Πρόσημο Τριωνύμου

1. $\alpha x^2+\beta x+\gamma$, $\alpha > 0$

$\Delta > 0$

x		x_1		x_2	
$\alpha x^2+\beta x+\gamma$	$+$	0	$-$	0	$+$

εκτός των ριζών ομόσημο του α.

2. $\alpha x^2+\beta x+\gamma$

$\alpha > 0$

$\Delta = 0$

x		x_1	
$\alpha x^2+\beta x+\gamma$	$+$	0	$+$

ομόσημο του σε παντού

3. $\alpha x^2+\beta x+\gamma$

$\alpha > 0$

$\Delta < 0$

x	
$\alpha x^2+\beta x+\gamma$	$+$

ομόσημο του α παντού

Ανισώσεις

Άσκηση 1

(α) $x^2 - 6x + 5 < 0$

x		1		5
x^2-6x+5	$+$	$-$	$+$	

$x \in (1, 5)$.

(β) $x^2 + x + 1 > 0$

$\Delta < 0$

x	
x^2+x+1	$+$

$x \in \mathbb{R}$.

(γ) $x^2 - 1 > 0$

$(x-1)(x+1) > 0$

$1 \quad -1$

x		-1		1	
x^2-1	$+$	$-$	$+$		

$x \in (-\infty, -1) \cup (1, +\infty)$

(8) $4x^2 - 4x + 1 \leq 0$

$\Delta = 0$

$x = \frac{1}{2}$

x		$\frac{1}{2}$	
$4x^2-4x+1$	+	0	+

Λύση: $x = \frac{1}{2}$

(9) $x^2 - 2x + 1 < 0$

$\Delta = 0$

$x = 1$

x		1	
x^2-2x+1	+		+

Αδύνατη.

(7) $x^2 + 2x + 1 > 0$

$\Delta = 0$

$x = -1$

x		-1	
x^2+2x+1	+		+

$x \in (-\infty, -1) \cup (-1, +\infty)$

ή $x \in \mathbb{R} - \{-1\}$

1.

(8) $4x^2 - 4$

$4(x^2 - 1)$

$4(x-1)(x+1)$

$x = 1$ $\quad x = -1$

x	-1		
$4x^2-4$	$+$	$-$	$+$

(29) $x - 2x^2 = 0$

$x(1-2x)$

$x = 0$ $\quad 1 - 2x = 0$

$-2x = -1$

$x = \frac{1}{2}$

x	0		$\frac{1}{2}$	
$x-2x^2$	$-$	$+$	0	$-$

2. (β) $-x^2+2x-1$

$\Delta = 4 - 4(-1)(-1) = 0$

$\boxed{\Delta = 0}$

$x = \frac{-\beta}{2a} = \frac{-2}{2} = -1$

x		-1	
$-x^2+2x-1$	$-$	ϕ	$-$

(δ) $-2x^2+x-1$

$\Delta = \beta^2 - 4\alpha\gamma = 1 - 4\cdot(-2)\cdot(-1) = -8$

$\Delta = -8$

x	
$-2x^2+x-1$	$-$

1.

б) $-x^2+x+2$

$\Delta = b^2 - 4ac$

$\Delta = 1^2 - 4 \cdot (-1) \cdot 2$

$\Delta = 1 + 8$

$\Delta = 9 > 0$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$

$x_{1,2} = \frac{-1 \pm 3}{-2}$ — -1

\ 2

x	$-\infty$		-1		2		$+\infty$
$-x^2+x+2$		$+$		$-$		$+$	

г) $-x^2+9$

$\Delta = b^2 - 4ac$

$\Delta = 0^2 - 4 \cdot (-1) \cdot 9$

$\Delta = 36$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$

$x_{1,2} = \frac{0 \pm \sqrt{36}}{-2}$ — 3

\ -3

x	$-\infty$		-3		3		$+\infty$
$-x^2+9$		$-$		$+$		$-$	

2. (52) $-x^2-1$

$a=-1 \quad b=0 \quad y=-1$

$\Delta = b^2 - 4ay = 0 - 4(-1)(-1) = -4$

$x \in$

x	
$-x^2-1$	$-$

4. ⓑ $(x-1)^2 \leq x+1$

$$x^2 - 2x + 1 - x - 1 \leq 0$$

$$x^2 - 3x \leq 0$$

$\Delta = b^2 - 4ac$

$\Delta = (-3)^2 - 4 \cdot 1 \cdot 0$

$\Delta = 9 - 0$

$\Delta = 9 > 0$

$x_{1,2} = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$x_{1,2} = \dfrac{3 \pm 3}{2}$ — 3 — 0

x	$-\infty$		0		3		$+\infty$
x^2-3x		$+$	0	$-$	0	$+$	

$x \in [0, 3]$

ⓒ $4x^2 > 9$

$4x^2 - 9 > 0$

$(2x-3)(2x+3) > 0$

$2x-3=0$ ou $2x+3=0$

$2x=3$ ou $2x=-3$

$x=\dfrac{3}{2}$ ou $x=-\dfrac{3}{2}$

x	$-\infty$		$-\frac{3}{2}$		$\frac{3}{2}$		$+\infty$
$4x^2-9$		$+$	0	$-$	0	$+$	

$x \in (-\infty, -\frac{3}{2}) \cup (\frac{3}{2}, +\infty)$

6. (B) $4x^2 > 4x - 1 \Leftrightarrow$

$4x^2 - 4x + 1 \Leftrightarrow$

$\Delta = \beta^2 - 4\alpha\gamma = 16 - 4 \cdot 4$

$\boxed{\Delta = 0}$

$\frac{-\beta}{2a} = \frac{4}{8} = \frac{1}{2}$

$a > 0$

$\Delta = 0$

x	x_1	
$4x^2 - 4x + 1$	$+$	$+$

$x \in \mathbb{R}$

(δ) $-x^2 < 2x + 1$

$-x^2 - 2x - 1 < 0$ $\quad a$

$\Delta = \beta^2 - 4\alpha\gamma = 4 - 4 \cdot (-1) \cdot (-1) = 0$

$\boxed{\Delta = 0}$

$x = \frac{-\beta}{2a} = \frac{2}{-2} = -1$

x	-1	
$-x^2 - 2x - 1$	$-$	$-$

$x \in (-\infty, -1) \cup (-1, +\infty)$.

7. (8) $3x - 1 < 5x^2$

$3x - 1 - 5x^2 < 0$

$-5x^2 - 3x - 1 < 0$

$\Delta = b^2 - 4ac$

$\Delta = (-3)^2 - 4 \cdot (-5) \cdot (-1)$

$\Delta = 9 - 20$

$\Delta = -11$

x	
$-5x^2 - 3x - 1$	$-$

$x \in R$

8. $f(x) = 3x^2 - 7x + 2$

(α) Ρίζες.

$\Delta = \beta^2 - 4\alpha\gamma = 49 - 4 \cdot 3 \cdot 2 = 49 - 24 = 25$

$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha} = \frac{7 \pm 5}{6}$ $\begin{matrix} x_1 = 2 \\ x_2 = \frac{1}{3} \end{matrix}$

(β) Παραγοντοποίηση

$3x^2 - 7x + 2 = 3(x-2)(x-\frac{1}{3})$

(γ) Πίνακας προσήμων

x		$\frac{1}{3}$		2	
$3x^2-7x+2$	$+$	0	$-$	0	$+$

$x \in (\frac{1}{3}, 2)$.

(δ) Ανίσωση $3x^2 < 7x - 2$

$3x^2 - 7x + 2 < 0$

$x \in (\frac{1}{3}, 2)$.

Ενότητα 19

3. (Β) $3x^2 - x < 2$

$3x^2 - x - 2 < 0$

$\Delta = b^2 - 4ac$

$(-1)^2 - 4 \cdot 3 \cdot (-2)$

$1 + 24$

$\Delta = 25$

$25 > 0$

$\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm 5}{2 \cdot 3}$ → 1, $\frac{-4}{6} = -\frac{2}{3}$

x	$-\frac{2}{3}$		1	
$3x^2 - x - 2$	+	−	+	

$x \in (-\frac{2}{3}, 1)$

(Ε) $(x-1)^2 > 5 - x(2x+1)$

$(x-1)^2 - 5 + x(2x+1) > 0$.

$x^2 - 2x + 1 - 5 + 2x^2 + x > 0$

$3x^2 - x - 4 > 0$

$a = 3 \quad b = -1 \quad \gamma = -4$

$\Delta = b^2 - 4ac$

$\Delta = (-1)^2 - 4 \cdot 3 \cdot (-4)$

$\Delta = 1 + 48$

$\Delta = 49$

$49 > 0$

$\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm 7}{2 \cdot 3}$ → $\frac{8}{6}$, -1

x	-1		$\frac{8}{6}$	
$3x^2 - x - 4$	−	+	−	

$x \in (-1, \frac{8}{6})$.

κοινές λύσεις ανισώσεων

$x^2 - 4x + 3 \leq 0$

$\Delta = 16 - 12 = 4$

$x = \frac{4 \pm 2}{2}$ → (3), (1)

x		1		3
x^2-4x+3	$+$	0	$-$	0 $+$

$x \in [1, 3]$

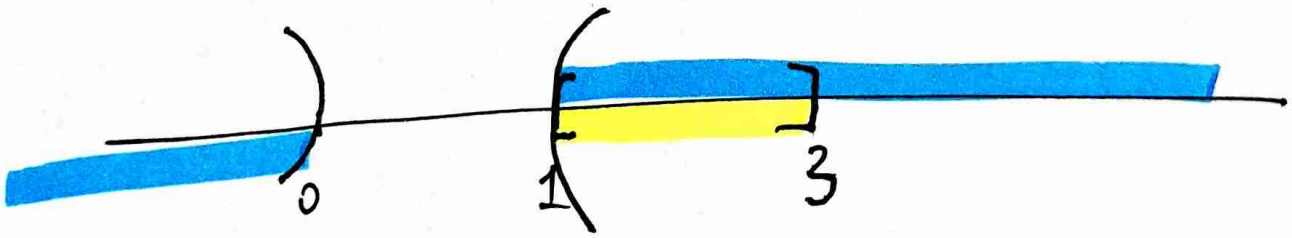
$x^2 - x > 0$

$x(x-1) > 0$

$0 \quad 1$

x		0		1
x^2-x	$+$	0	$-$	0 $+$

$x \in (-\infty, 0) \cup (1, +\infty)$



κοινές: $x \in (1, 3]$.

Εποχαο Μαθημα

19

(1) α ε

(2) α γ ε

(3) α γ δ σ τ

(4) α β δ

(5) α β γ

(6) α γ

(7) α β