Ενότητα 17

2. (α) $f(x) = x^2 - 3x + 1$ $\qquad x_0 = 2$

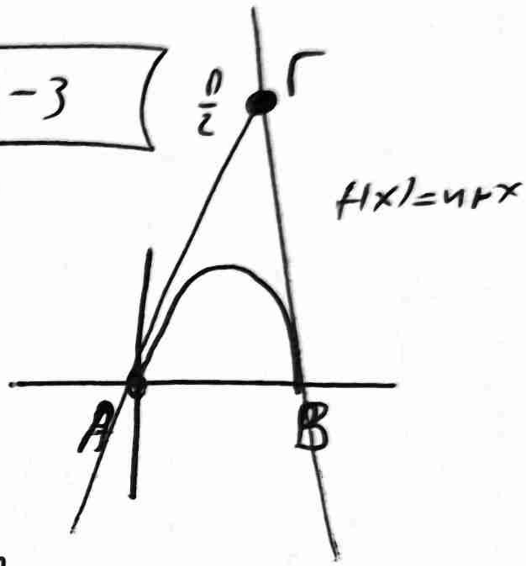
$$y - f(2) = f'(2)(x-2)$$

$$y - (-1) = 1 \cdot (x-2)$$

$$y + 1 = x - 2$$

$$\boxed{y = x - 3}$$

$$\boxed{f'(x) = 2x - 3}$$



4. $\varepsilon_A : y = x$

$\varepsilon_B : y = -x + \pi$

$$E_{\tau\rho\iota\gamma} = \frac{B \cdot \upsilon}{2} = \frac{\pi \cdot \frac{\pi}{2}}{2} = \frac{\pi^2}{4}$$

$$\begin{cases} y = x \\ y = -x + \pi \end{cases} \quad (+) \quad 2y = \pi$$

$$y = \frac{\pi}{2}$$

5. (a) $f(x) = \begin{cases} \eta\mu 2x, & x \le 0 \\ 2x + \sigma\upsilon\nu x - 1, & x > 0 \end{cases}$

$y - f(0) = f'(0)(x-0)$

- $f(0) = \eta\mu \cdot 0 = 0$
- $f'(0) = 2$

$y - 0 = 2(x-0)$ — $y = 2x$.

$$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-} \frac{\eta\mu 2x - 0}{x} = \lim_{x\to 0^-} \frac{\eta\mu 2x}{x} =$$

$$= \lim_{x\to 0^-} \frac{2\sigma\upsilon\nu 2x}{1} = 2\cdot 1 = 2$$

$$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} \frac{2x + \sigma\upsilon\nu x - 1}{x} =$$

$$= \lim_{x\to 0^+} \frac{2 - \eta\mu x}{1} = 2$$

(B) $f(x) = \begin{cases} x^2 ημ\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$y - f(0) = f'(0)(x-0)$$

$$y - 0 = 0(x-0)$$

$$y = 0 \quad x'x$$

$$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{x^2 ημ\frac{1}{x} - 0}{x} = \lim_{x\to 0} x ημ\frac{1}{x},$$

$$-1 \leq ημ\frac{1}{x} \leq 1$$

$$\left|ημ\frac{1}{x}\right| \leq 1$$

$$|x|\left|ημ\frac{1}{x}\right| \leq |x|$$

$$\left|x ημ\frac{1}{x}\right| \leq |x| \qquad f'(0) = 0$$

$$\boxed{-|x| \leq x ημ\frac{1}{x} \leq |x|}$$

$$\left.\begin{aligned} &\lim_{x\to 0} -|x| = 0 \\ &\lim_{x\to 0} |x| = 0 \end{aligned}\right\} \lim_{x\to 0} x ημ\frac{1}{x} = 0.$$

6. $f(x) = x^2 - 2x + 3$

① $y - f(x_0) = f'(x_0)(x - x_0) \perp y = x$

$f'(x_0) \cdot 1 = -1$

$2x_0 - 2 = -1$

$2x_0 = 1$

$x_0 = \frac{1}{2}$

$y - f(\frac{1}{2}) = f'(\frac{1}{2})(x - \frac{1}{2})$

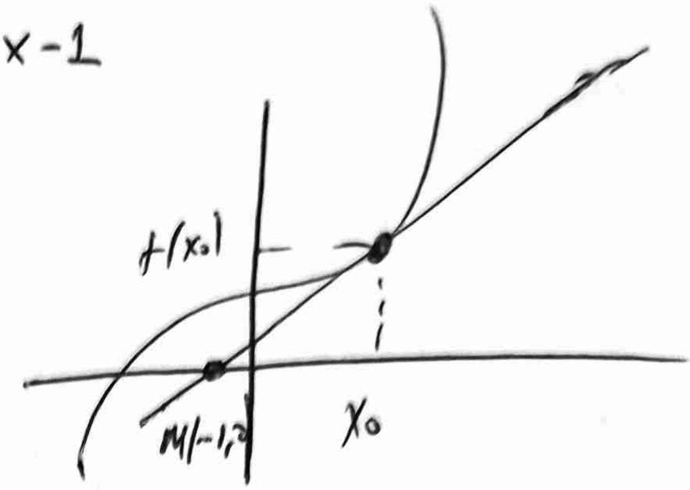
$y - (\frac{1}{4} + 2) = -(x - \frac{1}{2})$

$y - \frac{9}{4} = -x + \frac{1}{2}$

$y = -x + \frac{9}{4} + \frac{1}{2}$

$\boxed{y = -x + \frac{11}{4}}$

7. $f(x) = x^2 - x - 1$



$y - f(x_0) = f'(x_0)(x - x_0)$

$y - f(x_0) = f'(x_0)(x - x_0) \longrightarrow M(-1,0)$

$0 - f(x_0) = f'(x_0)(-1 - x_0)$,

$-f(x) = f'(x)(-1 - x)$

$-(x^2 - x - 1) = (2x - 1)(-1 - x)$

$-x^2 + \cancel{x} + \cancel{1} = -2x - 2x^2 + \cancel{1} + \cancel{x}$

$x^2 + 2x = 0$

$x(x + 2) = 0$

$x = 2$

$x = 0$

$y - f(2) = f'(2)(x + 2)$

$y = -5x - 5$

$y - f(0) = f'(0)(x - 0)$

$y = -x - 1$

8. $f(x) = \frac{2x-1}{x}$ $\qquad f'(x) = \frac{1}{x^2}$

$y - f(x_0) = f'(x_0)(x - x_0) \longrightarrow (0,0)$

$0 - f(x) = f'(x)(-x)$

$-\frac{2x-1}{x} = \frac{2x-(2x-1)}{x^2}(-x)$

$\frac{2x-1}{\cancel{x}} = \frac{\cancel{2x}-\cancel{2x}+1}{\cancel{x}}$

$2x - 1 = 1$

$2x = 2$

$x = 1$

$y - f(1) = f'(1)(x-1)$

$y - 1 = 1(x-1)$

$\underline{y = x}$

11. $f(x) = x^2 - x + 2$

(α) $y - f(x_0) = f'(x_0)(x - x_0) \; // \; y = 3x - 1$

$\Rightarrow f'(x_0) = 3$

$2x_0 - 1 = 3$

$2x_0 = 4$

$x_0 = 2$

$y - f(2) = f'(2)(x-2)$

$y - 4 = 3(x-2)$

$\boxed{y = 3x - 2}$

$M(2,4)$.

(β) $y - f(x_0) = f'(x_0)(x - x_0) \perp \varepsilon_{AB}$.

$f'(x_0) \cdot \lambda_{AB} = -1$

$(2x_0 - 1) \cdot \dfrac{y_B - y_A}{x_B - x_A} = -1$

$(2x_0 - 1) \cdot \dfrac{f(2) - f(0)}{2 - 0} = -1$

$M(0,2)$

$(2x_0 - 1) \cdot \dfrac{4 - 2}{2} = -1$

$2x_0 - 1 = -1 \Rightarrow x_0 = 0$

γ) $y - f(x_0) = f'(x_0)(x - x_0) \parallel x'x$

$f'(x_0) = 0$

$2x_0 - 1 = 0$

$x_0 = \frac{1}{2}$

$M\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$.

12. $f(x) = \alpha x^3 + \beta x - 1$ $\qquad$ $f'(x) = 3\alpha x^2 + \beta$

$y - f(1) = f'(1)(x-1) \perp 3x - y + 1 = 0$

$\Rightarrow f'(1) \cdot 3 = -1$

$(3\alpha + \beta) \cdot 3 = -1$

$\boxed{9\alpha + 3\beta = -1}$

$M(1, -1) \Rightarrow f(1) = -1.$

$\alpha + \beta - 1 = -1$

$\alpha = -\beta$

$-9\beta + 3\beta = -1$

$-6\beta = -1$

$\beta = \frac{1}{6} \qquad \alpha = -\frac{1}{6},$

13. $f(x) = \alpha x + \frac{\beta}{x}$

$y - f(1) = f'(1)(x-1) \parallel x'x \Rightarrow \underline{\underline{f'(1) = 0}}$

$y - f(2) = f'(2)(x-2)$ γωνία με $x'x$ $\frac{3\pi}{4}$.

$f'(2) = \varepsilon\varphi \frac{3\pi}{4} = \varepsilon\varphi 135 = -1$.

$\underline{\underline{f'(2) = -1}}$

$f'(x) = \alpha - \frac{\beta}{x^2}$

$f'(1) = \boxed{\alpha - \beta = 0} \Rightarrow \underline{\underline{\alpha = \beta}}$

$f'(2) = \alpha - \frac{\beta}{4} = -1$

$\alpha - \frac{\alpha}{4} = -1$

$4\alpha - \alpha = -4$

$\Rightarrow \alpha = -\frac{4}{3}$. $\beta = -\frac{4}{3}$

14. (A) $f(x) = x^3 + ax^2 - x - 1$ $\quad f'(x) = 3x^2 + 2ax - 1$

$y - f(0) = f'(0)(x - 0) \perp y - f(1) = f'(1)(x - 1)$

$f'(0)\ f'(1) = -1$

$-1 \cdot (3 + 2a - 1) = -1$

$2 + 2a = 1$

$2a = -1$

$a = -\frac{1}{2}.$

(B) $f'(0) = -1$

$\omega = 135$.

$f'(1) = 3 + 2a - 1 = 2 + 2a = 2 + 2\left(-\frac{1}{2}\right)$

$= 2 - 1 = 1$

45°

15. $f(x) = \alpha \ln x + \beta x^2 - \ln\alpha$.

$y - f(1) = f'(1)(x-1)$

$\underline{\underline{f'(1) = 3}}$

$f'(x) = \dfrac{\alpha}{x} + 2\beta x$

$f'(1) = \underline{\underline{\alpha + 2\beta = 3}}$

$\underline{\underline{f(1) = 1}}$

$\beta - \ln\alpha = 1$

$\beta = 1 + \ln\alpha$.

$\alpha + 2(1 + \ln\alpha) = 3$

$\alpha + 2 + 2\ln\alpha = 3$

$\underline{\underline{\alpha + 2\ln\alpha - 1 = 0}} \Rightarrow$

$\varphi(\alpha) = 0$

$\varphi(\alpha) = \varphi(1)$

φ 1-1

$\underline{\underline{\alpha = 1}}$

$\underline{\underline{\beta = 1}}$

$\varphi(x) = x + 2\ln x - 1$

$\varphi \nearrow$

16. $f(x) = x^4 - 14x^2 + 24x$

$f'(x) = 4x^3 - 28x + 24$

$f'(x) = 4(x^3 - 7x + 6)$.

$f'(x) = 0 \Rightarrow x^3 - 7x + 6 = 0$

$$\begin{array}{cccc|c} 1 & 0 & -7 & 6 & 1 \\ \downarrow & 1 & 1 & -6 & \\ \hline 1 & 1 & -6 & 0 & \end{array}$$

$(x-1)(x^2 + x - 6) = 0$

$x = 1$ $\quad x = -3$ $\quad x = 2$

Εψαχνα να δειξω οτι η $f'(x)$ εχει 3 ριζες και τις βρηκα.

17. $f(x) = e^x(x^2+2)$

Αρκει νδο η $f'(x) \neq 0$

$f'(x) = e^x(x^2+2) + e^x 2x$

$f'(x) = e^x(\overset{+}{x^2+2x+2}) > 0$

$\Delta < 0$

$f'(x) > 0$

$\Rightarrow f'(x) \neq 0.$

Ενότητα 23

2. ⓐ $f(x) = e^x + x^3 - 1$

$f'(x) = e^x + 3x^2 > 0$

$f \nearrow$

ⓨ $f(x) = -x^3 + x^2 - x + 1$

$f'(x) = -3x^2 + 2x - 1 < 0$

$\Delta = 4 - 4 \cdot (-3)(-1)$

$\Delta = 4 - 12 < 0$

$f \searrow$.

ⓔ $f(x) = 1 + (x-2)^3$

$f'(x) = 3(x-2)^2 \geq 0$

$f \nearrow$

6. E $f(x) = (x-1)^3 \cdot (x-3)^3$

$$f'(x) = 3(x-1)^2(x-3) + (x-1)^3\, 3(x-3)^2$$

$$f'(x) = 3(x-1)^2(x-3)^2(x-3+x-1)$$

$$f'(x) = 3(x-1)^2(x-3)^2(2x-4)$$

2

x		2	
f'	$-$	0	$+$
f	$\searrow$		$\nearrow$

1. ⓐ $f(x) = \dfrac{x}{x+1}$, $x \neq -1$.

$$f'(x) = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2} > 0$$

x		-1	
$f(x)$	$\nearrow$	‖	$\nearrow$

ⓑ $f(x) = \dfrac{x}{x^2+1}$ $D_f = \mathbb{R}$.

$$f'(x) = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

x		-1		1	
f'	$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$

ⓓ $f(x) = x + \dfrac{1}{x}$, $x \neq 0$

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x}$$

x		-1		0		1	
f'	$+$		$-$	‖	$-$		$+$
f	$\nearrow$		$\searrow$	‖	$\searrow$		$\nearrow$

(ε) $f(x) = x^2 + \frac{2}{x}, \ x \neq 0$

$$f'(x) = 2x - \frac{2}{x^2} = \frac{2x^3-2}{x^2} = 2\,\frac{x^3-1}{x^2}$$

x		0		1	
f'	$-$	$\|$	$-$		$+$
f	$\searrow$	$\|$	$\searrow$		$\nearrow$

8. ⓐ $f(x) = \dfrac{x}{(x-1)^2}$, $x \neq 1$

$$f'(x) = \frac{(x-1)^2 - 2x(x-1)}{(x-1)^4}$$

$$f'(x) = \frac{x^2 - \cancel{2x} + 1 - 2x^2 + \cancel{2x}}{(x-1)^4}$$

$$\boxed{f'(x) = \frac{-x^2+1}{(x-1)^4}}$$

x		-1		1	
f'	$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$

γ) $f(x) = \dfrac{2-x^2}{x^4}$, $x \neq 0$

$$f'(x) = \frac{-2x \cdot x^4 - (2-x^2)4x^3}{x^8}$$

$$f'(x) = \frac{-2x^5 - 4x^3(2-x^2)}{x^8}$$

$$f'(x) = \frac{-2x^5 - 8x^3 + 4x^5}{x^8}$$

$$f'(x) = \frac{2x^5 - 8x^3}{x^8} = \frac{2x^3(x^2-4)}{x^8}$$

x	-2		0		2	
$2x^3$	$-$	$-$	‖	$+$		$+$
x^2-4	$+$	$-$	‖	$-$		$+$
f'	$-$	$+$	‖	$-$		$+$
f	↘	↗	‖	↘		↗

Εποµαιο Μαθηµ

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