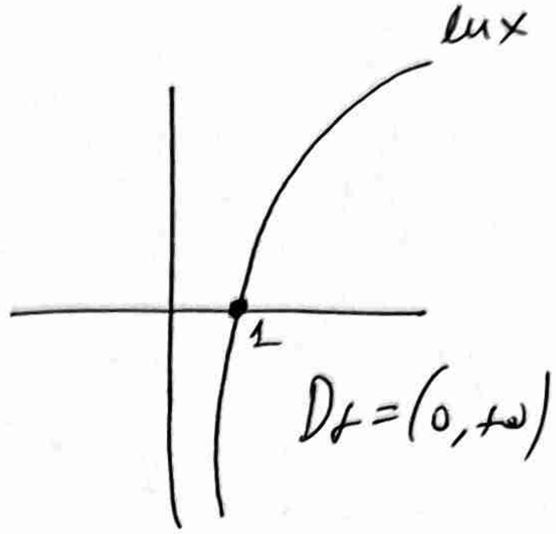
Ενότητα 26

1. (β) $\ln x - 1 = 0$

$\ln x = 1$

$e^{\ln x} = e^{1}$

$\boxed{x = e}$



(δ) $2\ln x - 1 = 0$

$2\ln x = 1$

$\ln x = \frac{1}{2}$

$e^{\ln x} = e^{1/2}$

$x = \sqrt{e}$

(στ) $\frac{1 - 2\ln x}{2} - 1 = 0$

$1 - 2\ln x - 2 = 0$

$-2\ln x = 1 \Rightarrow \ln x = -\frac{1}{2}$

$e^{\ln x} = e^{-\frac{1}{2}} \qquad x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} = \frac{\sqrt{e}}{e}$

2. (B) $2\ln(3x-1)+1=0$

$3x-1>0$

$\ln(3x-1) = -\frac{1}{2}$

$e^{\ln(3x-1)} = e^{-\frac{1}{2}}$

$3x-1 = e^{-\frac{1}{2}}$

$3x = 1 + \frac{1}{\sqrt{e}}$

$$\boxed{x = \frac{1}{3} + \frac{1}{3\sqrt{e}}}$$

$3\cdot\left(\frac{1}{3}+\frac{1}{3\sqrt{e}}\right)-1>0$

$1+\frac{1}{\sqrt{e}}-1>0$

$\frac{1}{\sqrt{e}}>0$ ✓

(J) $\ln x^2 - 1 = 0$

$x^2>0$

$\ln x^2 = 1$

$e^{\ln x^2} = e^1$

$x^2 = e$

$\boxed{x=\sqrt{e}}$ $\boxed{x=-\sqrt{e}}$ ✓

(52) $\ln(1-x^2) + 1 = 0$ $\quad 1-x^2 > 0$

$\ln(1-x^2) = -1$

$e^{\ln(1-x^2)} = e^{-1}$

$1-x^2 = \frac{1}{e}$

$1 - \frac{1}{e} = x^2$

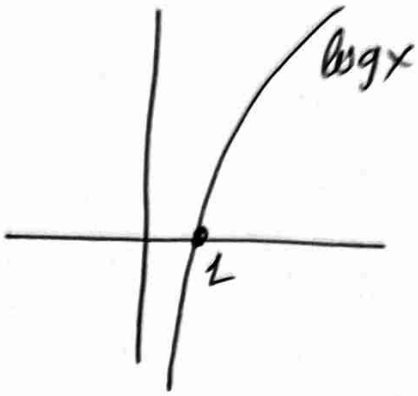
$x = \sqrt{1-\frac{1}{e}}$ y $x = -\sqrt{1-\frac{1}{e}}$

3. ① $\log(2x-1)+1=0$

$2x-1>0$

$\log(2x-1) = -1$

$10^{\log(2x-1)} = 10^{-1}$



$2x-1 = \frac{1}{10}$

$2x = \frac{1}{10}+1$

$x = \frac{1}{20}+\frac{1}{2}$

$x = \frac{11}{20}$ ✓

4. ① $\log(3-x^2) - \log(x^2-1) = 0$

$\log(3-x^2) = \log(x^2-1)$

$3-x^2>0$

$x^2-1>0$

$3-x^2 = x^2-1$

$4 = 2x^2$

$x^2 = 2$

$x = \sqrt{2}$

$x = -\sqrt{2}$

5. (8) $\log x + \log(x-1) = \log(x+2) - \log 2$

$\log(x(x-1)) = \log \frac{x+2}{2}$

$x^2 - x = \frac{x+2}{2}$

$x > 0$

$x-1 > 0$

$x+2 > 0$

$2x^2 - 2x = x + 2$

$2x^2 - 3x - 2 = 0$

$\Delta = 9 + 16 = 25$

$x = \frac{3 \pm 5}{4}$

(2)

$-\frac{1}{2}$

6. (8) $\ln x^2 - \ln(2x-2) = \ln 2e - 1$

$x^2 > 0$

$2x-2 > 0$

$$\ln \frac{x^2}{2x-2} = \ln 2e - \ln e$$

$$\ln \frac{x^2}{2x-e} = \ln \frac{2e}{e}$$

$$\frac{x^2}{2x-e} = 2$$

$$x^2 = 4x - 2e$$

$$x^2 - 4x + 2e = 0$$

$$\Delta = 16 - 8e < 0$$

Ответ.

7. (B) $\ln^2 x - 1 = 0$ $\quad x > 0$

$(\ln x)^2 - 1 = 0$ $\quad$ Θέτω $\ln x = t$

$t^2 - 1 = 0$

$t^2 = 1$

$t = 1$ ή $t = -1$

$\ln x = 1$ $\quad\quad$ $\ln x = -1$

$e^{\ln x} = e^1$ $\quad\quad$ $e^{\ln x} = e^{-1}$

$x = e$ ✓ $\quad\quad$ $x = \frac{1}{e}$ ✓

(Ε) $\frac{8\ln x}{\ln x - 3} = \ln x + 3$ $\quad x > 0$, $\ln x - 3 \neq 0$

Θέτω $\ln x = t$

$\frac{8t}{t-3} = t + 3 \Rightarrow 8t = t^2 - 9 \Rightarrow$

$t^2 - 8t - 9 = 0$

$t = 9$ $\quad t = -1$

$\ln x = 9$ $\quad \ln x = -1$

$\boxed{x = e^9}$ $\quad \boxed{x = \frac{1}{e}}$.

8. (B) $2^x = 3$

$$\log_2 2^x = \log_2 3$$

$$\boxed{x = \log_2 3}$$

(D) $e^{-2x} = 3^x$

$$\ln e^{-2x} = \ln 3^x$$

$$-2x = x \cdot \ln 3$$

$$0 = x \cdot \ln 3 + 2x$$

$$0 = x(\ln 3 + 2)$$

$$\underline{\underline{x = 0}}$$

$e^x - 3 > 0$

9. (B) $\ln(e^x - 3) + x = 2\ln 2$

$$e^{\ln(e^x-3)+x} = e^{2\ln 2}$$

$$e^{\ln(e^x-3)} \cdot e^x = e^{\ln 2^2}$$

$$(e^x - 3) \cdot e^x = 4$$

$e^x = t$

$$(t-3)\,t = 4$$

$$t^2 - 3t - 4 = 0$$

$t = 4$ $\qquad$ $t = -1$

$e^x = 4$ $\qquad$ $e^x = -1$

$\ln e^x = \ln 4$ $\qquad$ Absurdo.

$x = \ln 4$

25. ⑧ $\log(x-9) + 2\log\sqrt{2x-1} = 2$

$x-9>0$

$2x-1>0$

$\log(x-9) + \log(2x-1) = 2$

$\log(x-9)(2x-1) = \log 10^2$

$(x-9)(2x-1) = 100$

$2x^2 - 19x + 9 - 100 = 0$

$2x^2 - 19x - 91 = 0$

$x = \dfrac{19 \pm 33}{4}$

$\dfrac{52}{4} = 13$ ✓

$\dfrac{-14}{4} = -\dfrac{7}{2}$ ✗

⑧ $\frac{1}{2}\log(x+2) + \log\sqrt{x-3} = 1 + \log\sqrt{3}$

$\log\sqrt{x+2} + \log\sqrt{x-3} = \log 10 + \log\sqrt{3}$

$\log \sqrt{x+2}\sqrt{x-3} = \log 10\sqrt{3}$

$\sqrt{x+2}\sqrt{x-3} = 10\sqrt{3}$

$x+2 > 0$

$x-3 > 0$

$(x+2)(x-3) = 100 \cdot 3$

$x^2 - x - 6 = 300$

$x^2 - x - 306 = 0$

$x = \frac{1 \pm 35}{2}$

18 ✓

~~-17~~

26. (B) $\ln x = \ln^2 x$ $x > 0$

$\ln x = (\ln x)^2$

$\ln x = t$

$t = t^2$

$t^2 - t = 0$

$t(t-1) = 0$

$t = 0$ or $t = 1$

$\ln x = 0$ or $\ln x = 1$

$e^{\ln x} = e^0$ or $e^{\ln x} = e^1$

$x = 1$ ✓ $x = e$ ✓

$x > 0$

⑤ $\ln \sqrt{x} = \ln^2 x$

$\frac{1}{2} \ln x = (\ln x)^2$

$\ln x = t$

$\frac{1}{2} t = t^2$

$t = 2t^2$

$2t^2 - t = 0$

$t(2t-1) = 0$

$t = 0$ ou $t = \frac{1}{2}$

$\ln x = 0$

$e^{\ln x} = e^0$

$x = 1$

$\ln x = \frac{1}{2}$

$e^{\ln x} = e^{1/2}$

$x = \sqrt{e}$

28. $\begin{cases} x + \ln y - 1 = 0 \\ y = e^x + 1 - e \end{cases} \Rightarrow \boxed{x = 1 - \ln y}$

$$y = e^{1-\ln y} + 1 - e$$

$$y = e^{1} \cdot e^{-\ln y} + 1 - e$$

$$y = e \cdot \frac{1}{e^{\ln y}} + 1 - e$$

$$y = \frac{e}{y} + 1 - e$$

$$y^2 = e + y(1-e)$$

$$y^2 - y(1-e) - e = 0$$

$$y^2 - y + ye - e = 0$$

$$y(y-1) + e(y-1) = 0$$

$$(y-1)(y+e) = 0$$

$y = 1$

$y = -e$

$x = 1$

29. $\begin{cases} \ln\frac{x}{e} - 1 = 0 \\ \ln x - \ln^2 y = -1 \end{cases}$

$\ln\frac{x}{e} - 1 = 0$

$\ln x - \ln e - 1 = 0$

$\ln x - 1 - 1 = 0$

$\ln x = 2$

$e^{\ln x} = e^2$

$x = e^2$

$\ln e^2 - \ln^2 y = -1$

$2 - \ln^2 y = -1$

$3 = \ln^2 y$

$\ln y = t.$

$t^2 = 3$

$t = \sqrt{3}$ $\quad$ $t = -\sqrt{3}$

$\ln y = \sqrt{3}$ $\quad$ $\ln y = -\sqrt{3}$

$y = e^{\sqrt{3}}$ $\quad$ $y = e^{-\sqrt{3}}$

36. $3\sqrt{\ln x} + 2\ln\frac{1}{\sqrt{x}} = 2$

$3\sqrt{\ln x} + 2(\ln 1 - \ln\sqrt{x}) = 2$

$3\sqrt{\ln x} - 2\cdot\frac{1}{2}\ln x = 2$

$\ln x = t$

$3\sqrt{t} - t - 2 = 0$

$3\sqrt{t} = t + 2$

$t \geq 0 \qquad t + 2 \geq 0$

$9t = (t+2)^2$

$9t = t^2 + 4t + 4$

$0 = t^2 - 5t + 4$

$t = 4 \qquad t = 1$

$\ln x = 4 \qquad \ln x = 1$

$x = e^4 \qquad x = e$

$\ln x \geq 0$

$x > 0$

$\frac{1}{\sqrt{x}} > 0$

Επομενο Μαθημα

26

(4) α β

(5) α β

(6) α β

(7) α γ δ

(25) α γ

(26) α γ