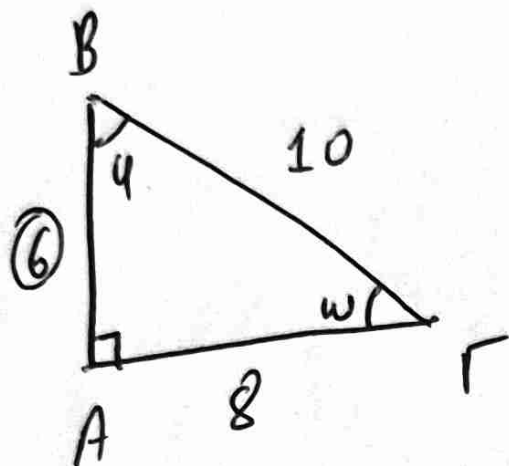


# Ενότητα 9

1.



$$AB^2 + A\Gamma^2 = B\Gamma^2$$

$$AB^2 + 8^2 = 10^2$$

$$AB^2 = 100 - 64$$

$$AB^2 = 36$$

$$AB = 6$$

$$\eta_{P\omega} = \frac{6}{10}$$

$$\sigma_{\omega\omega} = \frac{8}{10}$$

$$\epsilon_{\psi\omega} = \frac{6}{8}$$

$$\sigma_{\psi\omega} = \frac{8}{6}$$

$$\eta_{P\psi} = \frac{8}{10}$$

$$\sigma_{\omega\psi} = \frac{6}{10}$$

$$\epsilon_{\psi\psi} = \frac{8}{6}$$

$$\sigma_{\psi\psi} = \frac{6}{8}$$

$$11. \quad 0^\circ \longrightarrow 0 \text{ rad}$$

$$135^\circ \longrightarrow \frac{3\pi}{4}$$

$$210^\circ \longrightarrow \frac{7\pi}{6}$$

$$780^\circ \longrightarrow \frac{13\pi}{3}$$

$$-1125^\circ \longrightarrow -\frac{25\pi}{4}$$

$$12. \quad (a) \quad \frac{2\pi}{3} \longrightarrow 120$$

$$(b) \quad \frac{7\pi}{6} \longrightarrow 210$$

$$(c) \quad \frac{\pi}{16} \longrightarrow \frac{45}{4}$$

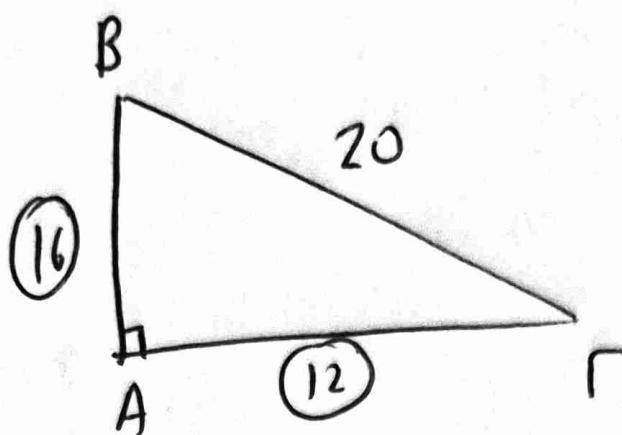
$$14. \quad (a) \quad A = 0 + 1 - 0 = 1$$

$$(b) \quad B = 1 - 0 = 1$$

$$(c) \quad C = 0 + (-1) - 0 = -1$$

$$(d) \quad \underline{\underline{D = -1}}$$

2.



$$\sin B = \frac{3}{5}$$

$$\sin B = \frac{AC}{BC}$$

$$\frac{3}{5} = \frac{AC}{20} \quad (\Rightarrow) \quad 60 = 5AC$$

$$\underline{\underline{AC = 12}}$$

$$12^2 + AB^2 = BC^2$$

$$144 + AB^2 = 400$$

$$AB^2 = 256$$

$$\underline{\underline{AB = 16}}$$

$$\sin B = \frac{12}{20}$$

$$\cos B = \frac{16}{20}$$

$$\tan B = \frac{12}{16}$$

$$\cot B = \frac{16}{12}$$

# Βασικές Τριγωνομετρικές Ταυτότητες

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$$1. \quad \eta\rho^2 x + \sigma\omega^2 x = 1.$$

$$2. \quad \varepsilon\varphi x = \frac{\eta\rho x}{\sigma\omega x}$$

$$3. \quad \sigma\varphi x = \frac{\sigma\omega x}{\eta\rho x}$$

$$4. \quad \varepsilon\varphi x \cdot \sigma\varphi x = 1.$$

$$5. \quad \sigma\omega^2 x = \frac{1}{1 + \varepsilon\varphi^2 x}$$

$$6. \quad \eta\rho^2 x = \frac{\varepsilon\varphi^2 x}{1 + \sigma\varphi^2 x}$$

$$-1 \leq \sigma\omega x \leq 1$$

$$-1 \leq \eta\rho x \leq 1$$

# Άσκηση 1

Εστω

$$\sigma \omega x = -\frac{4}{5}$$

και  $\frac{\pi}{2} < x < \pi$

$$\eta \rho^2 x + \sigma \omega^2 x = 1$$

$$\eta \rho^2 x + \left(-\frac{4}{5}\right)^2 = 1$$

$$\eta \rho^2 x + \frac{16}{25} = 1$$

$$\eta \rho^2 x = 1 - \frac{16}{25}$$

$$\eta \rho^2 x = \frac{25}{25} - \frac{16}{25}$$

$$\Rightarrow \eta \rho^2 x = \frac{9}{25}$$

$$\eta \rho x = \frac{3}{5}$$

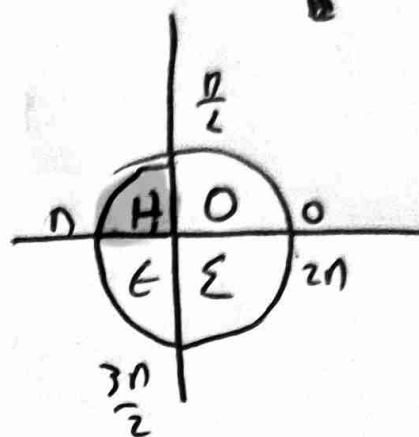
~~$$\eta \rho x = -\frac{3}{5}$$~~

$$\epsilon \varphi x = \frac{\eta \rho x}{\sigma \omega x}$$

$$\epsilon \varphi x = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$$

$$\Rightarrow \epsilon \varphi x = -\frac{3}{4}$$

$$\sigma \varphi x = -\frac{4}{3}$$



# Ασκηση 2

$$\sigma_{TW} \quad \sigma_{\psi X} = -\frac{5}{12}$$

$$\varepsilon_{\psi X} = -\frac{12}{5}$$

$$\sigma_{W^2 X} = \frac{1}{1 + \varepsilon_{\psi^2 X}}$$

$$\sigma_{W^2 X} = \frac{1}{1 + \left(-\frac{12}{5}\right)^2}$$

$$\sigma_{W^2 X} = \frac{1}{1 + \frac{144}{25}} = \frac{1}{\frac{25}{25} + \frac{144}{25}} = \frac{1}{\frac{169}{25}}$$

$$\sigma_{W^2 X} = \frac{25}{169}$$

$$\sigma_{WX} = \frac{5}{13}$$

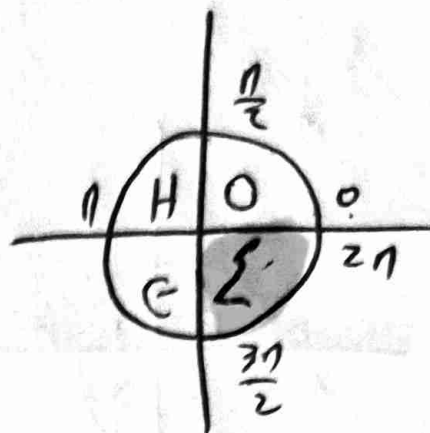
~~$$\sigma_{WX} = \frac{5}{13}$$~~

$$\varepsilon_{\psi X} = \frac{\eta_{\psi X}}{\sigma_{WX}}$$

$$\Rightarrow -\frac{12}{5} = \frac{\eta_{\psi X}}{\frac{5}{13}} \Rightarrow \eta_{\psi X} = -\frac{12}{5} \cdot \frac{5}{13}$$

$$\eta_{\psi X} = -\frac{12}{13}$$

$$\frac{3\pi}{2} < X < 2\pi$$

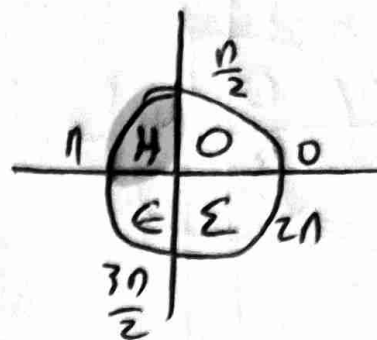


# Ασκηση 3

Έστω ότι  $2\sigma\omega^2x + 5\eta\mu x - 4 = 0$

και

$$0 < x < \frac{\pi}{2}$$



Γνωρίζουμε ότι  $\eta\mu^2x + \sigma\omega^2x = 1$

$$\sigma\omega^2x = 1 - \eta\mu^2x$$

$$2(1 - \eta\mu^2x) + 5\eta\mu x - 4 = 0$$

$$2 - 2\eta\mu^2x + 5\eta\mu x - 4 = 0$$

$$-2\eta\mu^2x + 5\eta\mu x - 2 = 0$$

Θέτω  $\eta\mu x = t$

$$-2t^2 + 5t - 2 = 0$$

$$\Delta = B^2 - 4\alpha\gamma$$

$$\Delta = 25 - 4(-2)(-2)$$

$$\Delta = 25 - 16 \quad \Delta = 9$$

$$t = \frac{-B \pm \sqrt{\Delta}}{2\alpha}$$

$$t = \frac{-5 \pm 3}{-4} \begin{matrix} \nearrow \frac{1}{2} \\ \searrow 2 \end{matrix}$$

$$t = \frac{1}{2}$$

↪

$$t = 2$$

$$\psi x = \frac{1}{2}$$

↪

$$\psi x = 2$$

Atom!

$$\psi^2 x + \sigma \omega^2 x = 1$$

$$\frac{1}{4} + \sigma \omega^2 x = 1$$

$$\sigma \omega^2 x = 1 - \frac{1}{4}$$

$$\sigma \omega^2 x = \frac{4}{4} - \frac{1}{4}$$

$$\sigma \omega^2 x = \frac{3}{4}$$

~~$$\sigma \omega x = \frac{\sqrt{3}}{2}$$~~

$$\sigma \omega x = -\frac{\sqrt{3}}{2}$$

$$\epsilon \psi x = \frac{\psi x}{\sigma \omega x} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$\epsilon \psi x = -\frac{1}{\sqrt{3}}$$

$$\sigma \psi x = -\sqrt{3}$$



# Ασκηση 4

Υπαρχει γωνια  $x$  ωστε

$$\eta\mu x = \frac{1}{2} \quad \text{και} \quad \sigma\mu x = \frac{1}{3}$$

Γνωριζω οτι  $\eta^2 x + \sigma^2 x = 1$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 = 1$$

$$\frac{1}{4} + \frac{1}{9} = 1$$

$$\frac{9}{36} + \frac{4}{36} = 1;$$

$$\frac{13}{36} \neq 1$$

Δα υπαρχει Τεωρια γωνι!

# Ενότητα 10

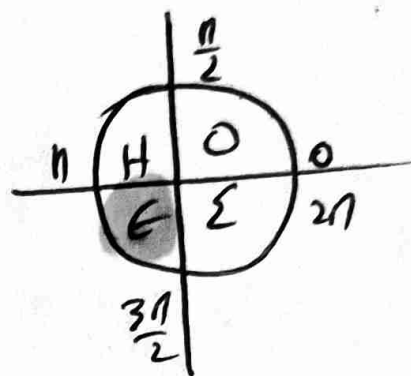
7.  $2\epsilon\psi^2x + \frac{3}{\sigma\omega^2x} = 8$

$$\pi < x < \frac{3\pi}{2}$$

Γνωρίζω ότι  $\sigma\omega^2x = \frac{1}{1+\epsilon\psi^2x}$

$$A = \epsilon\psi x - \sqrt{2}\sigma\omega x + \sigma\psi x$$

$$2\epsilon\psi^2x + \frac{3}{\frac{1}{1+\epsilon\psi^2x}} = 8$$



$$2\epsilon\psi^2x + 3(1+\epsilon\psi^2x) = 8$$

$$2\epsilon\psi^2x + 3 + 3\epsilon\psi^2x = 8$$

$$5\epsilon\psi^2x = 8 - 3$$

$$5\epsilon\psi^2x = 5$$

$$\epsilon\psi^2x = 1$$

$$\epsilon\psi x = 1$$

~~$$\epsilon\psi x = -1$$~~

$$\varepsilon\psi_X = 1$$

$$\sigma\omega^2 X = \frac{1}{1 + \varepsilon\psi^2 X}$$

$$\sigma\omega^2 X = \frac{1}{1+1} = \frac{1}{2}$$

~~$$\sigma\omega X = \sqrt{\frac{1}{2}}$$~~

$$\Rightarrow \sigma\omega X = -\sqrt{\frac{1}{2}}$$

$$\sigma\omega X = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{\sqrt{2}X}$$

$$\sigma\omega X = -\frac{\sqrt{2}}{2}$$

$$\varepsilon\psi_X = \frac{\eta\psi_X}{\sigma\omega X}$$

$$1 = \frac{\eta\psi_X}{-\frac{\sqrt{2}}{2}}$$

$$\Rightarrow \eta\psi_X = -\frac{\sqrt{2}}{2}$$

# Ενορρω Μαθημα

9.

(3)

(7)

(8)

(15)

(16)

(17)

(18)

(20) α

(22) α Β ε σζ.

10

(1)

(2)