27. Ενότητα

$$f(x) = x^{x} = e^{\ln x^{x}} = e^{x \ln x}$$

$$\boxed{f(x) = e^{x \ln x}}$$

$$f'(x) = e^{x \ln x} \, (x \ln x)'$$

$$f'(x) = e^{x \ln x} \left(\ln x + x \cdot \frac{1}{x} \right)$$

$$\boxed{f'(x) = x^{x} (\ln x + 1)}$$

27. (B) $f(x) = \left(x+\frac{1}{x}\right)^x$, $x > 0$ **16**

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$$f(x) = e^{\ln\left(x+\frac{1}{x}\right)^x} = e^{x\ln\left(x+\frac{1}{x}\right)}$$

$$f'(x) = e^{x\ln\left(x+\frac{1}{x}\right)} \left[x\ln\left(x+\frac{1}{x}\right)\right]'$$

$$f'(x) = \left(x+\frac{1}{x}\right)^x \cdot \left[\ln\left(x+\frac{1}{x}\right) + x \cdot \frac{1}{x+\frac{1}{x}}\left(x+\frac{1}{x}\right)'\right]$$

$$\left(e^{\square}\right)' = e^{\square} \cdot \square'$$

$$\left(e^x\right)' = e^x$$

29. (B) $f(x) = \begin{cases} x \ln x, & x \le 0 \\ \dfrac{x}{x+1}, & x > 0 \end{cases}$

$$f'(x) = \begin{cases} \ln x + x \cdot \frac{1}{x}, & x < 0 \\ \dfrac{x+1-x}{(x+1)^2}, & x > 0 \end{cases}$$

$$\boxed{f'(x) = \begin{cases} \ln x + x \cdot \frac{1}{x}, & x < 0 \\ \dfrac{1}{(x+1)^2}, & x > 0. \end{cases}}$$

- $\lim\limits_{x \to 0^-} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0^-} \dfrac{x \ln x}{x} = 0$

- $\lim\limits_{x \to 0^+} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0^+} \dfrac{\frac{x}{x+1} - 0}{x} =$

$$= \lim_{x \to 0^+} \frac{x}{x(x+1)} = 1$$

36. $f(x) = \begin{cases} \sin 2x, & x \le 0 \\ 2x \cos x, & x > 0 \end{cases}$

$$f'(x) = \begin{cases} \cos 2x \cdot (2x)', & x < 0 \\ 2\cos x - 2x\sin x, & x > 0 \end{cases}$$

$$\boxed{f'(x) = \begin{cases} 2\cos 2x, & x \le 0 \\ 2\cos x - 2x\sin x, & x > 0 \end{cases}}$$

$$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{\sin 2x}{x} = \left(\frac{0}{0}\right) = \text{DLH}$$

$$= \lim_{x \to 0^-} \frac{\cos 2x \cdot 2}{1} = 2$$

$$\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{2x \cos x - 0}{x} = 2$$

$$f''(x) = \begin{cases} 2(-\sin 2x)\cdot 2, & x < 0 \\ -2\sin x - (2\sin x + 2x\cos x), & x > 0 \end{cases}$$

$$\boxed{f''(x) = \begin{cases} -4\sin 2x, & x < 0 \\ -4\sin x - 2x\cos x, & x > 0 \end{cases}}$$

$$\lim_{x \to 0^-} \frac{f'(x) - f'(0)}{x - 0} = \lim_{x \to 0^-} \frac{2\cos 2x - 2}{x}$$

$$= \lim_{x \to 0^-} \frac{-2\sin 2x \cdot 2}{1} = 0$$

$$\lim_{x \to 0^+} \frac{f'(x) - f'(0)}{x - 0} = \lim_{x \to 0^+} \frac{2\cos x - 2x\sin x - 2}{x}$$

$$= \lim_{x \to 0^+} \frac{-2\sin x - 2\cos x - 2x\cos x}{1} = -2$$

1. ⓐ $\lim_{x\to 0} \frac{\ln(x+1)}{x} \overset{\left(\frac{0}{0}\right)}{\underset{DLH}{=}}$

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$= \lim_{x\to 0} \frac{\frac{1}{x+1}}{1} = 1$

ⓑ $\lim_{x\to 0} \frac{e^x - x - 1}{x^2} \overset{\left(\frac{0}{0}\right)}{\underset{DLH}{=}} \lim_{x\to 0} \frac{e^x - 1}{2x}$

$\overset{\left(\frac{0}{0}\right)}{\underset{DLH}{=}} \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$

2. ⓐ $\lim_{x\to 1} \frac{x - \ln x - 1}{e^{x-1} - x} \overset{\left(\frac{0}{0}\right)}{\underset{DLH}{=}} \lim_{x\to 1} \frac{1 - \frac{1}{x}}{e^{x-1} - 1}$

$\overset{\left(\frac{0}{0}\right)}{\underset{DLH}{=}} \lim_{x\to 1} \frac{\frac{1}{x^2}}{e^{x-1}} = \frac{1}{1} = 1$

2. (62) $\lim_{x\to 0} \frac{1}{x} - \frac{\ln(1+x)}{x^2} =$

$$= \lim_{x\to 0} \frac{x - \ln(x+1)}{x^2} \overset{\left(\frac{0}{0}\right)}{\underset{DLH}{=}} \lim_{x\to 0} \frac{1 - \frac{1}{x+1}}{2x}$$

$$\overset{\left(\frac{0}{0}\right)}{\underset{DLH}{=}} \lim_{x\to 0} \frac{-\frac{-1}{(x+1)^2}}{2} = \frac{1}{2}$$

3. (8) $\lim_{x\to+\infty} \frac{e^x}{\ln x} \overset{\frac{\infty}{\infty}}{\underset{DLH}{=}} \lim_{x\to+\infty} \frac{e^x}{\frac{1}{x}} =$

$$= \lim_{x\to+\infty} e^x \cdot x = +\infty,$$

3. (62) $\lim_{x\to+\infty} \frac{e^x}{x^2+1} \overset{\frac{\infty}{\infty}}{=}_{DLH} \lim_{x\to+\infty} \frac{e^x}{2x}$

$$= \lim_{x\to+\infty} \frac{e^x}{2} = +\infty$$

4. (8) $\lim_{x\to+\infty} \frac{1+\ln^2 x}{x} \overset{\frac{\infty}{\infty}}{=}_{DLH}$

$$\lim_{x\to+\infty} \frac{2\ln x\,(\ln x)'}{1} = \lim_{x\to+\infty} \frac{2\ln x}{x}$$

$$= \lim_{x\to+\infty} \frac{\frac{2}{x}}{1} = 0$$

4. (ОС) $\lim\limits_{x\to+\infty} \dfrac{\ln(x^2+1)}{e^x+1} =$

$$= \lim_{x\to+\infty} \frac{\frac{1}{x^2+1}\,2x}{\frac{e^x}{1}} =$$

$$= \lim_{x\to+\infty} \frac{2x}{(x^2+1)e^x} = \lim_{x\to+\infty} \frac{2}{2xe^x+(x^2+1)e^x}$$

$$= 0.$$