + 2x$

$g''(x) = 2e^x + 2 > 0$

x	$-\infty$ $\quad$ $+\infty$
g''	$+$
g'	$-\infty \nearrow +\infty$
g	

$\lim_{x \to -\infty} g'(x) = -\infty$

$\lim_{x \to +\infty} g'(x) = +\infty$

x		x_0
g''	$+$	$+$
g'	$\nearrow -$	$0 + \nearrow$
g	$\searrow$	$\nearrow$

• g' συνεχής

• $g' \nearrow$

• Το $\Sigma T g' = \mathbb{R}$

Το $0 \in \Sigma T g'$ άρα $\exists x_0$ $g'(x_0) = 0$

$$e^x \geq x+1$$

$$2e^x \geq 2x+2$$

$$2e^x + x^2 \geq x^2+2x+2$$

$$2e^x+x^2-1 \geq x^2+2x+2-1$$

$$2e^x+x^2-1 \geq x^2+2x+1$$

$$2e^x+x^2-1 \geq (x+1)^2 \geq 0$$

$$2e^x+x^2-1 \geq 0$$

(γ) i) $f(3x+1)+1 = f(x+1)+e^x$

$$f(3x+1)-f(x+1)+1-e^x=0$$

Προφανή ρίζα $x=0$

$x>0$

- $x+1<3x+1 \Rightarrow f(x+1)>f(3x+1) \Rightarrow f(3x+1)-f(x+1)<0$
- $x>0 \Rightarrow e^x>e^0 \Rightarrow e^x>1 \Rightarrow 1-e^x<0$ $(+)$

$$\boxed{f(3x+1)-f(x+1)+1-e^x<0}$$

$\underline{x < 0}$

- $x > 3x \Rightarrow x+1 > 3x+1 \Rightarrow f(x+1) < f(3x+1)$

$$\Rightarrow f(3x+1) - f(x+1) > 0 \quad \oplus$$

- $x < 0 \Rightarrow e^x < e^0 \Rightarrow e^x < 1 \Rightarrow 1 - e^x > 0$

$$\boxed{f(3x+1) - f(x+1) + 1 - e^x > 0}$$

11). $f(x^7) - \ln x = f(x^6)$

$$f(x^7) - \ln \frac{x^7}{x^6} = f(x^6)$$

$$f(x^7) - \ln x^7 = f(x^6) - \ln x^6$$

$$\boxed{\begin{array}{c} g(x) = f(x) - \ln x, \, x>0 \\ g \downarrow \end{array}}$$

$$g(x^7) = g(x^6)$$

g 1-1

$$x^7 = x^6$$

$$\underline{\underline{x = 1}}$$

III) $f(\eta\mu^2 x) + e^{x^2} = f(x^2) + 1$

$f(\eta\mu^2 x) - f(x^2) + e^{x^2} - 1 = 0$

Προφανής ρίζα $x=0$

• $|\eta\mu x| \leq |x|$

$\eta\mu^2 x \leq x^2$

$f(\eta\mu^2 x) \geq f(x^2)$

• $x^2 \geq 0$

$e^{x^2} \geq e^0$

$e^{x^2} \geq 1$

$$\boxed{f(\eta\mu^2 x) + e^{x^2} \geq f(x^2) + 1}$$

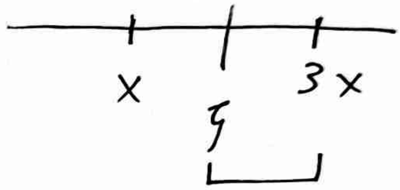
64. $f(x) = ημ3x - ημx \qquad x \in [0, \frac{\pi}{3})$

$$g(x) = \begin{cases} \frac{f(x)}{x}, & x \in (0, \frac{\pi}{3}) \\ 2, & x = 0 \end{cases}$$

(α) Ν.δ.ο. $f(x) > 2x\,συν3x \qquad \forall x \in (0, \frac{\pi}{3})$

$ημ3x - ημx > 2x\,συν3x \qquad$ στο $(0, \frac{\pi}{3})$

Θεωρώ $φ(x) = ημx$



$$φ'(ξ) = \frac{φ(3x) - φ(x)}{3x - x} = \frac{ημ3x - ημx}{2x}$$

$φ'(x) = συνx$

$φ''(x) = -ημx < 0$

$\forall x \in (0, \frac{\pi}{3})$

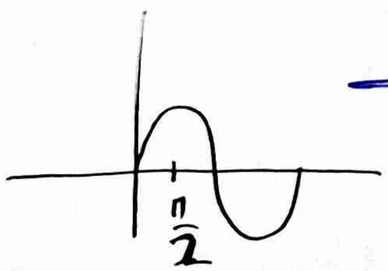
$φ' \downarrow$

$ξ < 3x$

$φ'(ξ) > φ'(3x)$

$$\frac{ημ3x - ημx}{2x} > συν3x$$

$ημ3x - ημx > 2x\,συν3x$



(B) $$g(x) = \begin{cases} \dfrac{f(x)}{x}, & x \in (0, \frac{\pi}{3}) \\ 2, & x = 0 \end{cases}$$

$f(x) = ημ3x - ημx$

$f'(x) = 3συν3x - συνx$

$$g'(x) = \frac{x f'(x) - f(x)}{x^2} = \frac{x(3συν3x - συνx) - ημ3x + ημx}{x^2}$$

$$g'(x) = \frac{3x\,συν3x - x\,συνx - ημ3x + ημx}{x^2} =$$

Γνωρίζω ότι

$f(x) > 2x\,συν3x$

$ημ3x - ημx > 2x\,συν3x$

$0 > 2x\,συν3x + ημx - ημ3x$

$x\,συν3x > 3x\,συν3x + ημx - ημ3x$

$x\,συν3x - x\,συνx > 3x\,συν3x + ημx - ημ3x - x\,συνx$

$x(συν3x - συνx) > 0$

$\oplus \quad \ominus$

$x < 3x$

συν ↓

$συνx > συν3x$

$0 > συν3x - συνx$

Άρα $\underbrace{3x\,συν3x + ημx - ημ3x - x\,συνx}_{\text{αριθμητής}} <$ Αρνητικό

του $g'(x)$

$$g'(x) = \frac{-}{x^2} < 0$$

g ↓.

65. $f(x) = e^x + x - 1 \quad , x \in \mathbb{R}.$

$F(x) = f(2x) - f(x) \quad , x \geq 0$

(α) Νδο f' και F ↗

$f'(x) = e^x + 1 > 0 \quad f$ ↗

$f''(x) = e^x > 0$

f' ↗

$F' = f = e^x + x - 1 \quad f(0)=0$

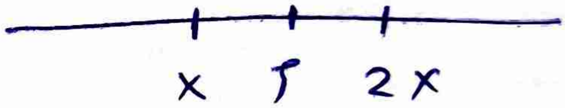
$F'' = f' = e^x + 1 > 0$

x		0	
F''	+		+
F'	↗ −	0	+ ↗
F	↘		↗

$\forall x \geq 0$ η F ↗

(β) νδο $F(x) < x f'(2x)$

$$f'(\xi) = \frac{f(2x) - f(x)}{2x - x}$$



$\xi < 2x \Rightarrow f'(\xi) < f'(2x)$

$$\frac{f(2x) - f(x)}{x} < f'(2x)$$

$$f(2x) - f(x) < x f'(2x)$$

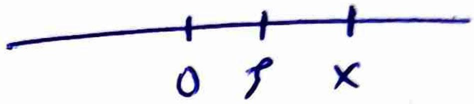
$$F(x) < x f'(2x).$$

(γ) $g(x) = \begin{cases} \dfrac{F(x)}{x}, & x > 0 \\ 2, & x = 0 \end{cases}$

$$g'(x) = \frac{x f(x) - F(x)}{x^2}$$

Αρκει νδο $x f(x) - F(x) > 0$

$$F'(\xi) = \frac{F(x) - F(0)}{x - 0} = \frac{F(x)}{x}$$



$\xi < x \Rightarrow F'(\xi) < F'(x) \Rightarrow \dfrac{F(x)}{x} < F'(x)$

$F' \nearrow$

$\dfrac{F(x)}{x} < f(x)$

$F(x) < x f(x)$

$$\boxed{0 < x f(x) - F(x)}$$

8. $f(2x^2) + f(x) < f(x^2) + f(2x)$ $\qquad (0, +\infty)$

$f(2x^2) - f(x^2) < f(2x) - f(x)$

$g(x) = f(2x) - f(x)$

$g(x) = F(x)$

$F(x^2) < F(x)$

$x^2 < x$

$\underline{x < 1}$

Ενότητα 26

11. $f(x)=(x-1)\ln(x+1)+x \,,\quad x>-1$

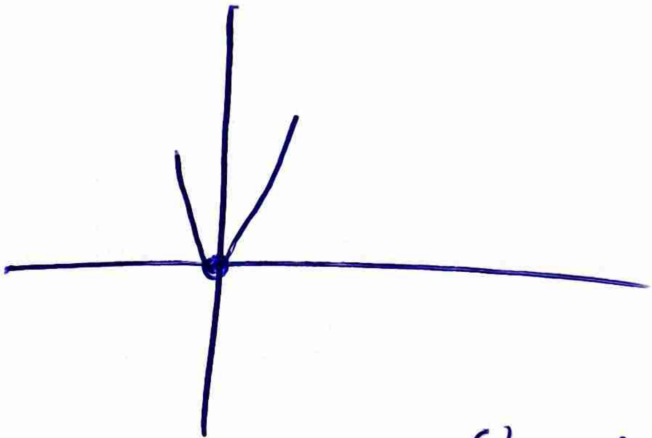
(α) Προφανή ρίζα $\underline{x=0}$

$$f'(x)= \ln(x+1)+(x-1)\frac{1}{x+1}+1$$

$$f'(x)= \ln(x+1)+\frac{x-1}{x+1}+1 \qquad \underline{\underline{f'(0)=0}}$$

$$f''(x)=\frac{1}{x+1}+\frac{x+1-(x-1)}{(x+1)^2}=\frac{x+1}{(x+1)^2}+\frac{2}{(x+1)^2}>0$$

x		0	
f''	$+$		$+$
f'	$\nearrow -$		$+\nearrow$
f	$\searrow$		$\nearrow$



$\boxed{\text{Ρίζα } x=0}$

$x>0 \Rightarrow f(x)>f(0) \Rightarrow f(x)>0$

$x<0 \Rightarrow f(x)>f(0) \Rightarrow f(x)>0$

$f(x)\geq f(0)$

$\underline{f(x)\geq 0}$

(B) $e^x (x+1)^{x-1} = 1$ $\qquad (-1, +\infty)$

$\ln e^x \cdot (x+1)^{x-1} = \ln 1$

$\ln e^x + \ln (x+1)^{x-1} = 0$

$x + (x-1) \ln(x+1) = 0$

$f(x) = 0$

$x = 0$

12. $f(x) = \dfrac{x}{e^{x-1} - 1 - \ln x}$

$\boxed{x=1 \ \text{προφανής}}$

$$\to \underbrace{e^{x-1} - 1 - \ln x}_{\varphi(x)} = 0$$

$\varphi'(x) = e^{x-1} - \dfrac{1}{x}$ $\qquad \varphi'(1) = 0$

$\varphi''(x) = e^{x-1} + \dfrac{1}{x^2} > 0$

x	0	1	
φ''	$+$		$+$
φ'	$\nearrow$ $-$	0	$+$ $\nearrow$
φ	$\searrow$		$\nearrow$

$D_f = \mathbb{R} - \{1\}$

$\varphi(x) \geq \varphi(1)$

$\varphi(x) \geq 0$

Το "=" $x = 1$

13. ⓐ $f(x) = e^x - x^2$

$f'(x) = e^x - 2x$

$f''(x) = e^x - 2$

$\longrightarrow f''(x) = 0 \Rightarrow e^x - 2 = 0$

$e^x = 2$

$x = \ln 2$

x		$\ln 2$	
f''	$-$		$+$
f'	$\searrow$		$\nearrow$
f	$\nearrow$		$\nearrow$

$f'(x) \geq f'(\ln 2)$

$f'(x) \geq e^{\ln 2} - 2\ln 2$

$f'(x) \geq 2 - \ln 4$

$f'(x) \geq \ln e^2 - \ln 4$

$f'(x) \geq \ln \frac{e^2}{4} > 0$

ⓑ $f(x) = e^x - (x+1)\ln(x+1)$, $x > -1$

$f'(x) = e^x - \ln(x+1) - \cancel{(x+1)}\frac{1}{\cancel{x+1}}$

$f'(x) = e^x - \ln(x+1) - 1$

$e^x \geq x+1$

$\ln(x) \leq x-1$

$\ln(x+1) \leq x+1-1$ $\quad \oplus$

$\ln(x+1) \leq x$

$-\ln(x+1) \geq -x$

$e^x - \ln(x+1) \geq x+1-x$

$e^x - \ln(x+1) - 1 \geq 0$

$f'(x) \geq 0$

$f \nearrow$

① $f(x) = ημx - \frac{x^3}{6} - x$

$f'(x) = συνx - \frac{1}{6}3x^2 - 1$

$f'(x) = συνx - \frac{1}{2}x^2 - 1$ $\qquad f'(0) = 0$

$f''(x) = -ημx - x$ $\qquad f''(0) = 0$

$f'''(x) = -συνx - 1 < 0$

$-1 \leq συνx \leq 1$

$1 \geq -συνx \geq -1$

$0 \geq -1 - συνx \geq -2$

x		0	
f'''	$-$		$-$
f''	$\searrow$ +	0	$\searrow$ $-$
f'	$\nearrow$	0	$\searrow$
f	$\searrow$		$\searrow$

20. $f(x) = e^{x-1} - x\ln x - 1 \quad , x>0$

ⓐ $f'(x) = e^{x-1} - \ln x - 1$

- $\ln x \leq x-1$

- $e^x \geqslant x+1$

$-\ln x \geqslant 1-x$

$e^{x-1} \geqslant x-1+1$

$e^{x-1} \geqslant x$ $\quad (+)$

$e^{x-1} - \ln x \geqslant x+1-x$

$e^{x-1} - \ln x - 1 \geqslant 0$

$f'(x) \geqslant 0 \quad f \nearrow$

ⓑ $\begin{cases} g(x) = h(x) \\ g'(x) = h'(x) \end{cases} \Rightarrow \begin{matrix} e^{x-1} - 1 = x\ln x \\ e^{x-1} = \ln x + 1 \end{matrix}$

$\begin{cases} f(x) = 0 \\ f'(x) = 0 \end{cases} \Rightarrow \underline{\underline{x=1}}$ $\quad \underline{\underline{x=1}}$

23. ⓐ $f(x) = x^6 + 6x + 7$

$f'(x) = 6x^5 + 6 = 6(x^5+1)$

x		-1	
f'	$-$		$+$
f	$\searrow$		$\nearrow$

$f(x) \geq f(-1)$

$f(x) \geq 2$

ⓑ $\lim\limits_{x \to -1} \dfrac{1}{f(x)-2} = +\infty$.

ⓒ $f(x) + (x+1)^2 = 2$

$f(x) - 2 + (x+1)^2 = 0$

• $f(x) \geq 2 \Rightarrow f(x) - 2 \geq 0$ Το "=" $x=-1$

• $(x+1)^2 \geq 0$ Το "=" $x=-1$

Γενικά $f(x) - 2 + (x+1)^2 \geq 0$

Το "=" $x=-1$

(δ) $f(f(x)) = 83$

$f(f(x)) = f(2)$

- $2 > -1$
- $f(x) \geq 2$

$\Big\}\ \forall x > -1$ u $f \nearrow$

$f(x) = 2$

$\underline{\underline{x = -1}}$

(ε) $3f(a) + 2f(\ln\beta) = 10$

$a = -1$ $\qquad$ $f(\ln\beta) \geq 2$ $\qquad$ $\ln\beta = -1$ $\qquad$ $\beta = \frac{1}{e}$

$f(a) \geq 2$ $\qquad$ $2f(\ln\beta) \geq 4$

$3f(a) \geq 6$

$3f(a) + 2f(\ln\beta) \geq 10$ ✓

25. $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x - 1$

(a) $f'(x) = \frac{1}{3}3x^2 - 4x + 3$

$f'(x) = x^2 - 4x + 3$

x		1	3
f'	$+$	$-$	$+$
f	$\nearrow$	$\searrow$	$\nearrow$

(b) $\forall x \le 3 \quad 3f(x) \le 1$

$\forall x \le 3 \quad f(x) \le f(1)$

$f(x) \le \frac{1}{3}$

$3f(x) \le 1$

(c) $f(\alpha) + f(\beta) = -2 \qquad \alpha, \beta \in [-1, +\infty$

$\forall x > 1 \quad f(x) \ge f(3)$

$\underline{f(x) \ge -1}$

$$\left.\begin{array}{l} f(\alpha) \ge -1 \\ f(\beta) \ge -1 \end{array}\right\} f(\alpha) + f(\beta) \ge -2$$

$\alpha = \beta = 3$

(δ) $3f(4-x)=1 \quad \forall (1, +\infty)$.

$x>1$

$-x<-1$

$4-x<3$

$\forall x<3 \quad f(x) \leq f(1)$

$f(x) \leq \frac{1}{3}$

$f(4-x) \leq \frac{1}{3}$ το "="

$4-x=1$

$\underline{\underline{x=3}}$

ε) $f(x^2+2)+f(x^4+2)=-2$

$\cdot\ x^2 \geq 0 \Rightarrow x^2+2 \geq 2$ $\quad \forall x \geq 2 \quad f(x) \geq f(3)$

$\cdot\ x^4 \geq 0 \Rightarrow x^4+2 \geq 2$ $\quad f(x) \geq -1$

$f(x^2+2) \geq -1$

$f(x^4+2) \geq -1$

$f(x^2+2)+f(x^4+2) \geq -2$

Το $\overset{''}{=}$ ''

$x^2+2=3$ και $x^4+2=3$

$x^2=1$ $\quad x^4=1$

$x=\pm 1$ $\quad x=\pm 1$

Ενότητα 27

1.

(γ) $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$

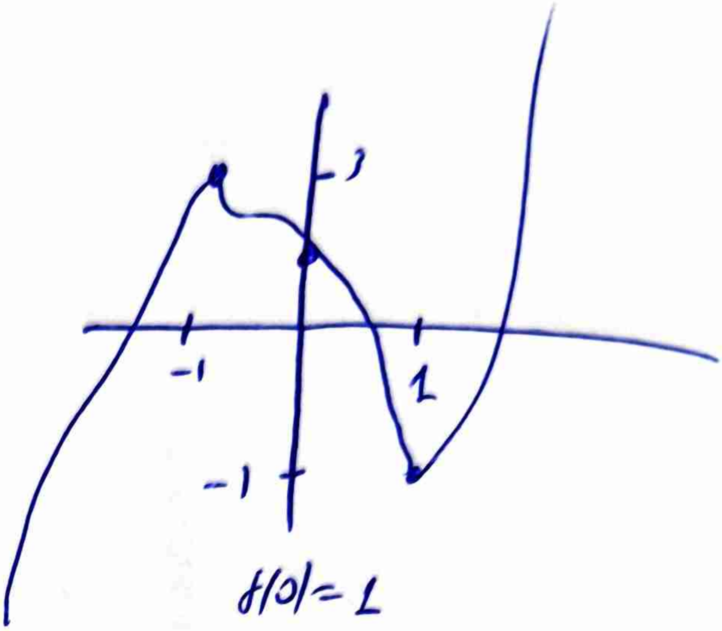
x	$-\infty$		-1		1		$+\infty$
f'		$+$		$-$		$+$	
f		$\nearrow$	3	$\searrow$	-1	$\nearrow$	

$\lim\limits_{x \to -\infty} f(x) = -\infty$

$f(-1) = 3$

$f(1) = -1$

$\lim\limits_{x \to +\infty} f(x) = +\infty$



$f(0) = 1$

$\Sigma T_f = \mathbb{R}$.

(6) $f(x) = x + \frac{1}{x}$, $x \neq 0$

$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$

x	-1		0		1	
f'	$+$	$-$	‖	$-$		$+$
f	$\nearrow$	$\searrow$	‖	$\searrow$		$\nearrow$

$-\infty$ $-\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

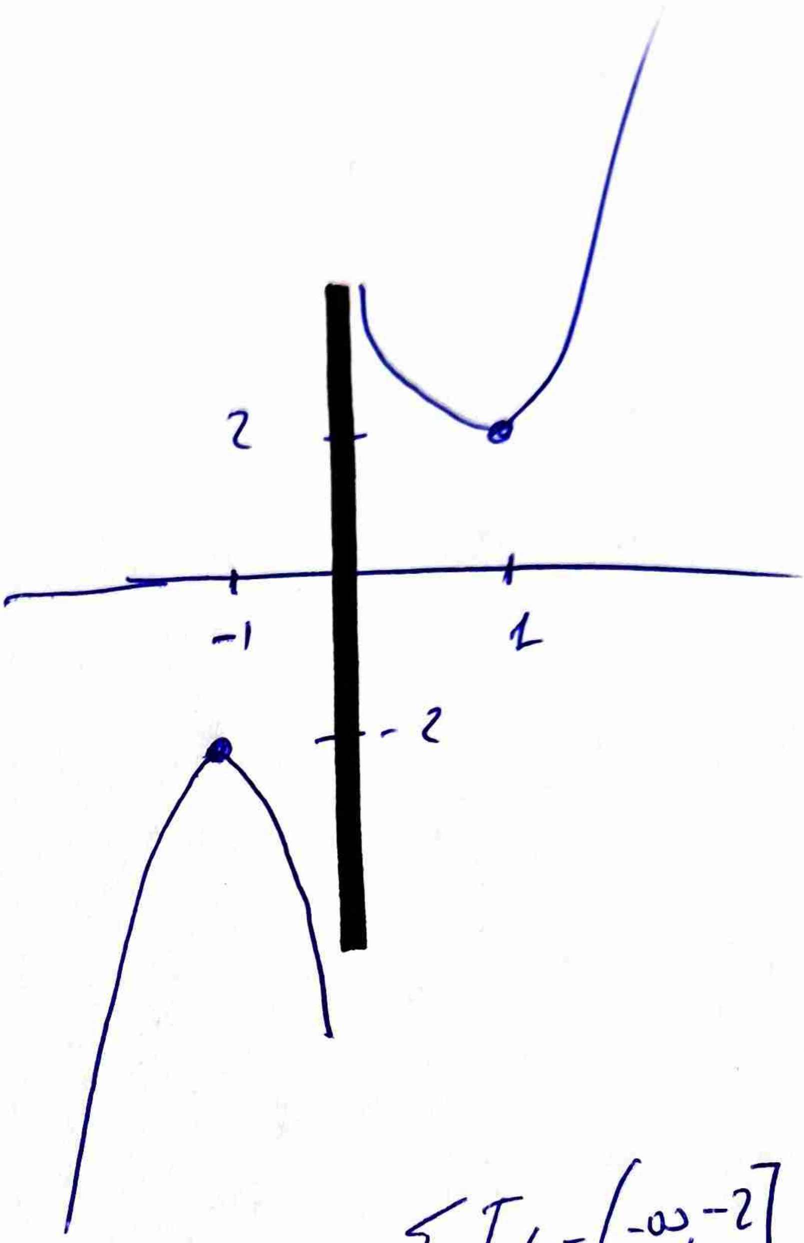
$f(-1) = -2$

$\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to 0^+} f(x) = +\infty$

$f(1) = 2$

$\lim_{x \to +\infty} f(x) = +\infty$



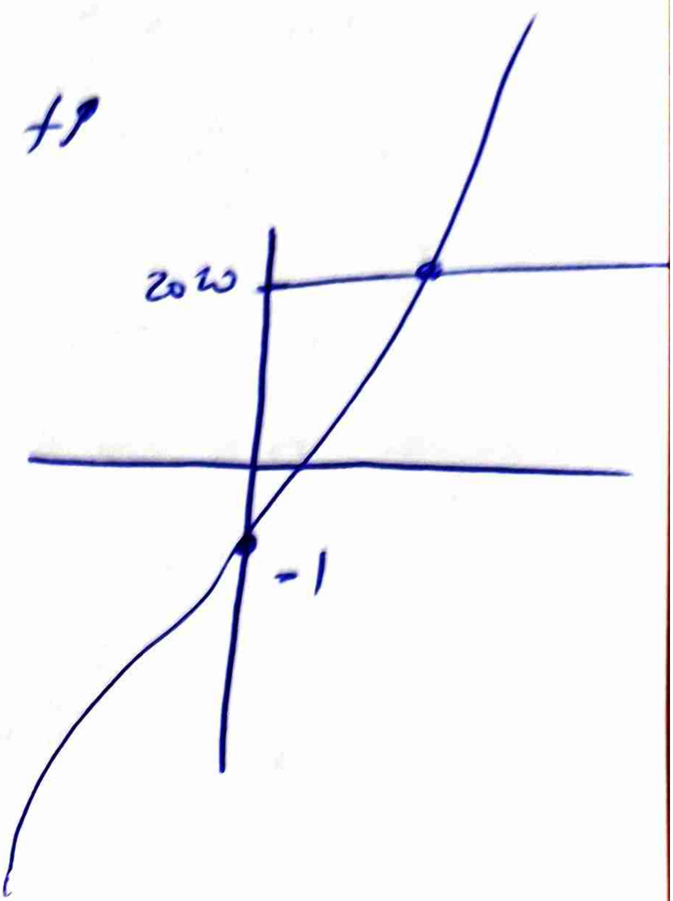
$T_f = (-\infty, -2] \cup [2, +\infty)$

2. $f(x) = x^3 - x^2 + 2x - 1$

(α) $f'(x) = 3x^2 - 2x + 2 > 0$ $\quad f \uparrow$

$\Delta = 4 - 24 < 0$

x	$-\infty$ $\qquad$ $+\infty$
$f(x)$	$\nearrow$



$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to +\infty} f(x) = +\infty$

(β) $f(f(x) - 2019) = 1$

$f(f(x) - 2019) = f(1)$

f 1-1

$f(x) - 2019 = 1$

$f(x) = 2020$

$$\left.\begin{array}{l} \bullet\ f \text{ συνεχής} \\ \bullet\ f \uparrow \\ \bullet\ \Sigma T_f = \mathbb{R} \end{array}\right\} \quad \text{Το } 2020 \in \Sigma T_f$$

άρα $\exists!\ \xi$ τ.ω. $f(\xi) = 2020$

Ⓐ Αφού $f\uparrow$ τότε η f αντιστρ.

$f(x)=x \Rightarrow x^3-x^2+2x-1=x$

$x^3-x^2+x-1=0$

$x^2(x-1)+x-1=0$

$(x-1)(x^2+1)=0$

$x=1$

Ⓑ $f^{-1}(f(x)-6)>1$

$f(x)-6>f(1)$

$f(x)>7$

$f(x)>f(2)$

$f\uparrow$

$x>2$.

4. $f(x) = x - \frac{1}{2}\ln^2 x$

(α) Νδο η $f \nearrow$

$f'(x) = 1 - \frac{1}{2}\, 2\ln x \, \frac{1}{x}$

$f'(x) = 1 - \frac{\ln x}{x} = \frac{x - \ln x}{x} > 0 \quad f \nearrow$

- $\ln x \leq x - 1$

  $1 \leq x - \ln x$

  $0 < x - \ln x$

(β) Αφού $f \nearrow$ αντιστρέφεται

$D_{f^{-1}} = \Sigma T_f$

x	0	$+\infty$
$f(x)$	$-\infty \nearrow$	$+\infty$

$\mathbb{R}$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x - \frac{1}{2}\ln^2 x = -\infty$

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x - \frac{1}{2}\ln^2 x = \lim_{x \to +\infty} x\left(1 - \frac{1}{2}\frac{\ln^2 x}{x}\right)$

$\to \lim_{x \to +\infty} \frac{\ln^2 x}{x} = \lim_{x \to +\infty} \frac{2\frac{\ln x}{x}}{} = +\infty$

γ $f^{-1}\left(f(x) - e + \frac{3}{2}\right) > 1$

$$f(x) - e + \frac{3}{2} > f(1)$$

$$f(x) - e + \frac{3}{2} > 1$$

$$f(x) > e - \frac{1}{2}$$

$$f(\cancel{x}) > f(e)$$

$$x > e.$$

5. $f: (1,+\infty) \to \mathbb{R}$

$$f(x) = \frac{e^x}{x}$$

(α) $f'(x) = \dfrac{e^x x - e^x}{x^2} = \dfrac{e^x(x-1)}{x^2} > 0$

$f \nearrow$

$\Sigma T_f = (e, +\infty)$

$\lim_{x \to 1^+} f(x) = e$

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \dfrac{e^x}{x} = +\infty$

(β) N.δ.ο. $f(x) + 1 > e + \ln f(x)$

$e^{f(x)+1} > e^{e + \ln f(x)}$

$e^{f(x)} \cdot e^1 > e^e \, e^{\ln f(x)}$

$e^{f(x)} e > f(x) \cdot e^e$

$\dfrac{e^{f(x)}}{f(x)} > \dfrac{e^e}{e} \Rightarrow f(f(x)) > f(e)$

$f(x) > e \Rightarrow f(x) > f(1)$
$x > 1$

7. (a) $f(x) = \begin{cases} \dfrac{x-1}{x\ln x}, & 0 < x \neq 1 \\ 1, & x = 1 \end{cases}$

$$f'(x) = \frac{x\ln x - (x-1)(\ln x + 1)}{x^2 \ln^2 x}$$

$$f'(x) = \frac{\cancel{x\ln x} - \cancel{x\ln x} - x + \ln x + 1}{x^2 \ln^2 x} \leq 0$$

$f \downarrow$

- $\ln x \leq x - 1$

$\ln x - x + 1 \leq 0$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x-1}{x\ln x} = \lim_{x \to 1} \frac{1}{\ln x + 1} = 1$$

$f(1) = 1$ Συνεχής στο 1.

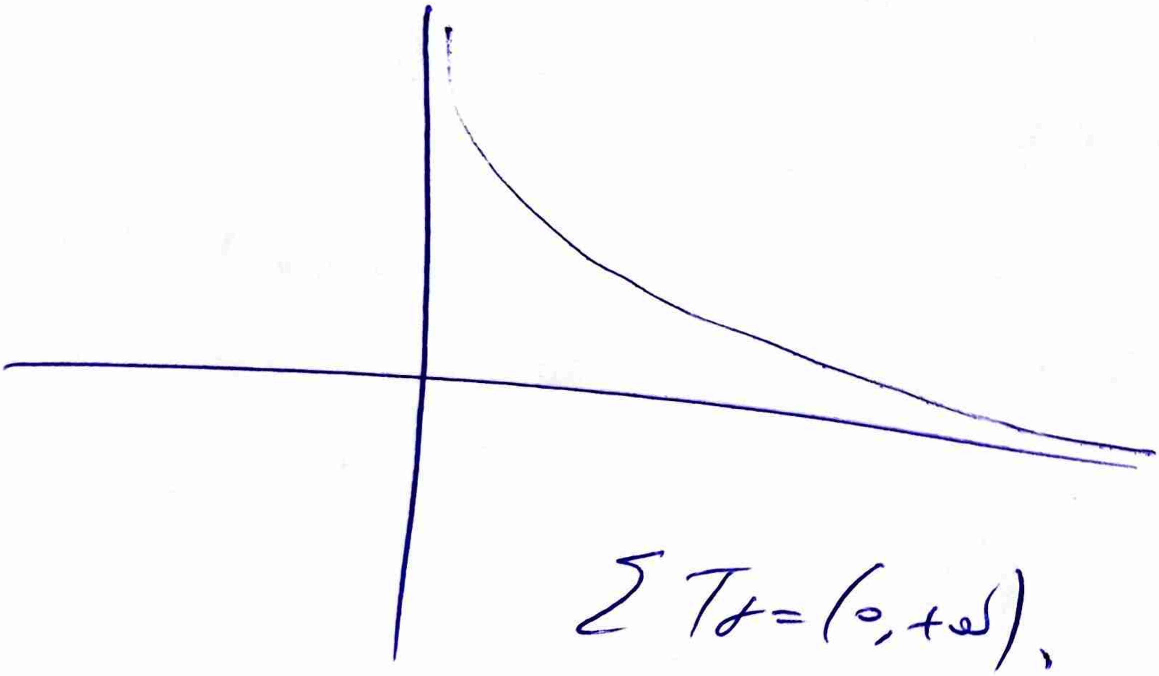
+f

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x-1}{x\ln x} = \lim_{x\to 0^+} (x-1)\frac{1}{x\ln x} = -1\cdot(-\infty) = +\infty$$

$$\to \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = 0$$

$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} \frac{x-1}{x\ln x} = \lim_{x\to +\infty} \frac{1}{\ln x + 1} = 0$$

x	0	$+\infty$
$f(x)$	$+\infty$ $\searrow$	0



$\Sigma\ T_f = (0, +\infty)$.

(3) $f(x)=\begin{cases} \dfrac{e^x}{\ln x - x}, & x>0 \\ 0, & x=0 \end{cases}$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^+} f(x)=0 \\ f(0)=0 \end{array}\right\} \text{ Συνεχής στο } 0!$$

$$f'(x)=\frac{e^x(\ln x - x) - e^x\left(\frac{1}{x}-1\right)}{(\ln x - x)^2}$$

$$f'(x)=\frac{e^x\left(\ln x - x - \frac{1}{x}+1\right)}{(\ln x - x)^2}=\frac{e^x\left(\overbrace{\ln x - x + 1}^{-} - \frac{1}{x}\right)}{(\ln x - x)^2}$$

$\ln x \leq x-1$

$\ln x - x + 1 \leq 0$ $\qquad$ $f'(x)<0$

$$\lim_{x\to +\infty} f(x)=\lim_{x\to +\infty}\frac{e^x}{\ln x - x}=\lim_{x\to +\infty}\frac{e^x}{\frac{1}{x}-1}=-\infty \qquad f \searrow$$

$ΣΤ_f=(-\infty, 0]$

$$\rightarrow \lim_{x\to +\infty} \ln x - x = \lim_{x\to +\infty} x\left(\frac{\ln x}{x}-1\right)=-\infty$$