Εναλλακτικά 7

5. (α) $x^2+y^2-2x+1 \geq 0$

$x^2-2x+1+y^2 \geq 0$

$(x-1)^2+y^2 \geq 0$ που ισχύει!

(β) $x^2+y^2-4x+6y+13 \geq 0$

$x^2-4x+4+y^2+6y+9 \geq 0$

$(x-2)^2+(y+3)^2 \geq 0$ ✓

19. (δ) Αν $\beta < -2\alpha$ τότε $\frac{1-\alpha}{2} - \frac{\beta-2}{4} > 1$

$4\frac{1-\alpha}{2} - 4\frac{\beta-2}{4} > 4 \cdot 1$

$2(1-\alpha) - (\beta-2) > 4$

$2-2\alpha-\beta+2 > 4$

$-2\alpha-\beta > 0$

$-2\alpha > \beta$ που ισχύει!

21. (B) $\alpha > 0 \quad \beta > 0$

II) $(\alpha-\beta)\left(\frac{1}{\alpha}-\frac{1}{\beta}\right) \leq 4$

$\cancel{\alpha}\frac{1}{\cancel{\alpha}} - \alpha\frac{1}{\beta} - \beta\frac{1}{\alpha} + \cancel{\beta}\frac{1}{\cancel{\beta}} \leq 4$

$1 - \frac{\alpha}{\beta} - \frac{\beta}{\alpha} + 1 \leq 4 \Rightarrow -\frac{\alpha}{\beta} - \frac{\beta}{\alpha} \leq 2$

$-\alpha\cancel{\beta}\frac{\alpha}{\cancel{\beta}} - \cancel{\alpha}\beta\frac{\beta}{\cancel{\alpha}} \leq 2\alpha\beta$

$-\alpha^2 - \beta^2 \leq 2\alpha\beta$

$0 \leq \alpha^2 + 2\alpha\beta + \beta^2$

$0 \leq (\alpha+\beta)^2$ ✓

III)

$\frac{\alpha}{1+\alpha} < \frac{\alpha+\beta}{1+\beta}$

$\alpha(1+\beta) < (\alpha+\beta)(1+\alpha)$

$\cancel{\alpha} + \alpha\cancel{\beta} < \cancel{\alpha} + \alpha^2 + \beta + \alpha\cancel{\beta}$

$0 < \alpha^2 + \beta$

πως ισχυει.

23. (a) Av $a<1$ vis $a(a-1)^2 < 1-a^2$

$a(a^2-2a+1) < 1-a^2$

$a^3-2a^2+a < 1-a^2$

$a^3-a^2+a-1 < 0$

$a^2(a-1)+a-1 < 0$

$(a-1)(a^2+1) < 0$

$(-)\quad\quad(+)$

$\cdot\ a<1 \Rightarrow a-1<0$

(B) Av $\alpha < \beta$ vsö $\alpha^3 - \beta^3 \leq \alpha\beta^2 - \alpha^2\beta$.

$$\alpha^3 - \beta^3 - \alpha\beta^2 + \alpha^2\beta \leq 0$$

$$\alpha(\alpha^2 - \beta^2) + \beta(\alpha^2 - \beta^2) \leq 0$$

$$(\alpha^2 - \beta^2)(\alpha + \beta) \leq 0$$

$$(\alpha - \beta)(\alpha + \beta)(\alpha + \beta) \leq 0$$

$$(\alpha - \beta)(\alpha + \beta)^2 \leq 0$$

$(-)$ $(+)$

- $\alpha < \beta \Rightarrow \alpha - \beta < 0$ ✓

24. (a) $(x+1)^3$

x		-1	
$(x+1)^3$	$-$	0	$+$

(β) $x^3+3x^2+3x+1<0$

$(x+1)^3<0$

$x+1<0$

$x<-1$

25. Αν $x>1$ νδο $x^2-3x+3>\frac{1}{x}$

$x^3-3x^2+3x>1$

$x^3-3x^2+3x-1>0$,

$(x-1)^3>0$

$x-1>0$

$x>1$ ✓

43. Av $a, B > 0$

⑥ $\dfrac{a^3+B^3}{a+B} \geq aB$

$a^3+B^3 \geq aB(a+B)$

$a^3+B^3 \geq a^2B+aB^2$

$a^3+B^3-a^2B-aB^2 \geq 0$

$a^2(a-B)+B^2(B-a) \geq 0$

$a^2(a-B)-B^2(a-B) \geq 0$

$(a-B)(a^2-B^2) \geq 0$

$(a-B)(a-B)(a+B) \geq 0$

$(a-B)^2(a+B) \geq 0$

(+) (F)

43. ⑤ $\frac{a^3+b^3}{(a+b)^3} \geq \frac{1}{4}$

$4(a^3+b^3) \geq (a+b)^3$

$4a^3+4b^3 \geq a^3+3a^2b+3ab^2+b^3$

$3a^3+3b^3-3a^2b-3ab^2 \geq 0$

$a^3+b^3-a^2b-ab^2 \geq 0$

$a^2(a-b)+b^2(b-a) \geq 0$

$a^2(a-b)-b^2(a-b) \geq 0$

$(a-b)(a^2-b^2) \geq 0$

$(a-b)(a-b)(a+b) \geq 0$

$(a-b)^2(a+b) \geq 0$

(+) (+)

Επαναληπτικά Μαθήματα

Ασκήσεις

1. Αν $2\cdot[5(a-3)-2(a-9)]<-3\cdot(1-a)$

   τότε $a<-3$

2. Αν $\alpha<3<\beta$ τότε $\alpha\beta+9<3(\alpha+\beta)$

3. Αν $\alpha<-2<\beta$ τότε $\alpha^2-2\beta>\alpha(\beta-2)$

4. Αν α,β θετικοί τότε $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\geq 2$

5. Να δείξετε ότι $\alpha^2+\beta^2-2(\alpha-\beta)+2\geq 0$

6. Να δείξετε ότι $2\alpha^2+\beta^2+9\geq 2\alpha(3-\beta)$.