Ενοτητα 4

3. $f(x) = x^5 + 2x - 3$

(α) $x_1 < x_2 \Rightarrow x_1^5 < x_2^5$

$x_1 < x_2 \Rightarrow 2x_1 - 3 < 2x_2 - 3$

$\left.\right\}$ (+)

$f(x_1) < f(x_2)$ $\quad f \nearrow$

(Β) $4 > \pi$

$f \nearrow$

$f(4) > f(\pi)$

9.

(α) $f(x) > f(2) \overset{f\downarrow}{\Rightarrow} \underline{\underline{x < 2}}$

(β) $f(x^2-3x) - f(6-4x) < 0$

$$f(x^2-3x) < f(6-4x)$$

$$f\downarrow$$

$$x^2-3x > 6-4x$$

$$x^2+x-6 > 0$$

x		-3		2	
x^2+x-6	$+$	|	$-$	|	$+$

$$x \in (-\infty, -3) \cup (2, +\infty)$$

(γ) $f(x^4+2) \geq f(x^2+2)$

$$f\downarrow$$

$$x^4+2 \leq x^2+2$$

$$x^4 \leq x^2$$

$$x^4 - x^2 \leq 0$$

$$\underset{+}{x^2}(x^2-1) \leq 0 \Rightarrow x^2-1 \leq 0$$

$$x^2 \leq 1$$

$$x^2 \leq 1^2$$

$$|x| \leq |1|$$

$$|x| \leq 1$$

$$\underline{\underline{-1 \leq x \leq 1}}$$

5 $f(f(5x-2)) < f(f(4x+1))$

$f\downarrow$

$f(5x-2) > f(4x+1)$

$f\downarrow$

$5x-2 < 4x+1$

$\underline{\underline{x < 3}}$

12. $f(x) = x^3 + x - 10$

ⓐ $x > \frac{10}{x^2+1} \Rightarrow x(x^2+1) > 10$

$x^3 + x - 10 > 0$

$f(x) > 0$

$f(x) > f(2)$

$f \nearrow$

$\underline{\underline{x > 2}}$

<u>Μονοτονία $f(x)$</u>

- $x_1 < x_2 \Rightarrow x_1^3 < x_2^3$ ⊕
- $x_1 < x_2 \Rightarrow x_1 - 10 < x_2 - 10$

$f \nearrow$

ⓑ $x^2 + \frac{10}{x} < -1$ Στο $(-\infty, 0)$

$x^3 + 10 < -x$

$x^3 + x + 10 < 0$

$x^3 + x - 10 < -10 - 10$

$f(x) < -20$

$f(x) < f(-2)$

$f \nearrow$

$x < -2$

(γ) $\ln^3 x + \ln x \geq 10$

$\ln^3 x + \ln x - 10 \geq 0$

$f(\ln x) \geq f(2)$

$f \nearrow$

$\ln x \geq 2$

$x \geq e^2$

(δ) $f(3x-4) + 10 < x^3 + x$

$f(3x-4) < x^3 + x - 10$

$f(3x-4) < f(x)$

$f \nearrow$

$3x - 4 < x$

$2x < 4$

$x < 2$.

(٤) $f(x^3+2x-2)+10-x > x^3$

$$f(x^3+2x-2) > x^3+x-10$$

$$f(x^3+2x-2) > f(x)$$

$$f \nearrow$$

$$x^3+2x-2 > x$$

$$x^3+x-2 > 0$$

$$x^3+x-10 > -10+2$$

$$f(x) > -8$$

$$f(x) > f(1)$$

$$f \nearrow$$

$$\underline{\underline{x > 1}}$$

(٥٢) $f^3(x)+f(x)-x > x^3$

$$f^3(x)+f(x)-10 > x^3+x-10$$

$$f(f(x)) > f(x)$$

$$f \nearrow$$

$$f(x) > x \Rightarrow x^3+\cancel{x}-10 > \cancel{x}$$

$$x^3 > 10 \Rightarrow x > \sqrt[3]{10}$$

14. $f(x) = e^{-x} - x - 1$

ⓐ $x_1 < x_2 \Rightarrow e^{-x_1} > e^{-x_2}$

$x_1 < x_2 \Rightarrow -x_1 - 1 > -x_2 - 1$ ⊕ $f \downarrow$

ⓑ . i) $x e^x < 1 - e^x$

$x < \frac{1}{e^x} - 1$

$x < e^{-x} - 1$

$0 < e^{-x} - x - 1$

$0 < f(x)$

$f(0) < f(x)$

$f \downarrow$

$0 > x$

ii) $e^x < \frac{1}{x+1}$ $\qquad x > -1$

$e^x(x+1) < 1$

$e^x x + e^x < 1$

$x < 0$ απο πριν ✓

iii). $f(3x-2) + x > e^{-x} - 1$

$f(3x-2) > e^{-x} - x - 1$

$f(3x-2) > f(x)$

$f \downarrow$

$3x - 2 < x$

$2x < 2$

$\underline{\underline{x < 1}}$

iv) $e^{-f(x)} - f(x) - 1 > 0$

$f(f(x)) > f(0)$

$f \downarrow$

$f(x) < 0$

$f(x) < f(0)$

$f \downarrow$

$x > 0$.

15. $f(x) = \ln x + x - 1,$

(α) $x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2$

$x_1 < x_2 \Rightarrow x_1 - 1 < x_2 - 1$

$\left.\right\} \oplus \quad f \nearrow$

(β) i) $x^2 + \ln(x^2+1) > 0$

$\ln(x^2+1) + x^2+1 - 1 > 0$

$f(x^2+1) > 0$

$f(x^2+1) > f(1)$

$f \nearrow$

$x^2+1 > 1$

$x^2 > 0$

$x \in (-\infty, 0) \cup (0, +\infty).$

11). $\ln \frac{3x}{x^2+2} < x^2-3x+2$

$f(x) = \ln x + x - 1$

$\ln 3x - \ln(x^2+2) < x^2-3x+2$.

$\ln 3x + 3x - 1 < \ln(x^2+2) + x^2+2-1$

$f(3x) < f(x^2+2)$

$f \nearrow$

$3x < x^2+2$

$0 < x^2-3x+2$

x		1		2	
x^2-3x+2	+		−		+

$x \in (-\infty, 1) \cup (2, +\infty)$

23. a) $(2x+1)^5 + (2x+1)^3 < 2$

$$\boxed{f(x) = x^5 + x^3}$$

- $x_1 < x_2 \Rightarrow x_1^5 < x_2^5$

$+$

- $x_1 < x_2 \Rightarrow x_1^3 < x_2^3$

$f \nearrow$

$f(2x+1) < f(1)$

$f \nearrow$

$2x+1 < 1$

$2x < 0$

$\underline{\underline{x < 0}}$

γ) $(x^2+1)e^{x^2+1} > e \Rightarrow f(x^2+1) > f(1)$

$f \nearrow$

$x^2+1>1$

$x^2>0$

$x \in \mathbb{R}^*$

$\boxed{f(x) = x e^{x},\ x>0}$

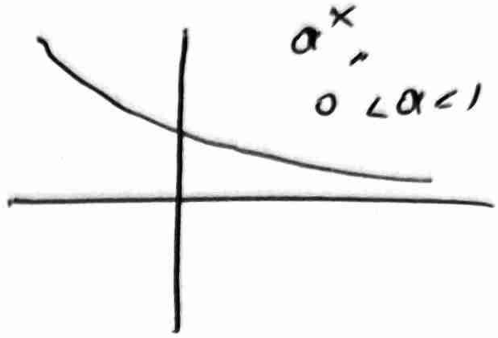
$$\left.\begin{array}{l} \bullet\ x_1<x_2 \Rightarrow e^{x_1}<e^{x_2} \\ \\ \bullet\ x_1<x_2 \end{array}\right\} \odot$$

$$x_1 e^{x_1} < x_2 e^{x_2}$$

$$f(x_1) < f(x_2)$$

$f \nearrow$

18. $f(x) = a^{x} - x, \quad 0 < a < 1$



ⓐ $x_1 < x_2 \Rightarrow a^{x_1} > a^{x_2}$

(+)

$x_1 < x_2 \Rightarrow -x_1 > -x_2$

$f \downarrow$

ⓑ $a^{x^2+4} - a^{5x-2} < x^2 - 5x + 6, \quad 0 < a < 1.$

$$a^{x^2+4} - (x^2+4) < a^{5x-2} - (5x-2)$$

$$f(x^2+4) < f(5x-2)$$

$f \downarrow$

$$x^2+4 > 5x-2$$

$$x^2 - 5x + 6 > 0$$

$x \in (-\infty, 2) \cup (3, +\infty)$

x		2	3	
x^2-5x+6	+	−		+

20. (a) $x^7+2x-3<0 \longrightarrow f(x)<0$

$f(x)<f(1)$

$f \nearrow$

$x<1$

$$\boxed{f(x)=x^7+2x-3}$$

- $x_1<x_2 \Rightarrow x_1^7<x_2^7$

(+) $f \nearrow$

- $x_1<x_2 \Rightarrow 2x_1-3<2x_2-3$

(B) $e^x<1-2x$

$\underbrace{e^x+2x-1}_{f(x)}<0 \Rightarrow f(x)<0$

$f(x)<f(0)$

$f \nearrow$

$x<0$

$x_1<x_2 \Rightarrow e^{x_1}<e^{x_2}$

(+)

$x_1<x_2 \Rightarrow 2x_1-1<2x_2-1$

$f \nearrow$

22. $f(x) = e^{x-1}$, $x > 0$

$g(x) = \frac{1}{x}$, $x > 0$

(a) $f(x) = g(x) \Leftrightarrow e^{x-1} = \frac{1}{x}$

$\underbrace{e^{x-1} - \frac{1}{x}}_{h(x)} = 0 \Rightarrow h(x) = 0$

$h(x) = h(1)$

h : 1-1

$\underline{\underline{x = 1}}$

• $x_1 < x_2 \Rightarrow e^{x_1 - 1} < e^{x_2 - 1}$

(+)

• $x_1 < x_2 \Rightarrow \frac{1}{x_1} > \frac{1}{x_2} \Rightarrow -\frac{1}{x_1} < -\frac{1}{x_2}$

$A(1, 1)$

$h(x_1) < h(x_2)$

$h \nearrow \Rightarrow h$ 1-1.

(B) , Έστω $f(x) < g(x) \Rightarrow e^{x-1} < \frac{1}{x} \Rightarrow e^{x-1} - \frac{1}{x} < 0$

$h(x) < 0$

$h(x) < h(1)$

$h \nearrow$

$x < 1$

<u>Συμπέρασμα</u>

Όταν $x < 1$ η $f(x) < g(x)$

Όταν $x > 1$ η $f(x) > g(x)$.

24. (α) Αν $\alpha, \beta \in (0, \frac{\pi}{2})$ και

$$\eta\mu\alpha - \eta\mu\beta < \sigma\upsilon\nu\alpha - \sigma\upsilon\nu\beta$$

ν.δ.ο. $\alpha < \beta$.

$$\eta\mu\alpha - \sigma\upsilon\nu\alpha < \eta\mu\beta - \sigma\upsilon\nu\beta$$

$$f(\alpha) < f(\beta)$$

$f \nearrow$

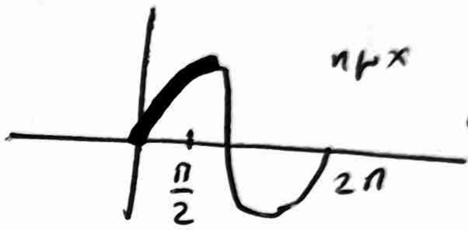
$\alpha < \beta$

$\boxed{f(x) = \eta\mu x - \sigma\upsilon\nu x}$

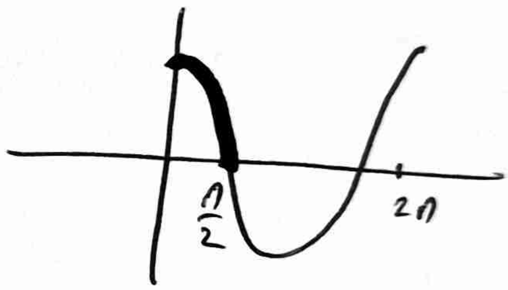
- $x_1 < x_2 \Rightarrow \eta\mu x_1 < \eta\mu x_2$
- $x_1 < x_2 \Rightarrow \sigma\upsilon\nu x_1 > \sigma\upsilon\nu x_2$

$-\sigma\upsilon\nu x_1 < -\sigma\upsilon\nu x_2$

(+)



$f \nearrow$



(β) i) $e^x - e^{x^2} > x^2 - x$

$$e^x + x > e^{x^2} + x^2$$

$\varphi(x) = e^x + x$

- $x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2}$
- $x_1 < x_2$ (+)

$\varphi \nearrow$

$$\varphi(x) > \varphi(x^2)$$

$\varphi \nearrow$

$$x > x^2$$

$$0 > x^2 - x$$

x		0		1	
$x^2 - x$	+		−		+

$x \in (0, 1)$.

1). $e^{2x+1} - e^{x^2+2} < \ln \frac{x^2+2}{2x^2+1}$

$$e^{2x+1} - e^{x^2+2} < \ln(x^2+2) - \ln(2x^2+1)$$

$$e^{2x+1} + \ln(2x^2+1) < e^{x^2+2} + \ln(x^2+2)$$

$$f(x) = e^x + \ln x$$

$f \nearrow$

$$f(2x^2+1) < f(x^2+2)$$

$f \nearrow$

$$2x^2+1 < x^2+2$$

$$x^2 < 1$$

$$x^2 < 1^2$$

$$|x| < |1|$$

$$|x| < 1$$

$$x \in (-1, 1).$$

28. $f \downarrow$ $g \uparrow$ $f(x) > 0$ $g(x) > 0$

ⓐ $h(x) = \left(\frac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

- $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ ⊙
- $x_1 < x_2 \Rightarrow g(x_1) < g(x_2) \Rightarrow \dfrac{1}{g(x_1)} > \dfrac{1}{g(x_2)}$

$$f(x_1) \frac{1}{g(x_1)} > f(x_2) \cdot \frac{1}{g(x_2)}$$

$$\frac{f(x_1)}{g(x_1)} > \frac{f(x_2)}{g(x_2)}$$

$$\left(\frac{f}{g}\right)(x_1) > \left(\frac{f}{g}\right)(x_2)$$

$$h(x_1) > h(x_2)$$

$h \downarrow$

③ $f(x^3) \cdot g(2x) - f(2x) \cdot g(x^3) < 0.$

$$f(x^3)\, g(2x) < f(2x)\, g(x^3)$$

$$\frac{f(x^3)}{g(x^3)} < \frac{f(2x)}{g(2x)}$$

$$\left(\frac{f}{g}\right)(x^3) < \left(\frac{f}{g}\right)(2x)$$

$$\cdot h(x^3) < h(2x)$$

$\cdot h$ [illegible]

$$x^3 > 2x$$

$$x^3 - 2x > 0$$

$$x(x^2 - 2) > 0$$

x		$-\sqrt{2}$		0		$\sqrt{2}$	
x	$-$	|	$-$	0	$+$	|	$+$
x^2-2	$+$	0	$-$	|	$-$	0	$+$
x^3-2x	$-$	|	$+$	|	$-$	|	$+$

$x \in (-\sqrt{2}, 0) \cup (\sqrt{2}, +\infty).$

Ενότητα 6

1-1

$f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$

2. (δ) $f(x) = \dfrac{2e^x - 1}{e^x + 2}$

Έστω $f(x_1) = f(x_2)$

$$\frac{2e^{x_1} - 1}{e^{x_1} + 2} = \frac{2e^{x_2} - 1}{e^{x_2} + 2}$$

$$(2e^{x_1} - 1)(e^{x_2} + 2) = (2e^{x_2} - 1)(e^{x_1} + 2)$$

$$\cancel{2e^{x_1} e^{x_2}} + 4e^{x_1} - e^{x_2} - \cancel{2} = \cancel{2e^{x_2} e^{x_1}} + 4e^{x_2} - e^{x_1} - \cancel{2}$$

$$5e^{x_1} = 5e^{x_2}$$

$$e^{x_1} = e^{x_2}$$

$$x_1 = x_2$$

f 1-1.

02 $f(x) = x - \ln(1-e^{x})$

$\cdot\ x_1 < x_2$

$\cdot\ x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2} \Rightarrow -e^{x_1} > -e^{x_2}$

$$1-e^{x_1} > 1-e^{x_2}$$

$(+)$ $$\ln(1-e^{x_1}) > \ln(1-e^{x_2})$$

$$-\ln(1-e^{x_1}) < -\ln(1-e^{x_2})$$

$$x_1 - \ln(1-e^{x_1}) < x_2 - \ln(1-e^{x_2})$$

$$f(x_1) < f(x_2)$$

$f \nearrow \Rightarrow f\ 3/-1/.$

7. $f(x) = e^{x-2} + x - 3$

(α) $\cdot\ x_1 < x_2 \Leftrightarrow x_1 - 3 < x_2 - 3$

$\cdot\ x_1 < x_2 \Leftrightarrow x_1 - 2 < x_2 - 2 \Leftrightarrow e^{x_1 - 2} < e^{x_2 - 2}$.

$e^{x_1-2} + x_1 - 3 < e^{x_2-2} + x_2 - 3 \Leftrightarrow$

$f(x_1) < f(x_2)$ Άρα $f \uparrow \Rightarrow f$ 1-1

(β) i) $x + e^{x-2} = 3$.

$e^{x-2} + x - 3 = 0$

$f(x) = 0$

$f(x) = f(2)$

f 1-1

$\underline{\underline{x = 2}}$

ii).
$$e^{x^2-2}+x^2=3$$

$$e^{x^2-2}+x^2-3=0$$

$$f(x^2)=f(2)$$

f 1-1

$$x^2=2$$

$$x=\pm\sqrt{2}$$

iii)
$$e^{3x-2}-e^{x^2}=x^2-3x+2$$

$$e^{3x-2}+3x-3=e^{x^2+2-2}+x^2+2-3.$$

$$f(3x)=f(x^2+2)$$

f 1-1

$$3x=x^2+2$$

$$0=x^2-3x+2$$

$x=2$ $x=1$

iv). $f\left((9-3x)e^{2-x}-1\right)=0.$

$$f\left((9-3x)e^{2-x}-1\right)=f(2)$$

f is 1-1

$$(9-3x)e^{2-x}-1=2$$

$$(9-3x)\cdot e^{2-x}=3$$

$$9-3x=\frac{3}{e^{2-x}}$$

$$9-3x=3e^{x-2}$$

$$3-x=e^{x-2}$$

$$0=e^{x-2}+x-3$$

$$f(2)=f(x)$$

f is 1-1

$$x=2$$

v). $e^{f(x)} + f(x) - 1 = 0$

$$e^{f(x)+2 \; - 2} + f(x)+2 \; - 3 = 0$$

$$f\left(f(x)+2\right) = f(2)$$

f 1-1

$$f(x)+2 = 2$$

$$f(x) = 0$$

$$f(x) = f(2)$$

f 1-1

$$x = 2,$$

9. (B) $e^{x^3-x} + x^3 = x+1$

$e^{x^3-x} + x^3 - x - 1 = 0$

$$\boxed{f(x) = e^x + x - 1}$$

$f(x^3-x) = f(0)$

• $x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2}$

(+)

• $x_1 < x_2 \rightarrow x_1 - 1 < x_2 - 1$

$f \nearrow$

$\Rightarrow f$ 1-1.

f 1-1

$x^3 - x = 0$

$x(x^2-1) = 0$

$x = 0$ $x^2 - 1 = 0$

$x^2 = 1$

$x = 1$

$x = -1$

8. (B) $e^{\frac{1}{x}} - \ln x = e$

$$\underbrace{e^{\frac{1}{x}} - \ln x - e}_{f(x)} = 0 \Rightarrow f(x) = 0$$

$f(x) = f(1)$

f 3)-1

$x = 1$

• $x_1 < x_2 \Rightarrow \frac{1}{x_1} > \frac{1}{x_2} \Rightarrow e^{\frac{1}{x_1}} > e^{\frac{1}{x_2}}$

• $x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2 \Rightarrow -\ln x_1 > -\ln x_2$

$-\ln x_1 - e > -\ln x_2 - e$

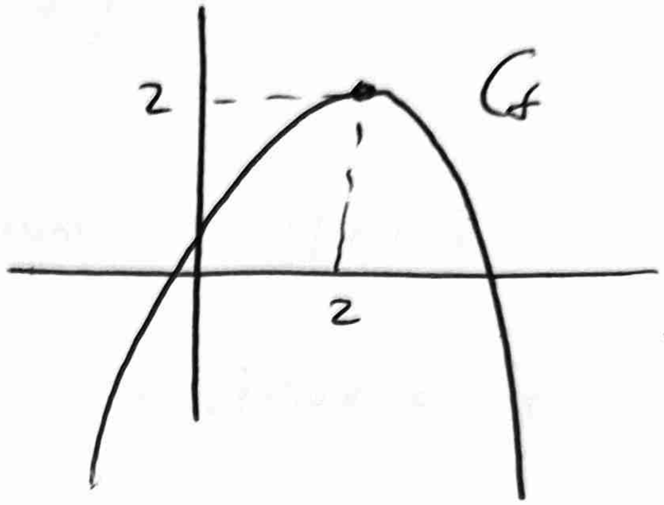
$$\underbrace{e^{\frac{1}{x_1}} - \ln x_1 - e}_{f(x_1)} > \underbrace{e^{\frac{1}{x_2}} - \ln x_2 - e}_{f(x_2)}$$

(+)

$f \downarrow$

$\Rightarrow$ f 3)-1

11.



ⓐ $f(x^6+4)=f(x^4+4)$

- $x^6+4>2$
- $x^4+4>2$

Στο $(2,+\infty)$ η f ~~f~~ $\Rightarrow f$ 1-1

$$x^6+4 = x^4+4$$

$$x^6=x^4$$

$$x^6-x^4=0$$

$$x^4(x^2-1)=0$$

$x=0$ $x=1$ $x=-1$

(B) $f(ημx) = f(\sqrt{3}\, συνx)$.

- $-1 \le ημx \le 1$ άρα $ημx < 2$

- $-1 \le συνx \le 1 \Rightarrow -\sqrt{3} \le \sqrt{3}\, συνx \le \sqrt{3}$

$\sqrt{3}\, συνx < 2$

Στο $(-\infty, 2)$ η $f \nearrow \Rightarrow f$ 1-1

Άρα $ημx = \sqrt{3}\, συνx$

$$\frac{ημx}{συνx} = \sqrt{3}$$

$$εφx = \sqrt{3}$$

$$εφx = εφ\frac{\pi}{3}$$

$$x = κ\pi + \frac{\pi}{3}.$$

Έστω $συνx = 0$

Τότε $ημx = 0$

όμως $ημ^2x + συν^2x = 1$

$0 + 0 = 1$

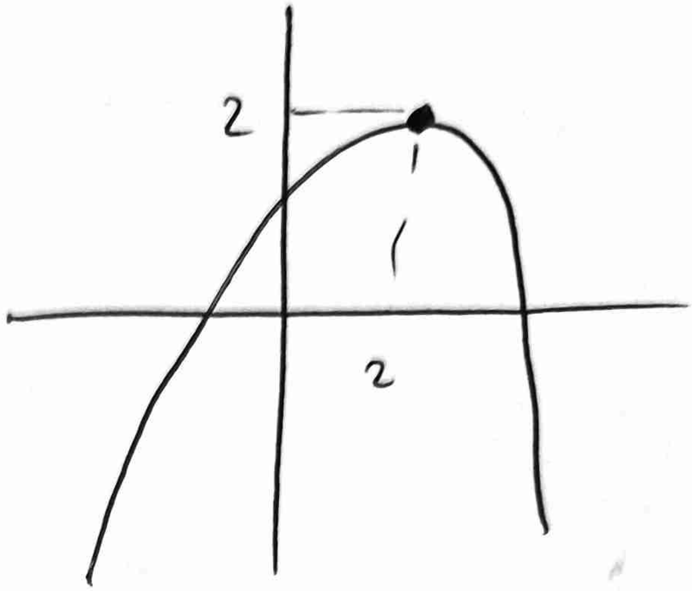
Άτοπο

Άρα $συνx \neq 0$

γ) $f(f(x)) = 2$

$f(x) = 2$

$x = 2$



Το $A(2,2)$ ειναι ολικο μεγιστο.

ΜΟΝΟ το $f(2)=2$ και κανενας αλλος.

13.

$$g(x) = f(x+3) - f(x)$$

g γν. μονότονη.

(α) $f(x+2) + f(0) = f(x-1) + f(3)$

$$\underbrace{f(x+2) - f(x-1)}_{g(x-1)} = \underbrace{f(3) - f(0)}_{g(0)}$$

$$g(x-1) = g(0)$$

g : $1-1$

$$x - 1 = 0$$

$$x = 1$$

(β) $f(x^2+3) + f(x+2) = f(x+5) + f(x^2)$

$$\underbrace{f(x^2+3) - f(x^2)}_{g(x^2)} = \underbrace{f(x+5) - f(x+2)}_{g(x+2)}$$

$$g(x^2) = g(x+2)$$

g : $1-1$

$x^2 = x+2 \Rightarrow x^2 - x - 2 = 0$ $x=2$, $x=-1$

14. $f: \mathbb{R} \to \mathbb{R}$ $\qquad f \downarrow$

(a) $f(x) + x < f(2x)$

$f(x) + 2x - x < f(2x)$

$f(x) - x < f(2x) - 2x$

$\longrightarrow$

$\boxed{g(x) = f(x) - x}$

$g(x) < g(2x)$

$g \downarrow$

- $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$
- $x_1 < x_2 \Rightarrow -x_1 > -x_2$ $\quad (+)$

$g \downarrow$

$x > 2x$

$0 > x$

(13) $f(x) + \ln x = f(x^2)$

$$f(x) + \ln \frac{x^2}{x} = f(x^2)$$

$$f(x) + \ln x^2 - \ln x = f(x^2)$$

$$f(x) - \ln x = f(x^2) - \ln x^2$$

$$g(x) = g(x^2)$$

g 81-1

$$\boxed{g(x) = f(x) - \ln x}$$

$x = x^2 \Rightarrow x - x^2 = 0$

$x(1-x) = 0$

(f) $x=0$ $x=1$

- $x_1 < x_2 \rightarrow f(x_1) > f(x_2)$

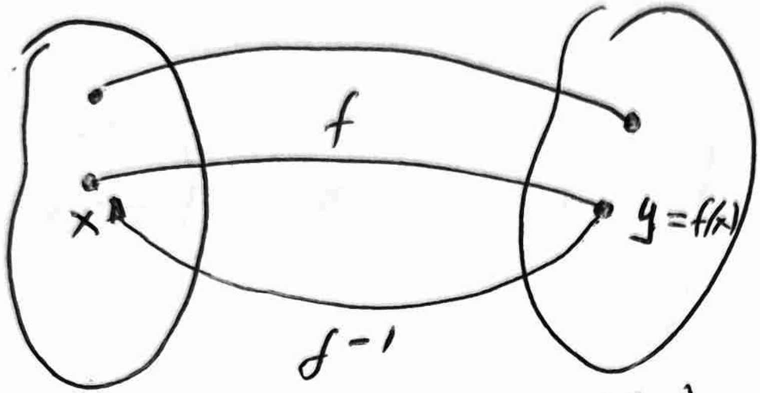
- $x_1 < x_2 \rightarrow \ln x_1 < \ln x_2 \Rightarrow -\ln x_1 > -\ln x_2$

$$g(x_1) > g(x_2)$$

$g \downarrow \Rightarrow g$ 81-1

Αντιστροφη συναρτηση

Εστω $f: 1-1$



A_f

Πεδιο ορισμου της f(x)

f(A)

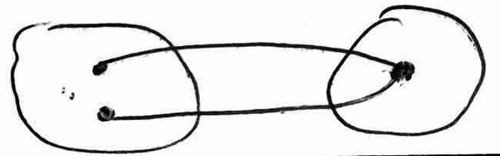
Συνολο Τιμων f(x)

→ $f(x) = y$ και $f^{-1}(y) = x$

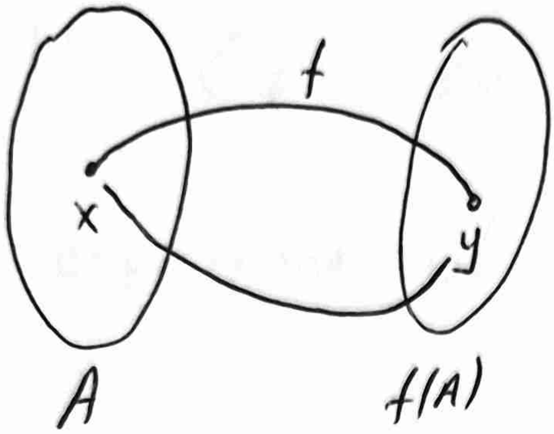
→ Το πεδιο ορισμου της αντιστροφης ειναι το συνολο τιμων της f(x)

Προσοχη

Αν η f <u>δεν</u> ηταν 1-1 τοτε



δεν οριζεται αντιστροφη



$y = f(x)$

$$f^{-1}(y) = x \iff f^{-1}(f(x)) = x$$

$$x \in A_f$$

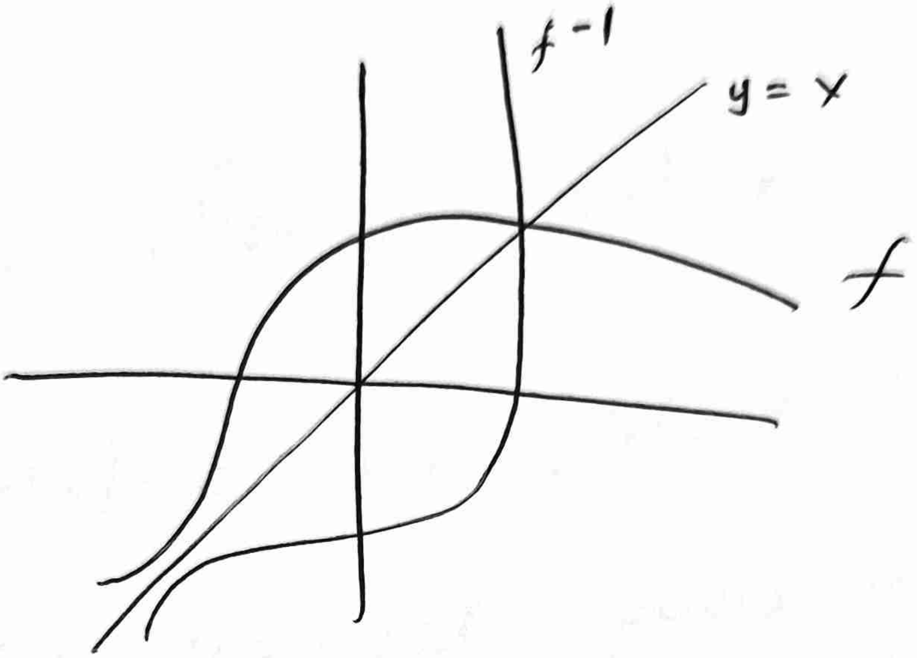
$$f(x) = y \Rightarrow f(f^{-1}(y)) = y$$

ή $$f(f^{-1}(x)) = x$$

$$x \in f(A)$$

1. Η C_f και $C_{f^{-1}}$ είναι

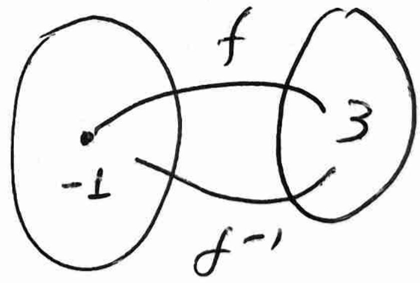
συμμετρικές ως προς την $y=x$



2. Επειδή $f(x)=y$ και $f^{-1}(y)=x$

τότε <u>π.χ</u> $f(-1)=3$

$\Rightarrow f^{-1}(3)=-1.$



3. Οι εξισωσεις $f(x) = x$ και $f^{-1}(x) = x$ ειναι ισοδυναμες.

4. Οι εξισωσεις $f(x) = x$ και $f^{-1}(x) = x$ και $f(x) = f^{-1}(x)$ ειναι ισοδυναμες μονο αν η $f \nearrow$ αυτη η εφαρμογη απαιτει αποδειξη.

Επομενο Μαθημα

Τεταρτη 9 - 11:30.

6

2 α β γ ε

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