Ενοτητα 25

2. $f(x) = \alpha x^3 + \beta x^2 - 9x + 1$

Αφου εχω ακροτατο στο -1 και 3

Fermat $f'(-1) = 0$ και $f'(3) = 0$.

$f'(x) = 3\alpha x^2 + 2\beta x - 9$

$f'(-1) = \boxed{3\alpha - 2\beta - 9 = 0}$

$f'(3) = \boxed{27\alpha + 6\beta - 9 = 0}$

$\alpha = 1$

$\beta = -3$

4. $f(x) = a^x - x$

$f(x) \geq 1$

$f(x) \geq f(0)$

Ακροτατο στο 0

Fermat $f'(0) = 0$

$f'(x) = a^x \ln a - 1$

$f'(0) = \ln a - 1 = 0$

$\ln a = 1$

$a = e$

5. (α) $\ln x + \frac{\alpha}{x} \geq \alpha$

$\ln x + \frac{\alpha}{x} - \alpha \geq 0$

$f(x) \geq 0$

$f(x) \geq f(1)$

Ακρότατο στο 1

Fermat $f'(1) = 0$.

$f'(x) = \frac{1}{x} - \frac{\alpha}{x^2}$

$f'(1) = 1 - \alpha = 0$

$\alpha = 1$

(B) $x^{a} \le a^{x}$

$\ln x^{a} \le \ln a^{x}$

$a \ln x \le x \ln a$

$a \ln x - x \ln a \le 0$

$f(x) \le 0$

$f(x) \le f(a)$

Ακροτατο στο α-

Fermat $f'(a) = 0$

$f'(x) = a \frac{1}{x} - \ln a$

$f'(a) = 1 - \ln a = 0$

$1 = \ln a$

$a = e$

6. $f(1) = 1$ παρ/γμ.

$f(x) \geq x \quad \forall x \in \mathbb{R}$.

Βρείτε την εφαπτομένη της C_f στο $x_0 = 1$

$$\underbrace{f(x) - x}_{g(x)} \geq 0$$

$$g(x) \geq 0$$

$$g(x) \geq g(1)$$

Ακρότατο στο 1.

$g'(1) = 0$ Fermat.

$$g'(x) = f'(x) - 1$$

$$g'(1) = f'(1) - 1 = 0.$$

$$f'(1) = 1$$

$$y - f(1) = f'(1)(x - 1)$$

$$y - 1 = x - 1 \qquad y = x.$$

14. (α) $f^3(x) + f(x) = x$.

Έστω ότι η f παρουσιάζει ακρότατο στο x_0.

Fermat $f'(x_0)=0$

$$3f^2(x)\,f'(x) + f'(x) = 1$$

$$\xrightarrow{x = x_0}$$

$$\cancel{3f^2(x_0)\,f'(x_0)} + \cancel{f'(x_0)} = 1$$

$0=1$ Άτοπο.

Άρα δεν μπορεί να έχει ακρότατα!

14. (B) $f^3(x)+2f(x)=x^3+x+1$

Εστω οτι η f παρουσιαζει ακροτατο στο x_0

Fermat $f'(x_0)=0$

$$3f^2(x)f'(x)+2f'(x)=3x^2+1$$

$\underline{x=x_0}$

$$3f^2(x_0)\cancel{f'(x_0)}+2\cancel{f'(x_0)}=3x_0^2+1$$

$$0=3x_0^2+1$$

Ατοπο!

Αρα δεν εχει ακροτατα

|

Ενότητα 24

32. $f(0) = 0$ παρ/μη.

$f(x) + f'(x) > e^{-x} \quad \forall x \in R$

(α) Νδο $f(x) < \dfrac{x}{e^x}$, $\forall x < 0$

$$\underbrace{e^x f(x) - x}_{g(x)} < 0$$

$g'(x) = e^x f(x) + e^x f'(x) - 1$

$g'(x) = e^x \left(f(x) + f'(x) \right) - 1 > 0$

$g \nearrow$

Αν $x < 0 \Rightarrow g(x) < g(0) \Rightarrow e^x f(x) - x < 0$

$f(x) < \dfrac{x}{e^x}$ ✓

$f(x) + f'(x) > e^{-x}$

$f(x) + f'(x) > \dfrac{1}{e^x}$

$e^x \left(f(x) + f'(x) \right) - 1 > 0$

B

$$f(x) < \frac{x}{e^x}$$

$$\lim_{x \to -\infty} f(x) < \lim_{x \to -\infty} \frac{x}{e^x}$$

$$\lim_{x \to -\infty} f(x) < \lim_{x \to -\infty} x \times \frac{1}{e^x}$$

$$\lim_{x \to -\infty} f(x) < -\infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

35. $f'(0)<0$ $\quad f$ συνεχ.

$f'(x)\neq 0$ $\quad f'$ συνεχ.

(α) Αφού $f'(x)\neq 0$ και συνεχ.

$\Rightarrow f'(x)>0$ ή $f'(x)<0$

αφού $f'(0)<0 \Rightarrow f'(x)<0$

$f \searrow$

(β) $f(\varepsilon\varphi x) > f(\eta\mu x)$ $\quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\varepsilon\varphi x < \eta\mu x$

$\underbrace{\varepsilon\varphi x - \eta\mu x}_{\varphi(x)} < 0 \Rightarrow \varphi(x)<0$

$\cancel{\varphi(x)<\varphi(0)}$

$-\frac{\pi}{2} < x < 0$

$\varphi'(x) = \frac{1}{\sigma\upsilon\nu^2 x} - \sigma\upsilon\nu x = \frac{1-\sigma\upsilon\nu^3 x}{\sigma\upsilon\nu^2 x} > 0$ (+)

$\varphi \nearrow$.

$\sigma\upsilon\nu x \leq 1$

$\sigma\upsilon\nu^3 x \leq 1^3$

$1-\sigma\upsilon\nu^3 x \geq 0$

5. $f(x^2) - \ln x = f(x)$

$f(x^2) - \ln \frac{x^2}{x} = f(x)$

$f(x^2) - \ln x^2 + \ln x = f(x)$

$f(x^2) - \ln x^2 = f(x) - \ln x$

$\boxed{g(x) = f(x) - \ln x}$

$g(x^2) = g(x)$

g 1-1

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

- $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

$\oplus$

- $x_1 < x_2 \Rightarrow -\ln x_1 > -\ln x_2$

$g \downarrow$

$\cancel{x=0}$ $\quad$ $x = 1$ ✓

38. $f(5)=f(4)$

$f' \nearrow$

ⓐ $f(x+1)-f(x)=0 \Rightarrow g(x)=0$

$g(x)=g(4)$

g 일-1

$x=4$

$$\boxed{g(x)=f(x+1)-f(x)}$$

$g'(x)=f'(x+1)-f'(x)>0 \quad g \nearrow$

$\cdot\ x<x+1$

$f' \nearrow$

$f'(x)<f'(x+1)$

$f'(x+1)-f'(x)>0$

ⓑ $f(2x+1)>f(2x)$

$f(2x+1)-f(2x)>0$

$g(2x)>g(4)$

$g \nearrow$

$2x>4$

$x>2$.

① $f(x^4+5) = f(x^4+4)$

$$f(x^4+5) - f(x^4+4) = 0$$

$$g(x^4+4) = 0$$

$$g(x^4+4) = g(4)$$

$$g : 1-1$$

$$x^4+4 = 4$$

$$x^4 = 0$$

$$x = 0$$

39. $f(x) = 2(x-2)\ln x + 2x^2 - 1 \quad , x > 0$

ⓐ $f'(x) = 2 \cdot \ln x + 2(x-2)\frac{1}{x} + 4x$

$$f''(x) = \frac{2}{x} + \frac{2x - 2(x-2)}{x^2} + 4$$

$$f''(x) = \frac{2}{x} + \frac{2}{x^2} + 4 > 0 \quad f' \nearrow$$

Ⓑ $f(x^3+2) + f(x^2) < f(x^2+2) + f(x^3)$

$$f(x^3+2) - f(x^3) < f(x^2+2) - f(x^2)$$

$$\boxed{g(x) = f(x+2) - f(x)}$$

$$g(x^3) < g(x^2) \longrightarrow x^3 < x^2$$

$g'(x) = f'(x+2) - f'(x) > 0 \quad \Rightarrow g \nearrow \quad \underline{0 < x < 1}$

- $x < x+2 \quad \Rightarrow f'(x) < f'(x+2) \quad \Rightarrow f'(x+2) - f'(x) > 0$

$g'(x) > 0$

46. $f(x) = e^{-x} + \ln(x+1) - 1$

(a) $f(x) = 0$

$f(x) = f(0)$

$f(0)-1$

$\boxed{x = 0}$

x		0	
$f(x)$	$\nearrow$ $-$	0	$\nearrow$ $+$

$$f'(x) = -e^{-x} + \frac{1}{x+1}$$

$$f'(x) = -\frac{1}{e^x} + \frac{1}{x+1} = \frac{\overset{+}{e^x - x - 1}}{e^x(x+1)} \geq 0$$

$f \nearrow$

(B) i) $\frac{1}{e^{x^2-1}} + 2\ln x = 1$

$$e^{1-x^2} + \ln x^2 - 1 = 0$$

$$e^{-(x^2-1)} + \ln x^2 - 1 = 0$$

$$f(x^2-1) = f(0)$$

$$x^2 - 1 = 0$$

$x = 1$ $\quad$ $\cancel{x = -1}$

11). $\ln \frac{x^2+1}{x+1} > \frac{e^{x^2}-e^{x}}{e^{x^2+x}}$

$$\ln(x^2+1) - \ln(x+1) > \frac{e^{x^2}}{e^{x^2+x}} - \frac{e^{x}}{e^{x^2+x}}$$

$$\ln(x^2+1) - \ln(x+1) > e^{-x} - e^{-x^2}$$

$$e^{-x^2} + \ln(x^2+1) - 1 > e^{-x} + \ln(x+1) - 1$$

$$f(x^2) > f(x)$$

$$f \nearrow$$

$$x^2 > x$$

$$x^2 - x > 0$$

$$x(x-1) > 0$$

x		0		1	
x^2-x	$+$		$-$		$+$

$$x \in (-\infty, 0) \cup (1, +\infty)$$

$\underline{D_f}$) $x \in (-1, 0) \cup (1, +\infty)$

① $\lim\limits_{x \to 0} \dfrac{1}{f(e^x) - f(x+1)} = +\infty$

⊕

- $e^x \geq x+1$

$f(e^x) \geq f(x+1)$

$f(e^x) - f(x+1) \geq 0$

Επομενο Μαθημα

Παλια	Νεα
24	24
47	7
	12
	17
	18 α γ
	19 α β
	20 α
	21
	56
	57