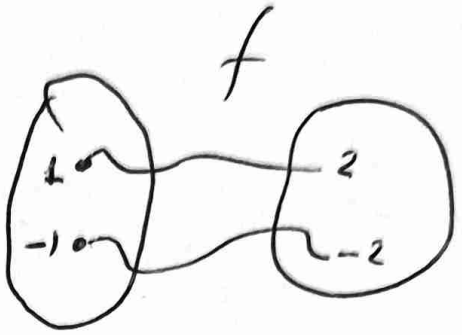
Συναρτήσεις

$$f(x) = \frac{x^2+1}{x}$$



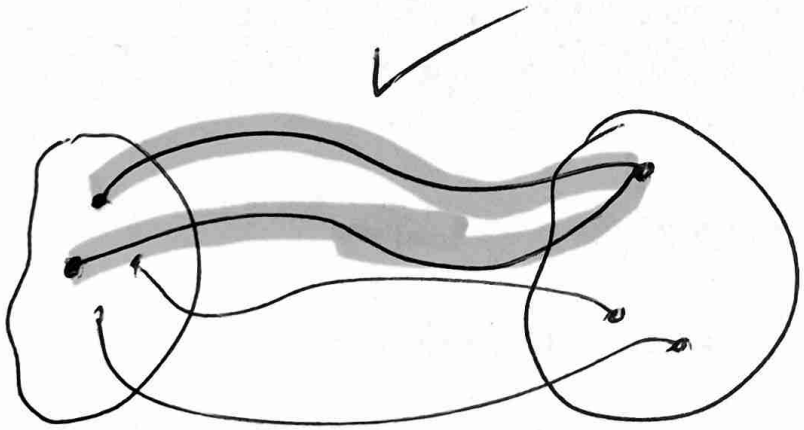
$\longrightarrow f(1) = \frac{1^2+1}{1} = 2$

$\longrightarrow f(-1) = \frac{(-1)^2+1}{-1} = -2$

Πεδίο ορισμού

Πρέπει όταν έχω $\sqrt{x}$ το $x \geq 0$

Πρέπει όταν έχω $\frac{1}{x}$ το $x \neq 0$.

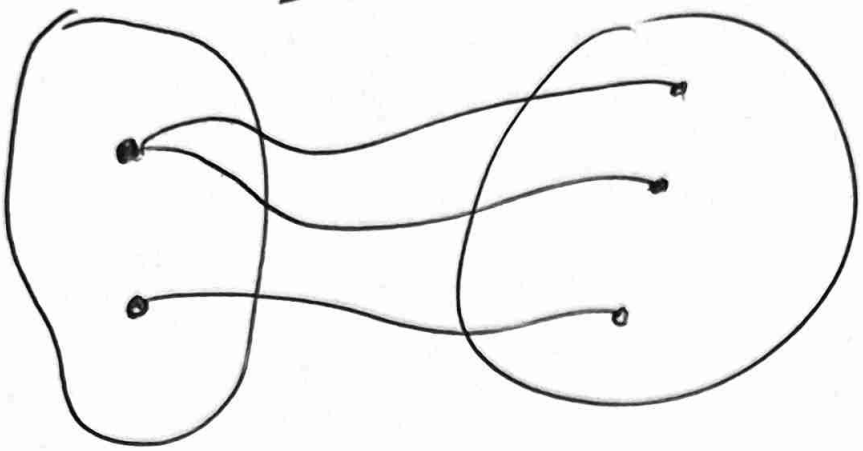


Πεδίο ορισμού

Σύνολο Τιμών

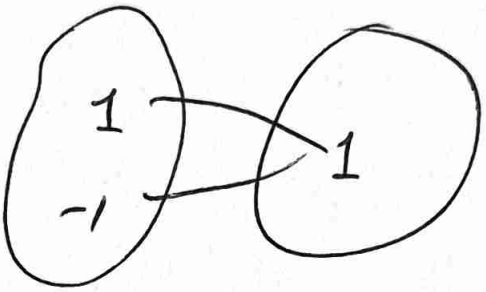
Αυτό είναι συνάρτηση

Δν είναι συναρτήση.



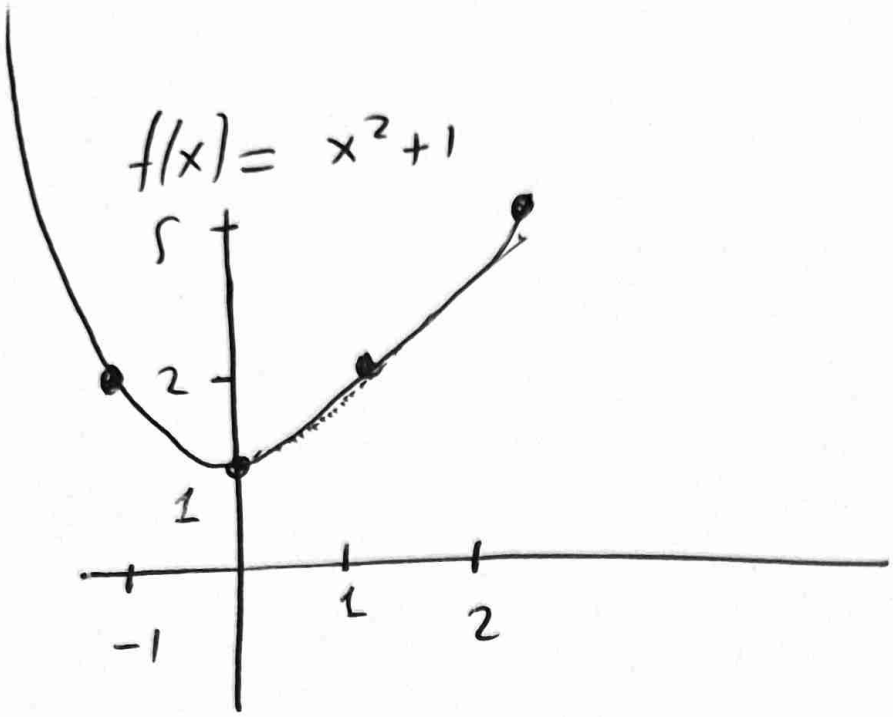
π.χ

$f(x) = x^2$



$f(1) = 1^2 = 1$

$f(-1) = (-1)^2 = 1$



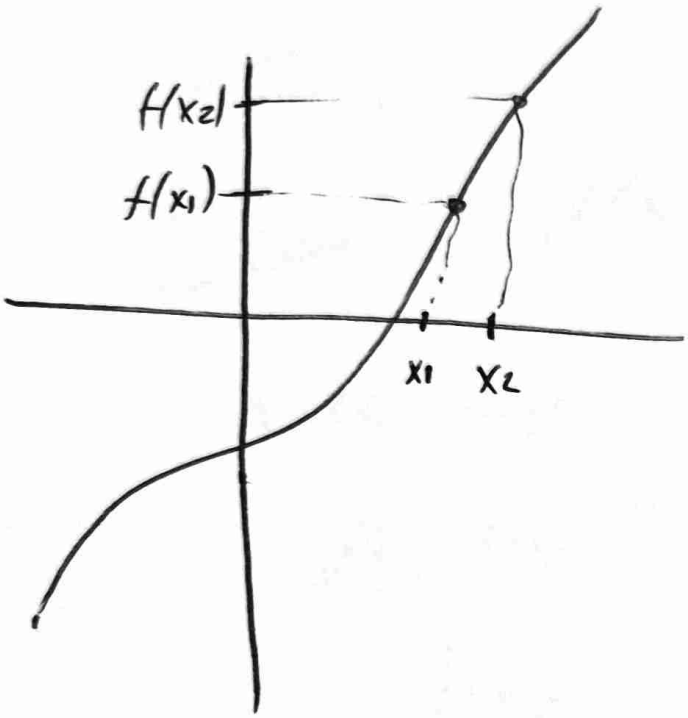
$f(0) = 0^2 + 1 = 1$

$f(1) = 1^2 + 1 = 2$

$f(2) = 2^2 + 1 = 5$

$f(-1) = (-1)^2 + 1 = 2$

f γνησίως αύξουσα.



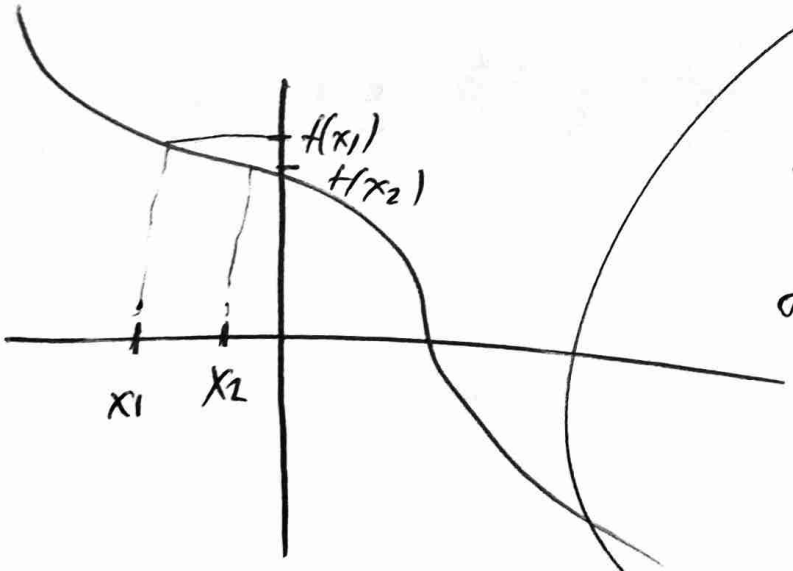
$f \nearrow$

$x_1 < x_2$

$f(x_1) < f(x_2)$

$\forall x_1, x_2$

f γν. φθίνουσα.

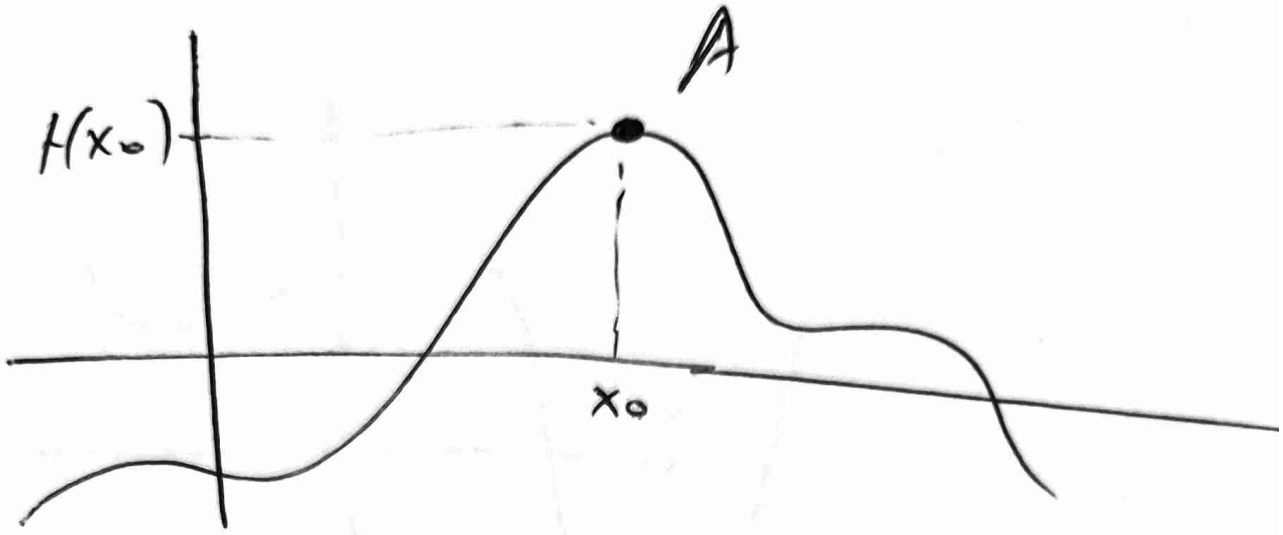


$f \searrow$

$x_1 < x_2$

$f(x_1) > f(x_2)$

$\forall x_1, x_2$



Όταν $f(x) \leq f(x_0)$, $\forall x \in A_f$

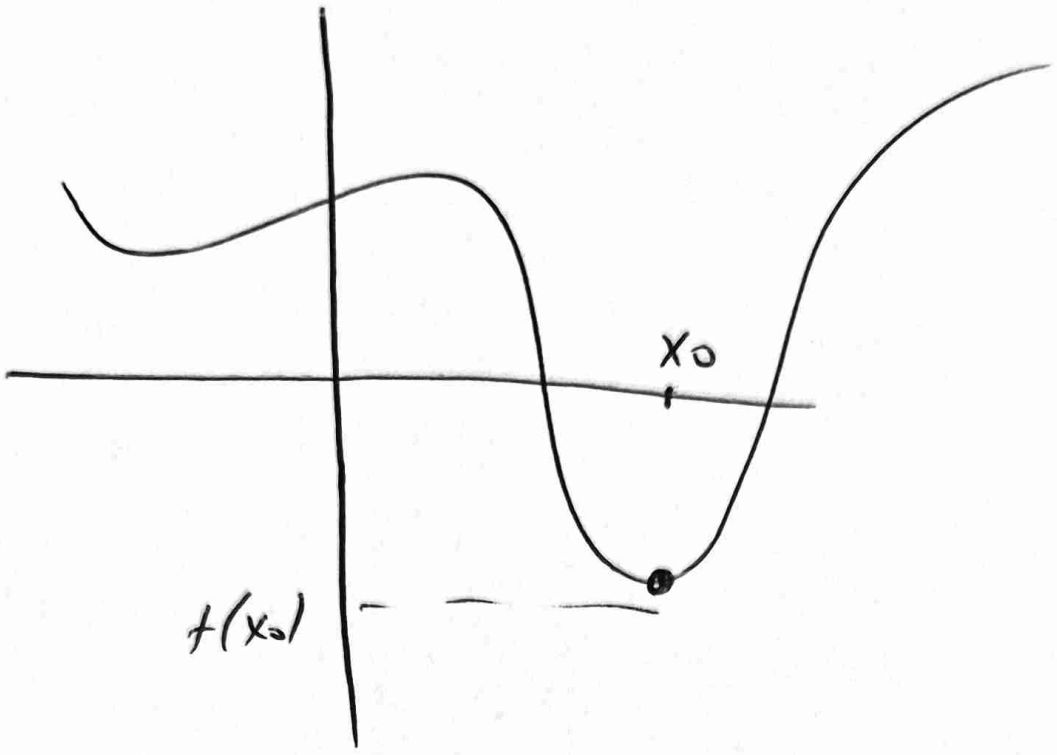
τότε λέμε ότι το σημείο

$A(x_0, f(x_0))$ λέγεται

ολικό μέγιστο.

x_0 : Θέση μεγίστου.

$f(x_0)$: Μέγιστη τιμή ή μέγιστο.

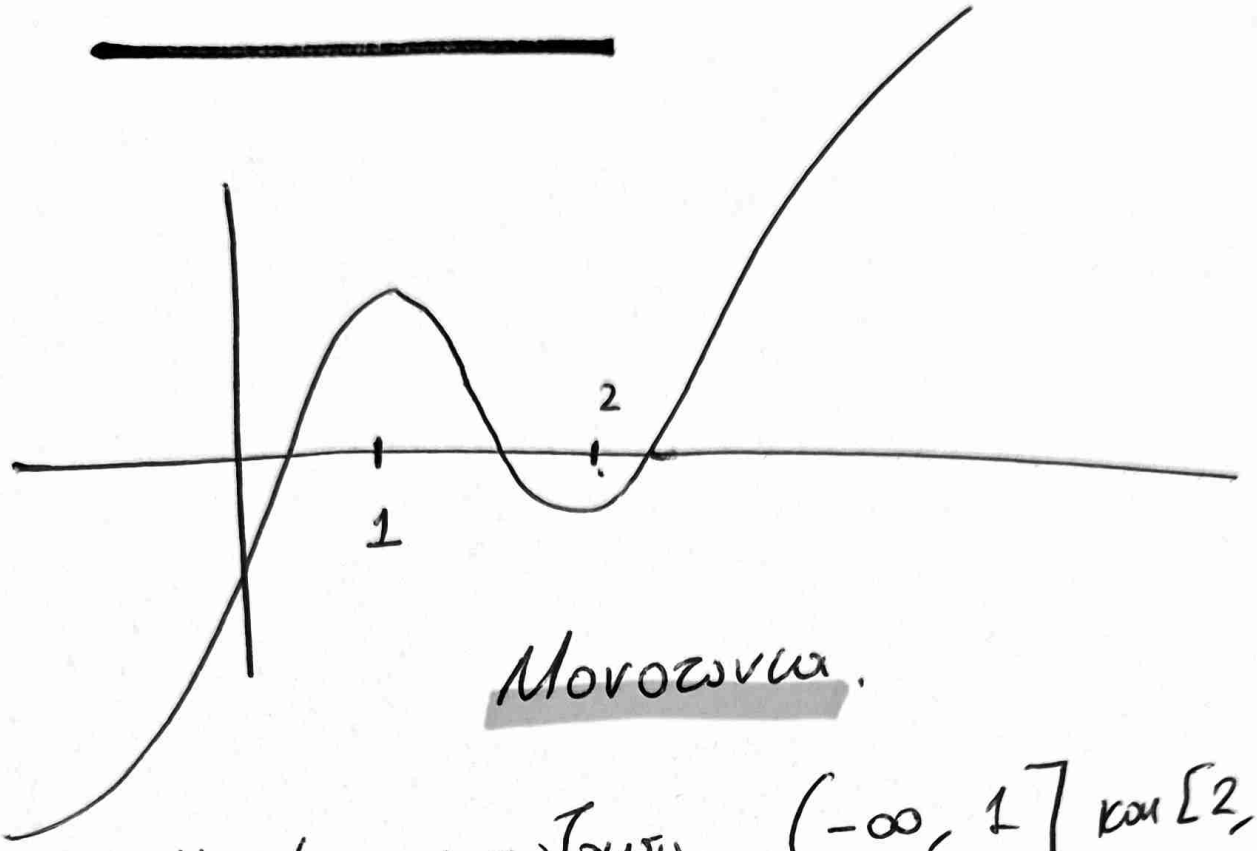


Το $A(x_0, f(x_0))$ είναι ολικό ελάχιστο.

όταν $f(x) \geq f(x_0)$ $\forall x \in A_f$.

Το $f(x_0)$ λέγεται ελάχιστη τιμή.

Ασκηση 1

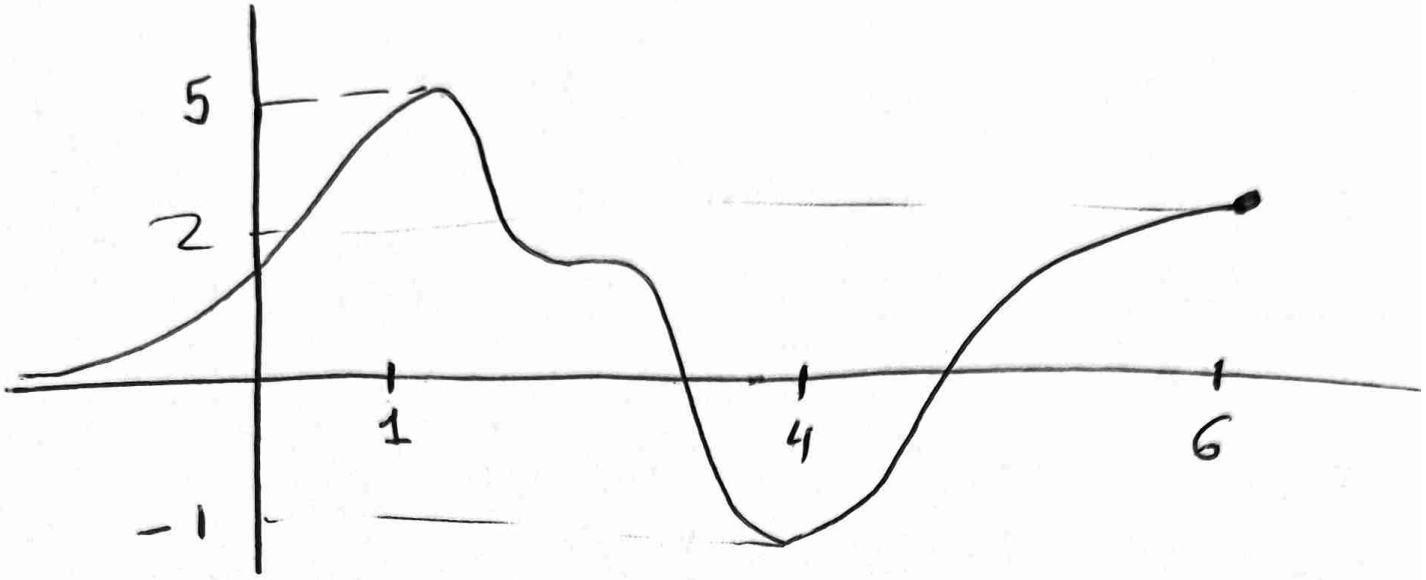


Μονοτονια.

Η f γν. αυξουσα $(-\infty, 1]$ και $[2, +\infty)$

Η f γν. φθινουσα $[1, 2]$

Άσκηση 2



Μονοτονία - ακρότατα.

Η f γν. αύξουσα $(-\infty, 1]$ και $[4, 6]$

Η f γν. φθίνουσα $[1, 4]$

Το $A(1, 5)$ ολικό μέγιστο.

Το $B(4, -1)$ ολικό ελάχιστο

Ασκηση 3

1. Νδο η $f(x) = x^2 - 6x + 10$ παρουσιαζει ελαχιστο για $x = 3$.

Αρκει νδο $f(x) \geq f(3)$

$$x^2 - 6x + 10 \geq 3^2 - 6 \cdot 3 + 10$$

$$x^2 - 6x + 10 \geq 19 - 18$$

$$x^2 - 6x + 10 \geq 1$$

$$x^2 - 6x + 9 \geq 0$$

$$(x-3)^2 \geq 0. \quad \checkmark$$

2. Νδο η $g(x) = \frac{2x}{x^2+1}$ παρουσιάζει μέγιστο για $x=1$.

Αρκεί νδο $g(x) \leq g(1)$

$$\frac{2x}{x^2+1} \leq \frac{2 \cdot 1}{1^2+1}$$

$$\frac{2x}{x^2+1} \leq \frac{2}{2}$$

$$\frac{2x}{x^2+1} \leq 1$$

$$2x \leq x^2+1$$

$$0 \leq x^2-2x+1$$

$$0 \leq (x-1)^2$$

✓

Ενοτητα 2

1.

$\varepsilon: y = 2x - 3$

(α) $\lambda_\varepsilon = 2$

(β) $x'x$ $\quad$ $y = 0$

$0 = 2x - 3$

$3 = 2x$

$x = \frac{3}{2}$

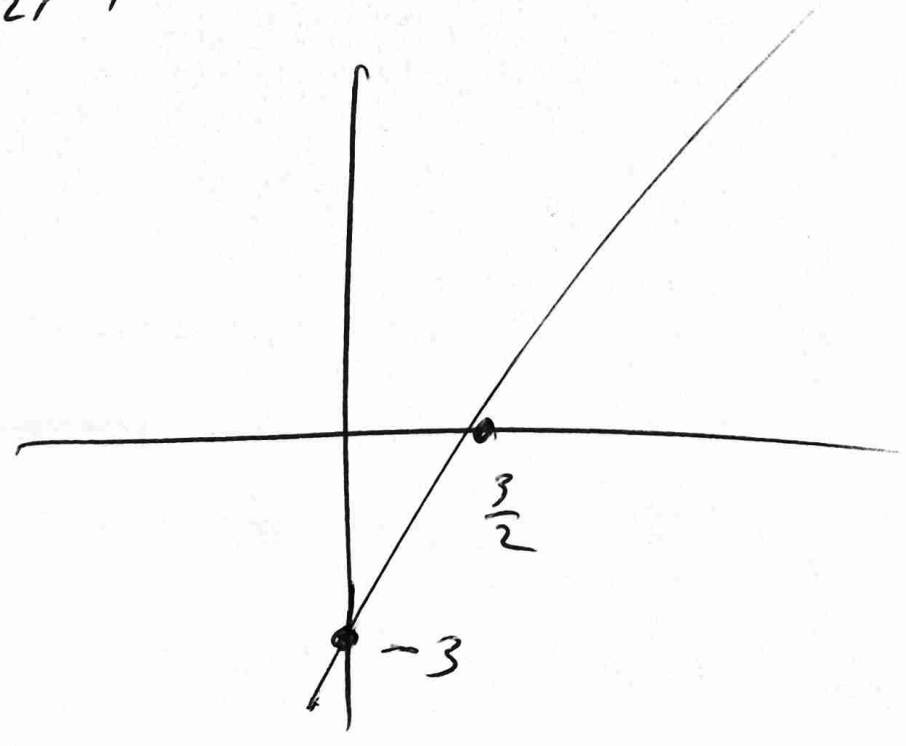
$A\left(\frac{3}{2}, 0\right)$

$y'y$ $\quad$ $x = 0$

$y = -3$

$B(0, -3)$.

(γ)



2. ① $y = ax + B$

$y = -2x + B \longrightarrow A(3,2)$

$2 = -2 \cdot 3 + B$

$2 + 6 = B$

$B = 8$

$y = -2x + 8$

② $y = ax + B.$

$\varepsilon\varphi 45 = a$

$1 = a$

$y = x + B \rightarrow (3,2)$

$2 = 3 + B$

$B = -1$

$y = x - 1$

3. $y = ax + B$ — $A(2, -3)$, $B(-2, 5)$

$-3 = 2a + B$

$5 = -2a + B$ } (+)

$-3 = 2a + 1$

$-4 = 2a$

$a = -2$

$2 = 2B$

$B = 1$

$y = -2x + 1$

5. (ii). $\varepsilon: y = (\alpha+\beta)x + \alpha \longrightarrow A(1,-1)$

Εχει κλιση -2

$\alpha+\beta=-2$

$-1 = \alpha+\beta+\alpha$

$2\alpha+\beta=-1$

$$\begin{cases} \alpha+\beta=-2 \\ 2\alpha+\beta=-1 \end{cases} \quad (-)$$

$-\alpha=-1$

$\alpha=1$

$\beta=-3$

$y=-2x+1$

(B)

x'x

$y=0$

$0=-2x+1$

$2x=1$

$x=\frac{1}{2}$

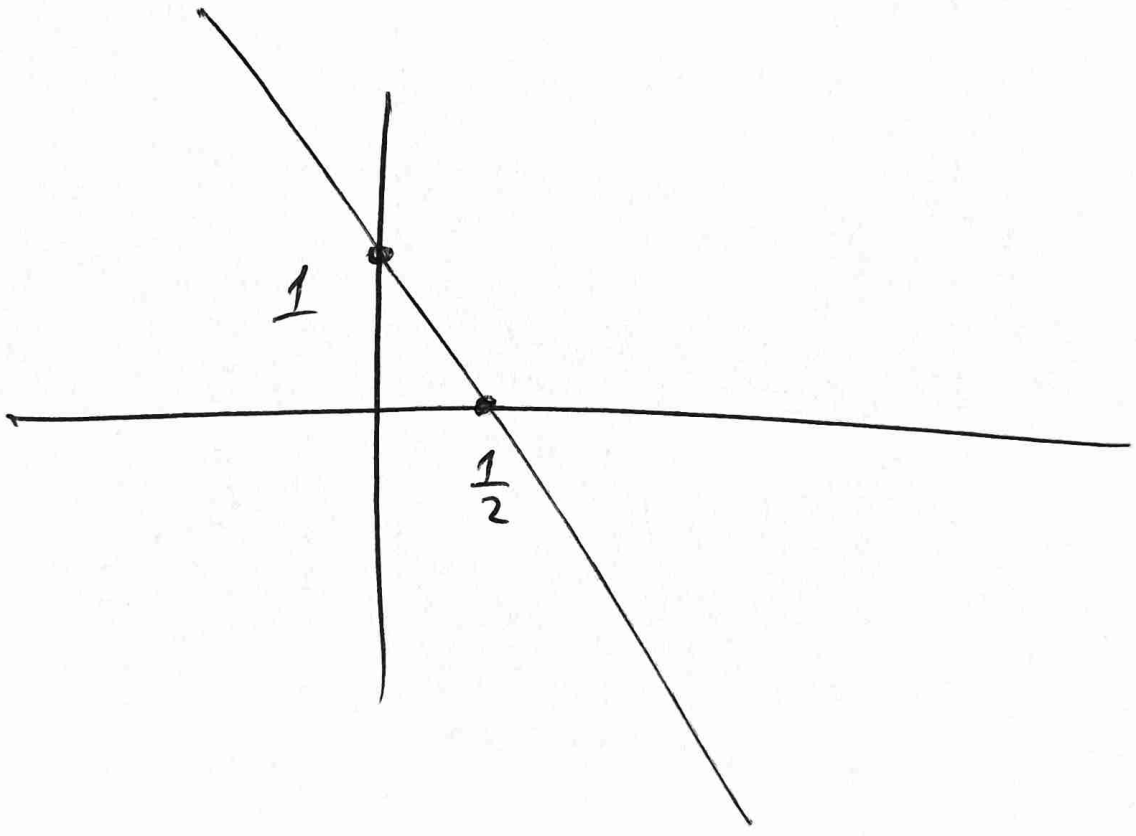
$K\left(\frac{1}{2},0\right)$

y'y

$x=0$

$y=1$

$A(0,1)$.



7. $\varepsilon : y = \alpha x + \beta \;//\; \zeta : 2x - y = 1$

$\varepsilon \to M(-1, 2)$

Αφού $\varepsilon // \zeta \Rightarrow \lambda_\varepsilon = \lambda_\zeta$

$\underline{\alpha = 2}$

$\varepsilon : y = 2x + \beta \to M(-1,2)$ $\qquad$ $y = 2x - 1$

$2 = -2 + \beta$

$\underline{\beta = 4}$

$\varepsilon : y = 2x + 4$

8. $\varepsilon: y = (a-1)x + \beta + 1 \longrightarrow M(1,3)$.

και $\varepsilon \parallel \zeta: y = 2x - 1$

$a - 1 = 2$

$a = 3$

$y = 2x + \beta + 1 \rightarrow (1,3)$

$3 = 2 + \beta + 1$

$\beta = \beta + \beta$

$\beta = 0$

$y = 2x + 1$

11. $\begin{cases} x+y=5 \\ x-y=1 \end{cases}$ $A(3,2)$.

12. ⓐ $\begin{cases} x-3y+1=0 \\ 2x-y-3=0 \end{cases}$ $A(2,1)$

ⓑ $\varepsilon: \lambda x - y = \lambda + 3 \longrightarrow A(2,1)$,

$2\lambda - 1 = \lambda + 3$

$\lambda = 4$

13. $\varepsilon_1 : x+y=3$

$\varepsilon_2 : x-y=1$

$\varepsilon_3 : x-ky=k+1$

$$\begin{cases} x+y=3 \\ x-y=1 \end{cases}$$

$A(2,1)$

$2-k=k+1$

$1=2k$

$k=\frac{1}{2}.$

14. ε_1: $2ax + By = a + 3g$ } $A(5;3)$

ε_2: $x + (a - B)y = B - 2$

$$\begin{cases} 10a + 3B = a + 3g \\ 5 + 3(a - B) = B - 2 \end{cases}$$

$a = 3$

$B = 4$

Εφορεία Μαΐνης

2	5	6
15	1	1.
16		2.
17.	(ΕΘΑ 60)	(ΕΘΑ 69)