Ενοτητα 35

2. (γ) $\int_0^{-1} x^2 e^x \, dx =$

$= \int_0^{-1} x^2 (e^x)' \, dx =$

$= (x^2 e^x)_0^{-1} - \int_0^{-1} e^x \cdot 2x \, dx =$

$= \frac{1}{e} - 2\int_0^{-1} (e^x)' x \, dx =$

$= \frac{1}{e} - 2\left((xe^x)_0^{-1} - \int_0^{-1} e^x \, dx \right)$

$= \frac{1}{e} - 2\left(-\frac{1}{e} - (e^x)_0^{-1} \right) = \frac{1}{e} + 2\frac{1}{e} + 2\left(\frac{1}{e} - 1\right)$

$= \frac{3}{e} + \frac{2}{e} - 2 = \frac{5}{e} - 2$

(δ) $\int_0^{\pi} x^2 \sigma\upsilon\nu x \, dx = \int_0^{\pi} x^2 (\eta\mu x)' \, dx$

$= \cancelto{0}{(x^2 \eta\mu x)_0^{\pi}} - \int_0^{\pi} 2x \, \eta\mu x \, dx =$

$= -2\int_0^{\pi} x (-\sigma\upsilon\nu x)' \, dx =$

$= -2\left((-x\,\sigma\upsilon\nu x)_0^{\pi} - 2\int_0^{\pi} -\sigma\upsilon\nu x \, dx \right)$

$= -2\left(-(-\pi) + 2\cancel{(\eta\mu x)_0^{\pi}} \right) = -2\pi.$

3. γ $\displaystyle\int_{-1}^{0} \ln(x+2)\,dx = \int_{-1}^{0} 1\cdot \ln(x+2)\,dx$

$$= \int_{-1}^{0} (x)' \ln(x+2)\,dx =$$

$$= \cancel{\Big(x\ln(x+2)\Big)_{-1}^{0}} - \int_{-1}^{0} x\,\frac{1}{x+2}\,dx$$

$$= -\int_{-1}^{0} \frac{x}{x+2}\,dx = -\int_{-1}^{0} \frac{(x+2)\cdot 1-2}{x+2}\,dx$$

$$\begin{array}{r|l} x & x+2 \\ \hline -(x+2) & 1 \\ \hline -2 & \end{array}$$

$$= -\int_{-1}^{0} \frac{x+2}{x+2} - \frac{2}{x+2}\,dx$$

$$= -\int_{-1}^{0} 1\,dx + 2\int_{-1}^{0} \frac{1}{x+2}\,dx$$

$$= -(x)_{-1}^{0} + 2\Big(\ln(x+2)\Big)_{-1}^{0}$$

$$= -(0+1) + 2(\ln 2)$$

$$= 2\ln 2 - 1.$$

4. (γ) $\int_1^a x^2 \ln x \, dx = \int_1^a \left(\frac{x^3}{3}\right)' \ln x \, dx =$

$= \left(\frac{x^3}{3}\ln x\right)_1^a - \int_1^a \frac{x^3}{3} \cdot \frac{1}{x} \, dx = \frac{1}{3} \cdot (x^3)_1^a - \frac{1}{3}\int_1^a x^2 \, dx$

$= \frac{1}{3}(a^3 \ln a) - \frac{1}{3} \frac{1}{3} (x^3)_1^a =$

$= \frac{1}{3} a^3 \ln a - \frac{1}{9}(a^3 - 1) = \frac{1}{3} a^3 \ln a - \frac{1}{9}a^3 + \frac{1}{9}$

(δ) $\int_1^t \ln^2 x \, dx = \int_1^t 1 \cdot \ln^2 x \, dx = \int_1^t (x)' \ln^2 x \, dx$

$= (x \ln^2 x)_1^t - \int_1^t x \frac{2 \ln x}{x} \, dx =$

$= t \ln^2(t) - 2 \int_1^t \ln x \, dx =$

$= t \ln^2 t - 2 \int_1^t (x)' \ln x \, dx =$

$= t \ln^2 t - 2 \left((x \ln x)_1^t - \int_1^t x \cdot \frac{1}{x} \, dx \right)$

$= t \ln^2 t - 2 \left(t \ln t - (x)_1^t \right) = t \ln^2 t - 2t \ln t$
$+ 2t - 2.$

(ε) $\displaystyle\int_1^2 \frac{1+\ln x}{x^2}\,dx = \int_1^2 \left(-\frac{1}{x}\right)'(1+\ln x)\,dx$

$\displaystyle = \left(-\frac{1}{x}(1+\ln x)\right)_1^2 - \int_1^2 -\frac{1}{x}\cdot\frac{1}{x}\,dx$

$\displaystyle = -\left(\frac{1}{2}(1+\ln 2) - 1\right) + \left(-\frac{1}{x}\right)_1^2$

$\displaystyle = -\frac{1}{2}(1+\ln 2+1) - \left(\frac{1}{2} - 1\right) =$

$\displaystyle = -\frac{1}{2} - \frac{\ln 2}{2} - \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} + \cancel{1}$

$\displaystyle = -\frac{\ln 2 + 1}{2},$

5. (γ) $I = \int_0^{\pi} \sigma\upsilon\nu x \; e^{-x} \, dx$

$$I = \int_0^{\pi} \sigma\upsilon\nu x \, (-e^{-x})' \, dx$$

$$I = \left(-\sigma\upsilon\nu x \, e^{-x}\right)_0^{\pi} - \int_0^{\pi} -e^{-x}(-\eta\mu x) \, dx$$

$$I = -\left(-e^{-\pi} - 1\right) - \int_0^{\pi} (-e^{-x})' \, \eta\mu x \, dx$$

$$I = e^{-\pi} + 1 - \left(\cancelto{0}{\left(-e^{-x} \eta\mu x\right)_0^{\pi}} - \int_0^{\pi} -e^{-x} \sigma\upsilon\nu x \, dx \right)$$

$$I = e^{-\pi} + 1 + \int_0^{\pi} -e^{-x} \sigma\upsilon\nu x \, dx$$

$$I = e^{-\pi} + 1 - I$$

$$2I = e^{-\pi} + 1$$

$$I = \frac{e^{-\pi} + 1}{2}$$

6. ① $\int_0^{-1} x e^{-x}\,dx = \int_0^{-1} x\,(-e^{-x})'\,dx$

$$= \left(-xe^{-x}\right)_0^{-1} - \int_0^{-1} -e^{-x}\,dx$$

$$= -(-e) + \left(-e^{-x}\right)_0^{-1}$$

$$= e - (e-1) = 1$$

② $\int_0^1 (x^2-x-1)e^{-x}\,dx = \int_0^1 (x^2-x-1)(-e^{-x})'\,dx$

$$= \left(-(x^2-x-1)e^{-x}\right)_0^1 - \int_0^1 -e^{-x}(2x-1)\,dx$$

$$= -\left(-\frac{1}{e}+1\right) + \int_0^1 (-e^{-x})'(2x-1)\,dx$$

$$= \frac{1}{e} - 1 + \left(\left(-e^{-x}(2x-1)\right)_0^1 - \int_0^1 -e^{-x}\,2\,dx\right)$$

$$= \frac{1}{e} - 1 - \left(\frac{1}{e} + 1\right) + 2\left(-e^{-x}\right)_0^1$$

$$= \cancel{\frac{1}{e}} - 1 - \cancel{\frac{1}{e}} - 1 - 2\left(\frac{1}{e} - 1\right) = -\frac{2}{e}.$$

15. (B) $\int_0^1 \frac{2}{4x+1}\,dy = 2\int_0^1 \frac{1}{4x+1}\,dx$

$= 2\,\frac{1}{4}\left(\ln|4x+1|\right)_0^1 = \frac{1}{2}\ln 5.$

(γ) $\int_0^1 \frac{dx}{\sqrt{2x+1}} = \int_1^{\sqrt{3}} \frac{1}{\cancel{t}}\,\cancel{t}\,dt$

$\sqrt{2x+1} = t$

$2x+1 = t^2$

$2dx = 2t\,dt$

$dx = t\,dt$

$= (t)_1^{\sqrt{3}} = \sqrt{3}-1.$

16. (α) $\int_0^1 x e^{x^2+1}\,dx = \int_1^2 e^u \frac{1}{2}\,du =$

$x^2+1 = u$

$2x\,dx = du$

$x\,dx = \frac{1}{2}\,du$

$= \frac{1}{2}\left(e^u\right)_1^2 =$

$= \frac{1}{2}\left(e^2 - e\right).$

(β) $\int_0^1 e^x \ln(1+e^x)\,dx = \int_2^{e+1} \ln t\,dt$

$1+e^x = t$

$e^x\,dx = dt$

$= \left(t\ln t\right)_2^{e+1} - \int_2^{e+1} t \cdot \frac{1}{t}\,dt$

$= (e+1)\ln(e+1) - 2\ln 2 - \left(t\right)_2^{e+1}$

$= (e+1)\ln(e+1) - 2\ln 2 - e - 1 + 2$

(δ) $\int_1^2 \frac{\sqrt{\ln x}}{x}\,dx = \int_0^{\sqrt{\ln 2}} t\,2t\,dt$

$\sqrt{\ln x} = t$

$\ln x = t^2$

$\frac{1}{x}\,dx = 2t\,dt$

$= 2\int_0^{\sqrt{\ln 2}} t^2\,dt$

$= \frac{2}{3}\left(t^3\right)_0^{\sqrt{\ln 2}} =$

$= \frac{2}{3}\left(\sqrt{\ln 2}^{\,3} - 0\right)$

$= \frac{2}{3}\ln 2\sqrt{\ln 2}.$

17. (a) $\int_0^1 (2x+1)(x+1)^5\,dx =$

$x+1 = t$

$x = t-1$

$dx = dt$

$$= \int_1^2 (2(t-1)+1)\,t^5\,dt$$

$$= \int_1^2 (2t-1)\,t^5\,dt$$

$$= \int_1^2 2t^6 - t^5\,dt$$

$$= \frac{2}{7}\left(t^7\right)_1^2 - \frac{1}{6}\left(t^6\right)_1^2$$

$$= \frac{2}{7}(2^7-1) - \frac{1}{6}(2^6-1)$$

18. (a) $\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (t^2+1)\,t\,2t\,dt$

$\sqrt{x-1} = t$

$x-1 = t^2$

$dx = 2t\,dt$

$x = t^2+1$

$= 2\int_0^1 t^2(t^2+1)\,dt$

$= 2\int_0^1 t^4 + t^2\,dt$

$= 2\left(\frac{1}{5}(t^5)\Big|_0^1 + \frac{1}{3}(t^3)\Big|_0^1\right)$

$= 2\left(\frac{1}{5}+\frac{1}{3}\right) =$

$= \frac{2}{5} + \frac{2}{3} = \frac{6}{15} + \frac{10}{15}$

$= \frac{16}{15}.$

(b) $\int_{-1}^0 x^2\sqrt{x+1}\,dx = \int_0^1 (t^2-1)^2\,t\,2t\,dt$

$\sqrt{x+1} = t$

$x+1 = t^2$

$x = t^2-1$

$dx = 2t\,dt$

$= \int_0^1 (t^4-2t^2+1)\,2t^2\,dt$

$= 2\int_0^1 t^6 - 2t^4 + t^2\,dt$

$= \sim$

⑧ $\int_0^3 (2x-1)\sqrt{x+1}\,dx = \int_1^2 \left(2(t^2-1)-1\right)t\,2t\,dt$

$\sqrt{x+1} = t$

$x+1 = t^2$

$x = t^2-1$

$dx = 2t\,dt$

$= 2\int_1^2 (2t^2-3)\,t^2\,dt$

$= 2\int_1^2 2t^4-3t^2\,dt$

KT?.

24. a) $\int_0^1 x^2 \ln(x+1)\, dx =$

$$= \int_0^1 \left(\frac{x^3}{3}\right)' \ln(x+1)\, dx =$$

$$= \left(\frac{1}{3} x^3 \ln(x+1)\right)\Big|_0^1 - \int_0^1 \frac{x^3}{3} \frac{1}{x+1}\, dx$$

$$= \frac{1}{3}\ln 2 - \frac{1}{3}\int_0^1 \frac{x^3}{x+1}\, dx =$$

$$\begin{array}{r|l} x^3 & x+1 \\ \hline -(x^3+x^2) & x^2-x+1 \\ \hline -x^2 & \\ -(-x^2-x) & \\ \hline x & \\ -(x+1) & \\ \hline -1 & \end{array}$$

$$= \frac{1}{3}\ln 2 - \frac{1}{3}\int_0^1 \frac{(x+1)(x^2-x+1)-1}{x+1}\, dx$$

$$= \frac{1}{3}\ln 2 - \frac{1}{3}\int_0^1 x^2-x+1 - \frac{1}{x+1}\, dx$$

$$= \frac{1}{3}\ln 2 - \frac{1}{3}\left(\frac{1}{3}(x^3)\Big|_0^1 - \frac{1}{2}(x^2)\Big|_0^1 + (x)\Big|_0^1 - (\ln(x+1))\Big|_0^1\right)$$

KT?.

(B) $\int_0^1 \ln(4-x^2)\,dx =$

$$= \int_0^1 (x)' \ln(4-x^2)\,dx =$$

$$= \left(x \ln(4-x^2)\right)_0^1 - \int_0^1 x \cdot \frac{-2x}{4-x^2}\,dx$$

$$= \ln 3 + 2\int_0^1 \frac{x^2}{4-x^2}\,dx =$$

$$\begin{array}{r|l} x^2 & 4-x^2 \\ \hline -(x^2-4) & -1 \\ \hline 4 & \end{array}$$

$$= \ln 3 + 2\int_0^1 \frac{-(4-x^2)+4}{4-x^2}\,dx$$

$$= \ln 3 + 2\int_0^1 -1 + \frac{4}{4-x^2}\,dx$$

$$= \ln 3 - 2\,(x)_0^1 + 8\int_0^1 \frac{1}{4-x^2}\,dx$$

$$= \ln 3 - 2 + 8 \cdot \int_0^1 \frac{1}{4-x^2}\,dx \quad \underline{\underline{(*)}}$$

$$\frac{1}{(2-x)(2+x)} = \frac{A}{2-x} + \frac{B}{2+x}$$

$$1 = A(2+x) + B(2-x)$$

$$1 = 2A + Ax + 2B - Bx$$

$$1 = (A-B)x + 2A + 2B$$

$$\begin{cases} A - B = 0 \quad \Rightarrow \underline{\underline{A = B}} \\ 1 = 2A + 2B \end{cases}$$

$$1 = 2A + 2A$$

$$A = \frac{1}{4} \qquad B = \frac{1}{4}$$

$$\underline{\underline{(*)}} \; \ln 3 - 2 + 8 \cdot \int_0^1 \frac{1}{4}\frac{1}{2-x} + \frac{1}{4}\frac{1}{2+x}\,dx$$

$$= \ln 3 - 2 - 2\left(\ln|2-x|\right)_0^1 + 2\left(\ln|x+2|\right)_0^1$$

$$= \ln 3 - 2 - 2(-\ln 2) + 2(\ln 3 - \ln 2) = 3\ln 3 - 2.$$

Εμβαδα 36

2. ⓐ $E : C_f,\ x'x,\ x=0,\ x=1$

$$E=\int_0^1 |f(x)|\,dx=\int_0^1 |\overset{+}{x^2+1}|\,dx$$

$$=\int_0^1 x^2+1\,dx = \text{κτλ}.$$

ⓑ $E : C_f,\ x'x,\ x=0,\ x=1$

$$E=\int_0^1 |f(x)|\,dx=\int_0^1 |\overset{-}{-\sqrt{x}}|\,dx=\int_0^1 \sqrt{x}\,dx$$

$$=\text{κτλ}.$$

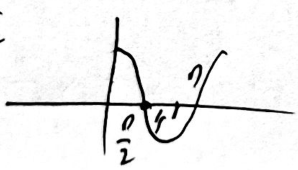
3. ⓐ $E : C_f,\ x'x,\ x=1,\ x=2$

$$E=\int_1^2 |f(x)|\,dx=\int_1^2 \left|\overset{+}{x+\tfrac{1}{x}}\right|\,dx=\int_1^2 x+\frac{1}{x}\,dx$$

κτλ.

ⓑ $E : C_f,\ x'x,\ x=\frac{\pi}{2},\ x=\pi$

$$E=\int_{\frac{\pi}{2}}^{\pi} |f(x)|\,dx=\int_{\frac{\pi}{2}}^{\pi} |\overset{-}{\sigma\upsilon\nu x}|\,dx=$$



$$=\int_{\frac{\pi}{2}}^{\pi} -\sigma\upsilon\nu x\,dx \quad \text{κτλ}.$$

4. ⓐ E: C_f, $x'x$, $x=0$, $x=2$

$$E=\int_0^2 |f(x)|\,dx=\int_0^2 \overset{-}{|x^2-3x|}\,dx=\int_0^2 -x^2+3x\,dx$$

x		0		3	
x^2-3x	+		−		+

ICT] .

Ⓑ E: C_f, $x'x$, $x=-1$, $x=0$.

$$E=\int_{-1}^{0} |f(x)|\,dx=\int_{-1}^{0} \overset{+}{|x^2-3x|}\,dx$$

$$=\int_{-1}^{0} x^2-3x\,dx = \text{ICT]}.$$

7. ⓐ E: C_f, $x'x$, $x=1$, $x=2$

$+$

$$E = \int_1^2 |f(x)|\,dx = \int_1^2 |x^3 - x^2 + 3x - 1|\,dx$$

$x > 1$

$f \nearrow$

$f(x) > f(1)$

$f(x) > 2$

$f'(x) = 3x^2 - 2x + 3 > 0$

$\Delta < 0$

$f \nearrow$

$$= \int_1^2 x^3 - x^2 + 3x - 1\,dx = \text{κτλ.}$$

Ενότητα 37

[redacted]

2. $f(x) = \dfrac{e^x}{x^2+1}$

ⓐ $f'(x) = \dfrac{e^x(x^2+1) - e^x\,2x}{(x^2+1)^2}$

$$f'(x) = \frac{e^x(x-1)^2}{(x^2+1)^2} \geq 0 \quad f \nearrow$$

ⓖ Νδο $\quad 2 < \displaystyle\int_0^2 f(x)\,dx < \frac{2e^2}{5}$.

$$0 < x < 2$$

$$f \nearrow$$

$$f(0) < f(x) < f(2)$$

$$1 < f(x) < \frac{e^2}{5}$$

$$\int_0^2 1\,dx < \int_0^2 f(x)\,dx < \int_0^2 \frac{e^2}{5}\,dx$$

$$(x)_0^2 < \int_0^2 f(x)\,dx < \frac{e^2}{5}(x)_0^2$$

$$2 < \int_0^2 f(x)\,dx < \frac{2e^2}{5} \quad \checkmark$$

11. $\left.\begin{array}{l} f(1)=2 \\ f'(1)=2 \end{array}\right\}$ $y-f(1)=f'(1)(x-1)$

$$y-2=2(x-1)$$

$$\boxed{y=2x}$$

Αφού f κοίλη

$$f(x)\leq 2x$$

$$\frac{f(x)}{x^2+1}\leq\frac{2x}{x^2+1}$$

$$\int_0^1\frac{f(x)}{x^2+1}\,dx<\int_0^1\frac{2x}{x^2+1}\,dx.$$

$$\int_0^1\frac{f(x)}{x^2+1}\,dx<\Big[\ln(x^2+1)\Big]_0^1$$

$$\int_0^1\frac{f(x)}{x^2+1}\,dx<\ln 2$$

5. $f(x) = \ln \frac{1+e^x}{2}$

(α) $f'(x) = \dfrac{1}{\frac{1+e^x}{2}} \left(\dfrac{1+e^x}{2}\right)'$

$f'(x) = \dfrac{2}{1+e^x} \, \dfrac{1}{2} e^x = \dfrac{e^x}{1+e^x}$

$f''(x) = \dfrac{e^x(1+e^x) - e^x e^x}{(1+e^x)^2} = \dfrac{e^x + \cancel{e^{2x}} - \cancel{e^{2x}}}{(1+e^x)^2}$

$f''(x) = \left(\dfrac{e^x}{1+e^x}\right)^2 > 0$ f κυρτή

(β) $y - f(0) = f'(0)(x - 0)$

$y - 0 = \frac{1}{2} x$

$y = \frac{1}{2} x$

Αφού f κυρτή $f(x) \geq \frac{1}{2} x$

(γ) i) νδο $\int_0^1 f(x)\,dx > \frac{1}{4}$

$$f(x) \geq \frac{1}{2}x$$

$$\int_0^1 f(x)\,dx > \int_0^1 \frac{1}{2}x\,dx$$

$$\int_0^1 f(x)\,dx > \frac{1}{4}.$$

ii). νδο $\int_1^2 \sqrt{x}\, f(x)\,dx > \frac{1}{5}(4\sqrt{2}-1)$

$$f(x) \geq \frac{1}{2}x$$

$$\sqrt{x}\, f(x) \geq \frac{1}{2}x\sqrt{x}$$

$$\int_1^2 \sqrt{x}\, f(x)\,dx > \int_1^2 \frac{1}{2}x\sqrt{x}\,dx$$

$$\int_1^2 \sqrt{x}\, f(x)\,dx > \frac{1}{5}(4\sqrt{2}-1).$$

3. (α) $f(x) = \dfrac{x}{\ln x}, \; x > 1$

$$f'(x) = \frac{\ln x - x \cdot \frac{1}{x}}{\ln^2 x} = \frac{\ln x - 1}{\ln^2 x}$$

x	1		e	
f'		$-$		$+$
f		$\searrow$		$\nearrow$

$f(x) \geq f(e)$

$f(x) \geq e$

(β) Νδο $\displaystyle\int_2^3 f(x)\,dx > e$

$$f(x) \geq e$$

$$\int_2^3 f(x)\,dx > \int_2^3 e\,dx$$

$$\int_2^3 f(x)\,dx > e\int_2^3 1\,dx$$

$$\int_2^3 f(x)\,dx > e\,(x)_2^3$$

$$\int_2^3 f(x)\,dx > e.$$

12. ε) $f(x) = 6x^5 - 2x + 1$

[illegible] 35

$$\int_0^{\pi/4} \frac{f(\varepsilon\varphi x)}{\sigma\upsilon\nu^2 x}\, dx = \int_0^1 f(t)\, dt$$

$\varepsilon\varphi x = t$

$\frac{1}{\sigma\upsilon\nu^2 x}\, dx = dt$

$$= \int_0^1 6t^5 - 2t + 1\, dt$$

$= \kappa\tau\lambda$

σε) $$\int_1^{1/2} \frac{f\left(\frac{1}{x}\right)}{x^2}\, dx = -\int_1^2 f(t)\, dt$$

$\frac{1}{x} = t$

$-\frac{1}{x^2}\, dx = dt$

$\frac{1}{x^2}\, dx = -dt$

$$= -\int_1^2 6t^5 - 2t + 1\, dt$$

$\kappa\tau\lambda.$

13. (γ) νδο $\int_1^3 f\left(\frac{3}{x}\right) dx = 3\int_1^3 \frac{f(x)}{x} dx$

$\longrightarrow \int_1^3 f\left(\frac{3}{x}\right) dx = -\int_3^1 f(t)\,\frac{3}{t^2}\, dt =$

$\frac{3}{x} = t$ $\qquad\qquad = -3\int_3^1 \frac{f(t)}{t^2}\, dt$

$-\frac{3}{x^2}\, dx = dt$ $\qquad\qquad = 3\int_1^3 \frac{f(x)}{x^2}\, dx.$

$-\frac{3}{x}\cdot\frac{1}{x}\, dx = dt$

$-t\cdot\frac{3}{x}\,\frac{1}{3}\, dx = dt$

$-t\cdot t\cdot\frac{1}{3}\, dx = dt$

$-t^2\,\frac{1}{3}\, dx = dt$

$$\boxed{dx = -\frac{3}{t^2}\, dt}$$

10. (β) $E: C_f, C_g, x=1, x=2$

Ενότητα 36

$$E=\int_1^2 |f(x)-g(x)|\,dx$$

$$E=\int_1^2 |\ln x - x|\,dx = \int_1^2 -\ln x + x\,dx$$

$$= -\int_1^2 \ln x\,dx + \int_1^2 x\,dx$$

- $\ln x \le x-1$

$\ln x - x \le -1$

$\ln x - x < 0$

κτλ.

11. (α) $E: C_f, C_g$

$$E=\int_{-2}^{2} |f(x)-g(x)|\,dx = \int_{-2}^{2} |x^3-4x|\,dx =$$

$\rightarrow f(x)=g(x) \Rightarrow x^3=4x \Rightarrow x^3-4x=0$

$x(x^2-4)=0$

$x=0$ $x=2$ $x=-2$

$$=\int_{-2}^{0} x^3-4x\,dx + \int_0^2 -x^3+4x\,dx$$

x		-2	0	2
x	$-$	$-$	0 $+$	$+$
x^2-4	$+$	$-$	$-$	$+$
x^3-4x	$-$	$+$	$-$	$+$

κτλ.

(B) i) E: C_f, C_g, $x=2$, $x=3$,

$$E = \int_2^3 |f(x) - g(x)|\, dx = \text{KT}\text{ე},$$

ii). E: C_f, C_g, $x=-2$, $x=3$

$$E = \int_{-2}^{3} |f(x) - g(x)|\, dx =$$

$$= \int_{-2}^{0} f(x) - g(x)\, dx + \int_0^2 -f(x) + g(x)\, dx$$

$$+ \int_2^3 f(x) - g(x)\, dx$$

$$= \text{KT}\text{ე}.$$

13. (ε) $E: C_f, C_g, x=2$

$$E = \int_1^2 |f(x)-g(x)|\,dx = \int_1^2 \left|\frac{1}{x} - e^{x-1}\right| dx \quad (*)$$

• $f(x)=g(x) \Rightarrow \frac{1}{x} = e^{x-1} \Rightarrow \underbrace{\frac{1}{x} - e^{x-1}}_{h(x)} = 0$

$h(x) = h(1)$

h 1-1

$x=1$

$$h'(x) = -\frac{1}{x^2} - e^{x-1} < 0$$

$h \downarrow$

$$(*)\quad \int_1^2 |\overset{-}{h(x)}|\,dx = \int_1^2 -h(x)\,dx = -\int_1^2 \frac{1}{x} - e^{x-1}\,dx$$

$1 < x < 2$

$h \downarrow$

$h(1) > h(x) > h(2)$

$0 > h(x)$

κτλ.

(52) E: $C_f, C_g, x=2$

$$f(x)=g(x) \Rightarrow \frac{1}{x} = 1+\ln x \Rightarrow \underbrace{\frac{1}{x}-1-\ln x=0}_{}$$

$$h'(x) = -\frac{1}{x^2}-\frac{1}{x}<0$$

$h\downarrow$

$h(x)=0$

$h(x)=h(1)$

h 1-1

$x=1$

$$E=\int_1^2 |f(x)-g(x)|\,dx = \int_1^2 \left|\frac{1}{x}-1-\ln x\right| dx$$

$$= \int_1^2 |\overset{(-)}{h(x)}|\,dx = -\int_1^2 h(x)\,dx =$$

$1<x<2$

$h\downarrow$

$h(1)>h(x)>h(2)$

$\underline{0>h(x)}$

$$= -\int_1^2 \frac{1}{x}-1-\ln x \; dx$$

κτλ.

17. $f(x) = x - 2 + \dfrac{e^x}{e^{2x}-1}$ $\quad x \neq 0$

$\in : C_f,\ \varepsilon,\ x=1,\ x=e$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} 1 - \frac{2}{x} + \frac{e^x}{xe^{2x}-x} = 1.$$

$$\to \lim_{x\to+\infty} \frac{e^x}{x(e^{2x}-x)} = 0$$

$$\to \lim_{x\to+\infty} \frac{e^x}{e^{2x}-x} = \lim_{x\to+\infty} \frac{e^x}{2e^{2x}-1} = \lim_{x\to+\infty} \frac{e^x}{4e^{2x}}$$

$$= \lim_{x\to+\infty} \frac{1}{4e^x} = 0$$

Αρα $\lim_{x\to+\infty} f(x) - x = \lim_{x\to+\infty} \cancel{x} - 2 + \dfrac{e^x}{e^{2x}-1} - \cancel{x}$

$= -2$

$y = x - 2$

$$E = \int_1^e \left| f(x) - (x-2) \right| dx$$

$$= \int_1^2 \left| \cancel{x-2} + \frac{e^x}{e^{2x}-1} - \cancel{x} + \cancel{2} \right| dx$$

$$= \int_1^2 \left| \frac{e^x}{e^{2x}-1} \right| dx = \int_1^2 \frac{e^x}{e^{2x}-1} dx$$

- $2x > 0 \Rightarrow e^{2x} > e^0 \Rightarrow e^{2x} - 1 > 0$

$e^x = t$

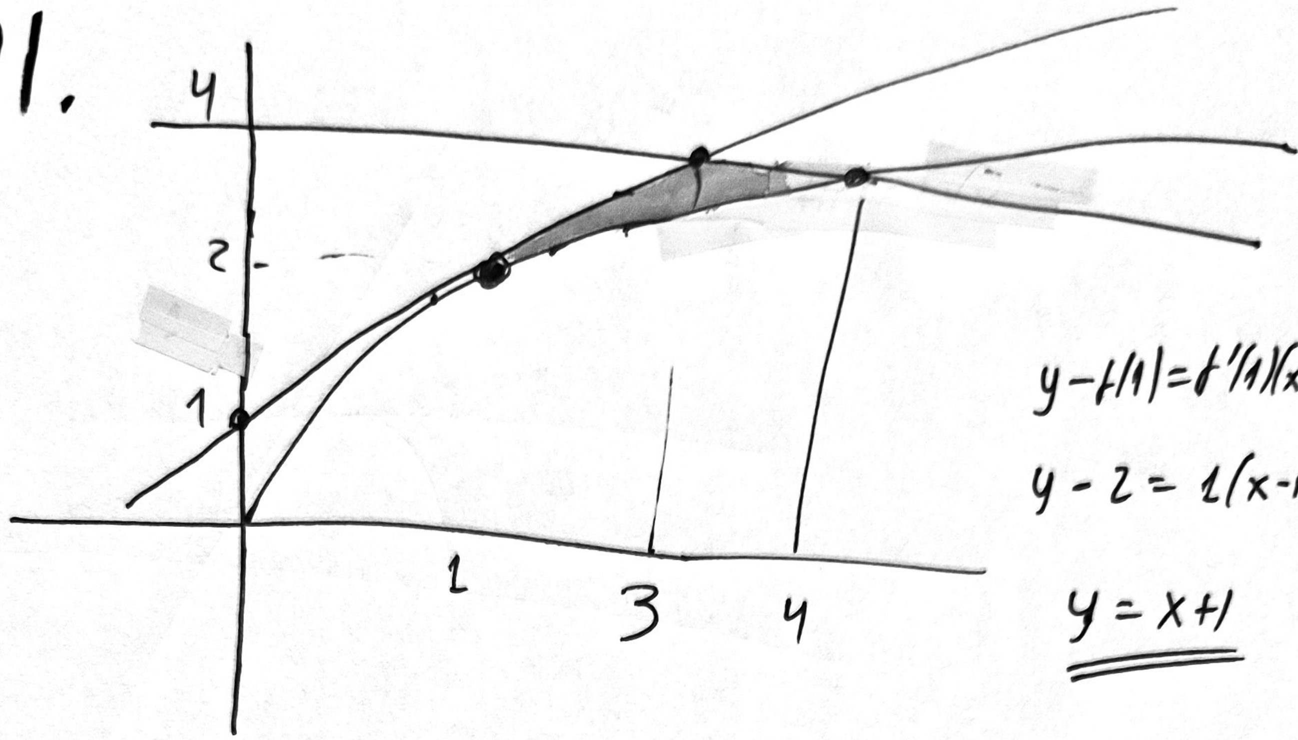
$e^x dx = dt$

$$= \int_e^{e^2} \frac{1}{t^2-1} dt$$

$$= \int_e^{e^2} \frac{1}{t^2-1} dt$$

A, B

21.



$y - f(1) = f'(1)(x-1)$

$y - 2 = 1(x-1)$

$\underline{\underline{y = x+1}}$

$2\sqrt{x} = 4$

$\sqrt{x} = 2$

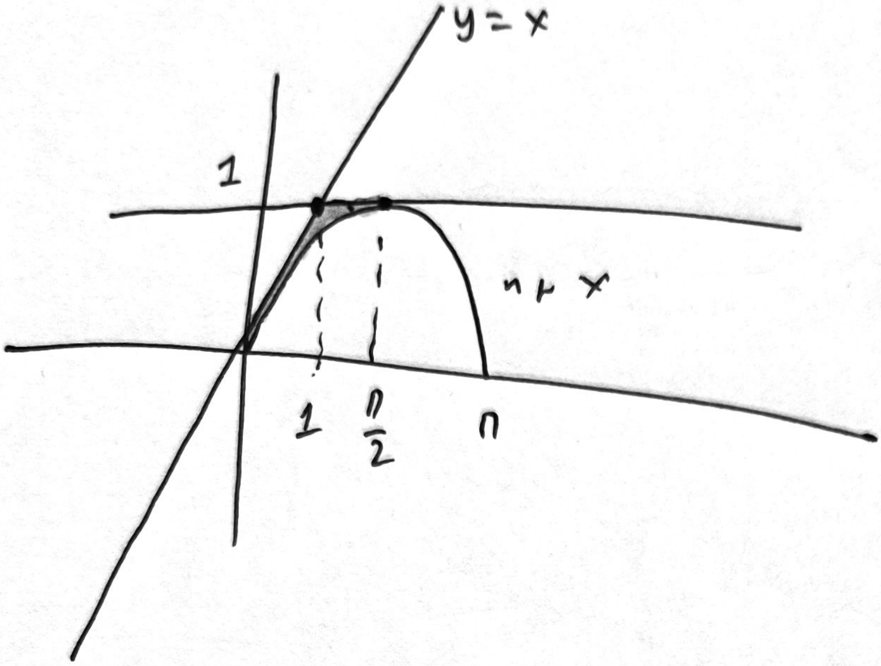
$x = 4$

$f'(x) = 2\,\frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$

$$E = \int_1^3 x+1 - 2\sqrt{x}\,dx + \int_3^4 4 - 2\sqrt{x}\,dx$$

κτλ.

20. $f(x) = \eta\mu x \quad x \in [0, \pi]$



$$E = \int_0^1 x - f(x)\, dx + \int_1^{\frac{\pi}{2}} 1 - f(x)\, dx$$

$$E = \int_0^1 x - \eta\mu x\, dx + \int_1^{\pi/2} 1 - \eta\mu x\, dx$$

κτλ

Επομενο Μαθημα

34	35	36	37
12 ολη	12 α β γ δ	10 α	6
13 ολη	13 α β	12	7
15	14	13 α β	8
		16	12
		18	13
		19	14.

Στο μαθημα της Δευτερας λυνουμε αυτα.
και ξεκιναμε επαναληψη.