

ΕΥΟΤΗΤΑ 5

10.

$$f(x) = x^5 + 5x - 2, \quad x > 0$$

α) $x_1 < x_2 \Rightarrow x_1^5 < x_2^5$

$x_1 < x_2 \Rightarrow 5x_1 < 5x_2 \Rightarrow 5x_1 - 2 < 5x_2 - 2$ ⊕

$$x_1^5 + 5x_1 - 2 < x_2^5 + 5x_2 - 2$$

$$\underline{f(x_1) < f(x_2)}$$

✓

β) $A > B$

✓

$$f(A) > f(B)$$

$$A^5 + 5A - 2 > B^5 + 5B - 2$$

$$A^5 - B^5 > 5B - 5A$$

$$A^5 - B^5 > 5(B - A)$$

✓

11. (a) $f(x) = 2\sqrt{x} - \frac{3}{x}$, $x > 0$

• $x_1 < x_2 \Rightarrow 2\sqrt{x_1} < 2\sqrt{x_2}$

• $x_1 < x_2 \Rightarrow \frac{1}{x_1} > \frac{1}{x_2} \Rightarrow -\frac{3}{x_1} < -\frac{3}{x_2}$ (+)

$$2\sqrt{x_1} - \frac{3}{x_1} < 2\sqrt{x_2} - \frac{3}{x_2}$$

$$f(x_1) < f(x_2)$$

f ↗

(b) $f(x) = x^2 + 3\sqrt{x}$, $x \geq 0$

• $x_1 < x_2 \Rightarrow x_1^2 < x_2^2$

• $x_1 < x_2 \Rightarrow \sqrt{x_1} < \sqrt{x_2} \Rightarrow 3\sqrt{x_1} < 3\sqrt{x_2}$ (+)

$$x_1^2 + 3\sqrt{x_1} < x_2^2 + 3\sqrt{x_2}$$

$$f(x_1) < f(x_2)$$

f ↗

12.

$$f(x) = -x^5 - 3x + 4$$

$$\textcircled{a} f(1) = -1^5 - 3 \cdot 1 + 4$$

$$\underline{\underline{f(1) = 0}}$$

$$\textcircled{B} x_1 < x_2 \Rightarrow x_1^5 < x_2^5 \Rightarrow -x_1^5 > -x_2^5$$

$$x_1 < x_2 \Rightarrow -3x_1 + 4 > -3x_2 + 4 \quad \textcircled{+}$$

$$f(x_1) > f(x_2) \quad f \downarrow$$

$$\textcircled{D} f(2x-1) < 0$$

$$f(2x-1) < f(1)$$

$$f \downarrow$$

$$2x-1 > 1$$

$$2x > 2$$

$$x > 1$$

$$\textcircled{a} \text{ i) } x^5 + 3x > 4$$

$$0 > -x^5 - 3x + 4$$

$$0 > f(x)$$

$$f(1) > f(x)$$

$$\begin{array}{c} f \downarrow \\ \underline{\underline{1 < x}} \end{array}$$

$$\text{ii) } x^5 = 4 - 3x$$

$$0 = -x^5 - 3x + 4$$

$$0 = f(x)$$

Παρατηρώ ότι $f(1) = 0$.

Εάν αρα η f γν. ποσότητα

η εξίσωση $f(x) = 0$ έχω

Το μόνο πιθανό αρα το

1 ποσότητες π.2

$$13. \quad f(x) = 3\sqrt{x} - \frac{\alpha}{x} - 5 \rightarrow A(1, -6)$$

$$(a) \quad -6 = 3 - \alpha - 5$$

$$-6 = -2 - \alpha$$

$$\underline{\underline{\alpha = 4}}$$

$$f(x) = 3\sqrt{x} - \frac{4}{x} - 5$$

$$(B) \quad x_1 < x_2 \Rightarrow 3\sqrt{x_1} < 3\sqrt{x_2}$$

$$x_1 < x_2 \Rightarrow \frac{4}{x_1} > \frac{4}{x_2} \Rightarrow -\frac{4}{x_1} < -\frac{4}{x_2}$$

$$-\frac{4}{x_1} - 5 < -\frac{4}{x_2} - 5$$

$$f(x_1) < f(x_2) \quad f \nearrow$$

$$\textcircled{8} \quad f(4) = 3\sqrt{4} - \frac{4}{4} - 5 = 6 - 6 = 0 \quad \checkmark$$

$$\textcircled{9} \quad 3\sqrt{x} > \frac{4}{x} + 5$$

$$3\sqrt{x} - \frac{4}{x} - 5 > 0$$



$$f(x) > 0$$

$$f(x) > f(4)$$

$$f \nearrow$$

$$x > 4$$

3.

Νόο $f(x) = \frac{x^2+1}{x^2+2}$ έχω ελάχιστο

τιμή το $\frac{1}{2}$ και να βρω τι

όσο ελάχιστο

Αρκεί να

$$f(x) \geq \frac{1}{2}$$

$$\frac{x^2+1}{x^2+2} \geq \frac{1}{2}$$

$$2(x^2+1) \geq x^2+2$$

$$2x^2 + 2 \geq x^2 + 2$$

$$x^2 \geq 0$$



$$f(x) = \frac{1}{2}$$

οοο

$$x^2 = 0$$

$$\underline{\underline{x=0}}$$

όσο,

7. (8) $f(x) = 3 - 2|x-1|$

Isuzu on $|x-1| \geq 0$

$$-2|x-1| \leq 0$$

$$\underbrace{3 - 2|x-1|}_{f(x)} \leq 3$$

→ $f(x) = 3$
 ~~$3 - 2|x-1| = 3$~~

$$-2|x-1| = 0$$

$$|x-1| = 0$$

$$x-1 = 0$$

$$\underline{\underline{x=1}}$$

$$A(1, 3)$$

$$O.M.$$

7. (52) $f(x) = x^2 \cdot (x-1)^4$

$$\begin{aligned} & \bullet x^2 \geq 0 \\ & \bullet (x-1)^4 \geq 0 \end{aligned} \quad \left. \vphantom{\begin{aligned} & \bullet x^2 \geq 0 \\ & \bullet (x-1)^4 \geq 0 \end{aligned}} \right\} \odot \quad \begin{aligned} & x^2(x-1)^4 \geq 0 \\ & f(x) \geq 0 \end{aligned}$$

$\rightarrow f(x) = 0$

$$x^2 \cdot (x-1)^4 = 0$$

$$x^2 = 0$$

u'

$$(x-1)^4 = 0$$

$$x-1 = 0$$

$$\boxed{x=0}$$

$$\boxed{x=1}$$

$$A(0,0)$$

$$0. \in$$

$$B(1,0)$$

$$0. \in$$

$$7. \quad \textcircled{a} \quad f(x) = \sqrt{x^2 + 1}$$

$$\bullet x^2 \geq 0$$

$$x^2 + 1 \geq 1$$

$$\sqrt{x^2 + 1} \geq \sqrt{1}$$

$$\underline{\underline{f(x) \geq 1}}$$

$$\rightarrow f(x) = 1$$

$$\sqrt{x^2 + 1} = 1$$

$$x^2 + 1 = 1$$

$$x^2 = 0$$

$$\underline{\underline{x = 0}}$$

$$A(0, 1) \quad 0 \in .$$

$$8. f: (0, +\infty) \rightarrow \mathbb{R}$$

$$\underline{\underline{D_f = (0, +\infty)}}$$

$$f(x) = \frac{x^2 + 5x + 1}{x}$$

$$\textcircled{\alpha} \quad f(x) = \frac{x^2}{x} + \frac{5x}{x} + \frac{1}{x}$$

$$f(x) = x + 5 + \frac{1}{x}$$

$$\boxed{f(x) = x + \frac{1}{x} + 5.}$$

$$\rightarrow (x-1)^2 \geq 0 \Rightarrow x^2 - 2x + 1 \geq 0 \Rightarrow$$

$$x - 2 + \frac{1}{x} \geq 0$$

$$x + \frac{1}{x} \geq 2.$$

$$x + \frac{1}{x} + 5 \geq 7$$

$$\underline{\underline{f(x) \geq 7}}$$

$$\rightarrow f(x) = 7$$

$$\frac{x^2 + 5x + 1}{x} = 7$$

$$x^2 + 5x + 1 = 7x$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x-1 = 0$$

$$\underline{\underline{x=1}}$$

$$A(1, 7) \in$$

$$\textcircled{B} \text{ NSo } f(2021) + \frac{3}{7} f(2022) > 10$$

$$f(2021) > 7$$

$$f(2022) > 7$$

$$\frac{3}{7} f(2022) > \cancel{7} \cdot \frac{3}{\cancel{7}}$$

$$\frac{3}{7} f(2022) > 3$$

+

$$f(2021) + \frac{3}{7} f(2022) > 10$$

Επορευο Μαθημα

Τεταρτη 1/10/25

5

(14)

(15)

(16)

(17).

6

(2) α.

(4) .

(7)

(11) .

≡ εκτινω το μαθημα

με αρτια - περιττα .

