39. (B) $\lim\limits_{x\to 1} \dfrac{f(x)-x^3}{x^2-1} = 2$

Βρείτε το $\lim\limits_{x\to 1} \dfrac{f(x)-x}{\sqrt{x}-1}$

Θέτω $\dfrac{f(x)-x^3}{x^2-1} = g(x)$ $\quad \lim\limits_{x\to 1} g(x) = 2$

$f(x) = (x^2-1)\,g(x) + x^3$

$$\lim_{x\to 1} \frac{(x^2-1)\,g(x) + x^3 - x}{\sqrt{x}-1} =$$

$$= \lim_{x\to 1} \frac{(x-1)(x+1)g(x) + x(x-1)(x+1)}{(\sqrt{x}-1)(\sqrt{x}+1)}(\sqrt{x}+1)$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}\left((x+1)g(x) + x(x+1)\right)}{\cancel{x-1}}(\sqrt{x}+1)$$

$= 6 \cdot 2 = 12.$

Β' τρόπος

$$\lim_{x\to 1} \frac{f(x)-x^3}{x^2-1} = 2$$

$$\lim_{x\to 1} \frac{f(x)-x}{\sqrt{x}-1} = \lim_{x\to 1} \frac{f(x)-x^3+x^3-x}{\sqrt{x}-1}$$

$$= \lim_{x\to 1} \frac{f(x)-x^3+x^3-x}{x^2-1} \cdot \frac{x^2-1}{\sqrt{x}-1}$$

$$= \lim_{x\to 1} \left(\frac{f(x)-x^3}{x^2-1} + \frac{x\cancel{(x^2-1)}}{\cancel{x^2-1}} \right) \frac{\cancel{(x-1)}(x+1)(\sqrt{x}+1)}{\cancel{x-1}}$$

$$= (2+1)\cdot 4$$

$$= 12.$$

34. (B) $\lim\limits_{x\to 0} \dfrac{f(x)}{x} = 5.$

$\to \lim\limits_{x\to 0} \dfrac{f(2x)}{x} = \lim\limits_{x\to 0} \dfrac{f(2x)}{2x} \cdot \dfrac{2\cancel{x}}{\cancel{x}}$

$= 5 \cdot 2$

$= 10$

$\to \lim\limits_{x\to 0} \dfrac{f(2x)}{2x} \underset{\substack{x\to 0 \\ \omega\to 0}}{\overset{2x=\omega}{=\!=}} \lim\limits_{\omega\to 0} \dfrac{f(\omega)}{\omega} = 5$

36. $\boxed{\lim\limits_{x\to 0} \dfrac{f(x)-2}{x} = 3}$

(α) $\lim\limits_{x\to 0} \dfrac{f(3x)-2}{x} = \lim\limits_{x\to 0} \dfrac{f(3x)-2}{3x} \cdot 3$

Θέτω $3x = t$

$x \to 0$

$t \to 0$

$= \lim\limits_{t\to 0} \dfrac{f(t)-2}{t} \cdot 3 = 3\cdot 3 = 9.$

(B) $\lim_{x\to 0} \frac{f(5x)-f(3x)}{x} \overset{(*)}{=\!=}$

$\boxed{\lim_{x\to 0} \frac{f(x)-2}{x} = 3}$

$\boxed{\lim_{x\to 0} \frac{f(3x)-2}{x} = 9}$

$\overset{(*)}{=\!=} \lim_{x\to 0} \frac{f(5x)-2-f(3x)+2}{x}$

$= \lim_{x\to 0} \frac{f(5x)-2}{x} - \frac{f(3x)-2}{x} = 15-9$

$= 6.$

$\rightarrow \lim_{x\to 0} \frac{f(5x)-2}{5x}\cdot 5 \; \underset{\substack{x\to 0\\ t\to 0}}{\overset{5x=t}{=\!=}} \; \lim_{t\to 0} \frac{f(t)-2}{t}\cdot 5$

$= 3\cdot 5 = 15$

1

Βοηθητικη Συναρτηση

SOS

Εστω οτι $\lim_{x\to 0} \frac{f(x)-x^2}{x} = 4$

1. Βρει $\lim_{x\to 0} f(x)$

Θεωρω Βοηθητικη συναρτηση

$$g(x) = \frac{f(x)-x^2}{x} \qquad \boxed{\lim_{x\to 0} g(x) = 4}$$

$$x\,g(x) = f(x) - x^2$$

$$f(x) = x\,g(x) + x^2$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(x\,g(x) + x^2\right)$$

$$\lim_{x\to 0} f(x) = 0\cdot 4 + 0$$

$$\boxed{\lim_{x\to 0} f(x) = 0}$$

2. Bpd to $\lim_{x \to 0} \frac{f(x) - \sqrt{x+1} + 1}{x}$

$$\lim_{x \to 0} \frac{f(x) - \sqrt{x+1} + 1}{x} = \lim_{x \to 0} \frac{x g(x) + x^2 + 1 - \sqrt{x+1}}{x}$$

$$= \lim_{x \to 0} \frac{\cancel{x} g(x)}{\cancel{x}} + \frac{x^{\cancel{2}}}{\cancel{x}} + \frac{1 - \sqrt{x+1}}{x} =$$

$$= 4 + 0 + \left(-\frac{1}{2}\right)$$

$$= \frac{7}{2}.$$

$$\rightarrow \lim_{x \to 0} \frac{1 - \sqrt{x+1}}{x} = \lim_{x \to 0} \frac{(1 - \sqrt{x+1})(1 + \sqrt{x+1})}{x(1 + \sqrt{x+1})} = \lim_{x \to 0} \frac{1 - (x+1)}{x(1 + \sqrt{x+1})}$$

$$= \frac{-1}{2}$$

$x \to 2$

40. $\lim_{x\to 1} \frac{f(x)}{x-1} = 3$

$|g(x)-1| \leq |f(x)|$

ⓐ $\lim_{x\to 1} f(x)$

Θεωρω Βοηθητικη συναρτηση

$h(x) = \frac{f(x)}{x-1}$

$\boxed{\lim_{x\to 1} h(x) = 3}$

$f(x) = h(x)(x-1)$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} h(x)(x-1)$

$\lim_{x\to 1} f(x) = 3 \cdot 0$

$\lim_{x\to 1} f(x) = 0$

(B) $\lim_{x \to 1} g(x)$

$$|g(x)-1| \le |f(x)|$$

$$-|f(x)| \le g(x)-1 \le |f(x)|$$

$$\boxed{1-|f(x)| \le g(x) \le 1+|f(x)|}$$

$\lim_{x \to 1} 1-|f(x)| = 1$

$\lim_{x \to 1} 1+|f(x)| = 1$

Από Κ.Π

$\lim_{x \to 1} g(x) = 1$

٢ $$\lim_{x\to 1} \frac{\overset{\ominus}{|g(x)-3|}-2}{g^2(x)-g(x)} =$$

$$= \lim_{x\to 1} \frac{-g(x)+3-2}{g^2(x)-g(x)} = \lim_{x\to 1} \frac{1-g(x)}{g(x)\,(g(x)-1)}$$

$$= \lim_{x\to 1} \frac{-\cancel{(g(x)-1)}}{g(x)\,\cancel{(g(x)-1)}} = \frac{-1}{1}$$

$$= -1$$

42. $\lim_{x\to 0} \frac{f(x)}{x} = 3$

ⓐ $\lim_{x\to 0} \frac{f(x^2+x)}{x} = \lim_{x\to 0} \frac{f(x^2+x)}{x^2+x} \cdot \frac{x^2+x}{x} \underset{=}{\overset{(*)}{=}}$

• $\lim_{x\to 0} \frac{f(x^2+x)}{x^2+x} \overset{x^2+x=t}{=\!=} \lim_{t\to 0} \frac{f(t)}{t} = 3$

• $\lim_{x\to 0} \frac{x^2+x}{x} = \lim_{x\to 0} \frac{\cancel{x}(x+1)}{\cancel{x}} = 1$.

$\overset{(*)}{=\!=\!=} 3 \cdot 1 = 3$.

(B) $\lim_{x\to 0} \dfrac{f^2(x) + x f(x) + x\,\eta\mu x}{f^2(x) + x^2 + \eta\mu^2 x} =$

$\lim_{x\to 0} \dfrac{f(x)}{x} = 3$

$= \lim_{x\to 0} \dfrac{\dfrac{f^2(x)}{x^2} + \dfrac{x f(x)}{x^2} + \dfrac{x\,\eta\mu x}{x^2}}{\dfrac{f^2(x)}{x^2} + \dfrac{x^2}{x^2} + \dfrac{\eta\mu^2 x}{x^2}}$

$= \lim_{x\to 0} \dfrac{\left(\dfrac{f(x)}{x}\right)^2 + \dfrac{f(x)}{x} + \dfrac{\eta\mu x}{x}}{\left(\dfrac{f(x)}{x}\right)^2 + 1 + \left(\dfrac{\eta\mu x}{x}\right)^2}$

$= \dfrac{3^2 + 3 + 1}{3^2 + 1 + 1^2} = \dfrac{13}{11}$

Ερώτημα 8

21. ⓐ $x^2-2x \le f(x) \le x^3-x^2-x$

- $\lim_{x\to 1} (x^2-2x) = -1$
- $\lim_{x\to 1} (x^3-x^2-x) = -1$

$\Big\}$ Από Κ.Π

$\lim_{x\to 1} f(x) = -1$

22. ⓐ $|f(x)-x| \le x^2$

$$-x^2 \le f(x)-x \le x^2$$

$$\boxed{x-x^2 \le f(x) \le x^2+x}$$

- $\lim_{x\to 0} x-x^2 = 0$
- $\lim_{x\to 0} x^2+x = 0$

$\Big\}$ Από Κ.Π

$\lim_{x\to 0} f(x) = 0$

23. $3x - 5x^2 \leq f(x) \leq 5x^2 + 3x$

$$\lim_{x \to 0} \frac{f(x)}{x} = 3 .$$

Για $x < 0$

$$3 - 5x \geq \frac{f(x)}{x} \geq 5x + 3$$

$$\left.\begin{aligned} \lim_{x \to 0^-} 3 - 5x &= 3 \\ \lim_{x \to 0^-} 5x + 3 &= 3 \end{aligned}\right\} \boxed{\lim_{x \to 0^-} \frac{f(x)}{x} = 3}$$

Για $x > 0$

$$3 - 5x \leq \frac{f(x)}{x} \leq 5x + 3$$

$$\left.\begin{aligned} \lim_{x \to 0^+} 3 - 5x &= 3 \\ \lim_{x \to 0^+} 5x + 3 &= 3 \end{aligned}\right\} \text{Απο Κ.Π}$$

$$\boxed{\lim_{x \to 0^+} \frac{f(x)}{x} = 3}$$

43. $\eta\mu x - x^2 \leq f(x) \leq \eta\mu x + x^2$

ⓐ $\lim\limits_{x\to 0} \dfrac{f(x)}{x} = 1.$

<u>Για $x<0$</u>

$$\frac{\eta\mu x}{x} - x \geq \frac{f(x)}{x} \geq \frac{\eta\mu x}{x} + x$$

$$\left.\begin{aligned} &\lim_{x\to 0^-}\left(\frac{\eta\mu x}{x} - x\right) = 1 - 0 = 1 \\ &\lim_{x\to 0^-}\left(\frac{\eta\mu x}{x} + x\right) = 1 + 0 = 1 \end{aligned}\right\} \lim_{x\to 0^-} \frac{f(x)}{x} = 1$$

<u>Για $x>0$</u> $\quad \dfrac{\eta\mu x}{x} - x \leq \dfrac{f(x)}{x} \leq \dfrac{\eta\mu x}{x} + x$

$$\left.\begin{aligned} &\lim_{x\to 0^+} \frac{\eta\mu x}{x} - x = 1 \\ &\lim_{x\to 0^+} \frac{\eta\mu x}{x} + x = 1 \end{aligned}\right\} \text{Από Κ.Π.} \quad \lim_{x\to 0^+} \frac{f(x)}{x} = 1$$

B

$$\lim_{x\to 0} \frac{f(x)+2x}{x+\eta\mu x} = \lim_{x\to 0} \frac{\frac{f(x)}{x}+\frac{2x}{x}}{\frac{x}{x}+\frac{\eta\mu x}{x}}$$

$$= \lim_{x\to 0} \frac{\frac{f(x)}{x}+2}{1+\frac{\eta\mu x}{x}} = \frac{1+2}{1+1} = \frac{3}{2}.$$

24. (a) $\lim\limits_{x\to 0}\left(x^2 \text{συν}\dfrac{2}{x}\right) \xlongequal[\text{χτισαμε}]{\text{συν}\infty}$

$$-1 \le \text{συν}\frac{2}{x} \le 1$$

$$\boxed{-x^2 \le x^2\,\text{συν}\frac{2}{x} \le x^2}$$

$$\left.\begin{aligned} &\lim_{x\to 0} -x^2 = 0 \\ &\lim_{x\to 0} x^2 = 0 \end{aligned}\right\}$$

Από Κ.Π

$$\lim_{x\to 0} x^2\,\text{συν}\frac{2}{x} = 0.$$

Επόμενο μάθημα

Ημερομηνία μαθήματος : 27/7

Ώρα μαθήματος : 11:30 .

Ασκήσεις

Ενοτητα 8

34 α

39 α

41

παρατηρήσεις

Ασκηση

Δινεται $f(x) = x^5 + x$

1. Νδο f αντιστρεψιμη
2. Βρει μονοτονια αντιστρ. οφης
3. Νδο f περιττη
4. εξισωση $f(x^5+2x-2) - x^5 = x$
5. Ανισωση $(f^{-1}(x))^5 + f^{-1}(x) < 1 - \ln x$
6. εξισωση $f^{-1}(f(e^{x+1}) - f^{-1}(34)) = 0$
7. εξισωση $f(2x) - f(x) + x = 0$
8. εξισωση $f(x) = 2 + f^{-1}(1-x)$
9. εξισωση $f(x^2) + x = f(x) + 1$ στο $[0, +\infty)$