

κανόνες διατάξης

1. $\forall \alpha < \beta \Leftrightarrow \alpha - \beta < 0$

2. $\forall \alpha > \beta \Leftrightarrow \alpha - \beta > 0$

3. $\forall \alpha < \beta \text{ και } \beta < \gamma \text{ τότε } \alpha < \gamma$

4. $\forall \alpha < \beta \Leftrightarrow \alpha + \gamma < \beta + \gamma$

5. $\forall \alpha < \beta \Leftrightarrow \alpha \cdot \gamma < \beta \cdot \gamma$
(μόνο αν $\gamma > 0$)

6. $\forall \alpha < \beta \Leftrightarrow \alpha \gamma > \beta \gamma$
(μόνο αν $\gamma < 0$)

$$7. \quad \forall \alpha < \beta \quad (\Rightarrow) \alpha^3 < \beta^3$$

$$8. \quad \forall \alpha < \beta \quad (\Rightarrow) \alpha^2 < \beta^2$$

(αν α, β θετικοί)

$$9. \quad \forall \alpha < \beta \quad (\Rightarrow) \alpha^2 > \beta^2$$

(αν α, β αρνητικοί)

$$10. \quad \forall \alpha < \beta \quad (\Rightarrow) \frac{1}{\alpha} > \frac{1}{\beta}$$

(αρκι α, β ομοσημ.)

Πράξεις ανισοτήτων

$$1. \left. \begin{array}{l} \alpha < \beta \\ \gamma < \delta \end{array} \right\} \oplus \alpha + \gamma < \beta + \delta$$

Μπορώ να προσθέτω αψοβα
ανισοτιτελ,

$$2. \left. \begin{array}{l} \alpha < \beta \\ \gamma < \delta \end{array} \right\} \odot \alpha \gamma < \beta \delta$$

Μπορώ να πολλαπλασιάζω ανισοτιτελ

$\ominus \in \mathbb{T} / \mathbb{K} \in \mathbb{Z}$

3. Δα αφαίρεση

Δα διαίρεση

κατά μέλη

ανωστρεφής



Σενάρια

1. Αν $a < -3$ να αποδείχθου ότι

$$2. [5(a-3) - 2(a-9)] < -3(1-a)$$

$$2. [5a - 15 - 2a + 18] < -3 + 3a$$

$$2. [3a + 3] < -3 + 3a$$

$$6a + 6 < -3 + 3a$$

$$6a - 3a < -3 - 6$$

$$3a < -9$$

$$a < -3 \quad \text{που ισχύει!}$$

2. $\forall \alpha > B > 1 \quad \forall \alpha \quad B(\alpha+1) > \alpha + B^2$

$$B\alpha + B > \alpha + B^2$$

$$B\alpha + B - \alpha - B^2 > 0$$

$$\alpha(B-1) + B(1-B) > 0$$

$$\alpha(B-1) - B(B-1) > 0$$

$$(B-1)^{\oplus} (\alpha-B)^{\oplus} > 0 \quad \checkmark$$

- $B > 1 \Rightarrow B-1 > 0$

- $\alpha > B \Rightarrow \alpha - B > 0$

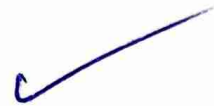
3. Ndo $2(x^2+9) \geq (x+3)^2$

$$2x^2+18 \geq x^2+6x+9$$

$$2x^2+18 - x^2 - 6x - 9 \geq 0$$

$$x^2 - 6x + 9 \geq 0$$

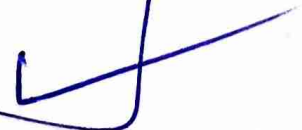
$$(x-3)^2 \geq 0$$



4. Ndo $2x^2 - 6x + 9 > 0$

$$x^2 + x^2 - 6x + 9 > 0$$

$$x^2 + (x-3)^2 > 0$$



5. Ndo $x^2 + 2x + 5 > 0$

$$x^2 + 2x + 1 + 4 > 0$$

$$\boxed{(x+1)^2 + 4 > 0} \quad \checkmark$$

6. Ndo $\frac{x}{x^2+1} \leq \frac{1}{2}$

$$2x \leq x^2 + 1$$

$$0 \leq x^2 - 2x + 1$$

$$0 \leq (x-1)^2$$

✓

7. Av $x \neq 1$ vđo $\frac{x^3-2}{x-1} > \frac{x^2-2x}{x-1}$

$$\frac{x^3-2}{x-1} - \frac{x^2-2x}{x-1} > 0$$

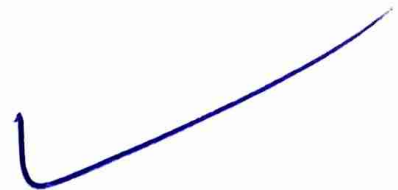
$$\frac{x^3-2-x^2+2x}{x-1} > 0$$

$$\frac{x^3-x^2+2x-2}{x-1} > 0$$

$$\frac{x^2(x-1)+2(x-1)}{x-1} > 0$$

$$\frac{\cancel{(x-1)}(x^2+2)}{\cancel{x-1}} > 0$$

$$x^2+2 > 0$$



$$8. \text{ Nfdo } aB \leq \frac{a^2+B^2}{2} \leq \frac{a^3-B^3}{a-B}$$

$$\underline{\underline{\text{or } a \neq B}}$$

$$aB \leq \frac{a^2+B^2}{2}$$

$$\text{kan } \frac{a^2+B^2}{2} \leq \frac{a^3-B^3}{a-B}$$

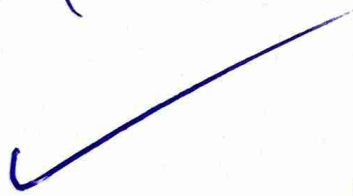
$$2aB \leq a^2+B^2$$

$$\frac{a^2+B^2}{2} - \frac{a^3-B^3}{a-B} \leq 0$$

$$0 \leq a^2 - 2aB + B^2$$

$$0 \leq (a-B)^2$$

$$\frac{(a^2+B^2)/(a-B)}{2(a-B)} - \frac{2(a^3-B^3)}{2(a-B)} \leq 0$$



$$\frac{(a^2+B^2)/(a-B) - 2(a^3-B^3)}{2(a-B)} \leq 0$$

$$\frac{(a^2+b^2)(a-b) - 2(a-b)(a^2+ab+b^2)}{2(a-b)} \leq 0$$

$$\frac{a^2+b^2 - 2(a^2+ab+b^2)}{2} \leq 0$$

$$\frac{a^2+b^2 - 2a^2 - 2ab - 2b^2}{2} \leq 0$$

$$\frac{-a^2 - 2ab - b^2}{2} \leq 0$$

$$\frac{a^2 + 2ab + b^2}{2} \geq 0 \Rightarrow \frac{(a+b)^2}{2} \geq 0 \quad \checkmark$$

Εξασκηση

1. Να δο $(a+b)^2 \geq 4ab$

$$a^2 + 2ab + b^2 \geq 4ab$$

$$a^2 + 2ab + b^2 - 4ab \geq 0$$

$$a^2 - 2ab + b^2 \geq 0$$

$$(a-b)^2 \geq 0.$$

2. Να δο $(a+3b)^2 \geq 2b(5a+2b).$

$$a^2 + 6ab + 9b^2 \geq 10ab + 4b^2$$

$$a^2 - 4ab + 5b^2 \geq 0.$$

$$a^2 - 4ab + 4b^2 + b^2 \geq 0$$

$$(a-2b)^2 + b^2 \geq 0$$

$$3. \text{ No } (a-b)^2 + 8B^2 \geq 4aB$$

$$a^2 - 2aB + B^2 + 8B^2 - 4aB \geq 0$$

$$a^2 - 6aB + 9B^2 \geq 0$$

$$(a - 3B)^2 \geq 0,$$

$$4. \text{ No } \left(\frac{a+B}{2}\right)^2 \geq aB$$

$$\frac{(a+B)^2}{2^2} \geq aB$$

$$\frac{a^2 + 2aB + B^2}{4} \geq aB$$

$$a^2 + 2aB + B^2 \geq 4aB$$

$$a^2 - 2aB + B^2 \geq 0 \Rightarrow (a-B)^2 \geq 0,$$

$$5. \text{ Ndo } 3(a^2 - b^2) + 2ab \geq -2(a+2b)^2$$

$$3a^2 - 3b^2 + 2ab \geq -2(a^2 + 4ab + 4b^2)$$

$$3a^2 - 3b^2 + 2ab \geq -2a^2 - 8ab - 8b^2$$

$$5a^2 + 10ab + 5b^2 \geq 0$$

$$a^2 + 2ab + b^2 \geq 0$$

$$(a+b)^2 \geq 0.$$

$$6. \text{ Ndo } a^2 + b^2 - 4a + 6b \geq -13.$$

$$a^2 - 4a + 4 + b^2 + 6b + 9 \geq 0$$

$$(a-2)^2 + (b+3)^2 \geq 0$$

7. Ndo $(a^2+b^2)(x^2+y^2) \geq (ax+by)^2$

$$\cancel{a^2x^2} + a^2y^2 + b^2x^2 + \cancel{b^2y^2} \geq \cancel{a^2x^2} + 2axby + \cancel{b^2y^2}$$

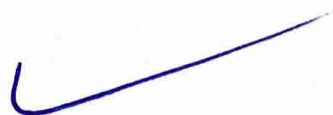
$$a^2y^2 + b^2x^2 - 2axby \geq 0$$

$$(ay - bx)^2 \geq 0,$$

8. Ndo $13a^2 - 12a + 13 > 0$

$$4a^2 + 9a^2 - 12a + 4 + 9 > 0$$

$$4a^2 + (3a-2)^2 + 9 > 0$$

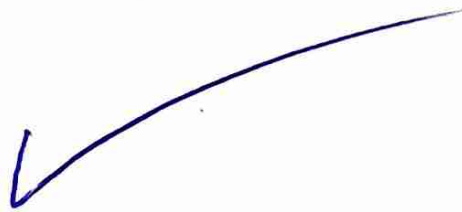


9. Ndo $x^2 + \frac{1}{x^2+1} \geq 1$

$$(x^2+1)x^2 + \cancel{(x^2+1)} \frac{1}{\cancel{x^2+1}} \geq x^2+1$$

$$\cancel{x^4} + \cancel{x^2} + \cancel{1} \geq \cancel{x^2} + \cancel{1}$$

$$x^4 \geq 0$$



Επόμενο μάθημα

Ημερομηνία μαθήματος: 25/9/26

Ώρα μαθήματος: 17:30.

Ασκήσεις

Ενοστά 7

(3)	(21)	(43)
(5)	(22)	
(19)	(23)	
(20)	(24)	
	(25)	

παρατηρήσεις