21. (β) $3x - x^2 \le f(x) + 2 \le 3x + x^2$

Ενότητα 8

ψαχνω το $\lim_{x\to 0} f(x)$

$$\boxed{3x - x^2 - 2 \le f(x) \le 3x + x^2 - 2}$$

$\lim_{x\to 0} (3x - x^2 - 2) = -2$

$\lim_{x\to 0} (3x + x^2 - 2) = -2$

Από Κ.Π.

$\lim_{x\to 0} f(x) = -2$

(γ) $f(R) = (1,2)$

$A_f = R$ και $T_f = (1,2)$ $\Rightarrow$ $1 < f(x) < 2$

$x^4 < x^4 f(x) < 2x^4$

Βρω $\lim_{x\to 0} x^4 f(x)$

$\lim_{x\to 0} x^4 = 0$

$\lim_{x\to 0} 2x^4 = 0$

Από Κ.Π.

$\lim_{x\to 0} f(x) = 0$

24. (B) $\lim\limits_{x \to 1} (x-1)\,ημ\dfrac{3x}{x-1}$

$$-1 \leq ημ\frac{3x}{x-1} \leq 1$$

$$\left| ημ\frac{3x}{x-1} \right| \leq 1$$

$$|x-1|\left| ημ\frac{3x}{x-1} \right| \leq |x-1|$$

$$\left| (x-1)\,ημ\frac{3x}{x-1} \right| \leq |x-1|$$

$$\boxed{-|x-1| \leq (x-1)\,ημ\frac{3x}{x-1} \leq |x-1|}$$

$$\left.\begin{aligned} &\bullet \lim_{x \to 1} -|x-1| = 0 \\ &\bullet \lim_{x \to 1} |x-1| = 0 \end{aligned}\right\} \text{Από Κ.Π}$$

$$\lim_{x \to 1} (x-1)\,ημ\frac{3x}{x-1} = 0$$

44.

$$x - x^2 \le f(x) \le x^2 + x$$

ⓐ Βρείτε $\lim\limits_{x \to 0} \dfrac{f(x)}{x}$

Για $x < 0$

$$1 - x \ge \frac{f(x)}{x} \ge x + 1$$

$$\left.\begin{aligned} &\cdot \lim_{x \to 0^-} 1 - x = 1 \\ &\cdot \lim_{x \to 0^-} x + 1 = 1 \end{aligned}\right\} \text{Από К.П } \lim_{x \to 0^-} \frac{f(x)}{x} = 1$$

Για $x > 0$

$$1 - x \le \frac{f(x)}{x} \le x + 1$$

$$\left.\begin{aligned} &\cdot \lim_{x \to 0^+} 1 - x = 1 \\ &\lim_{x \to 0^+} x + 1 = 1 \end{aligned}\right\} \text{Από К.П}$$

$$\lim_{x \to 0^+} \frac{f(x)}{x} = 1$$

$$\lim_{x \to 0} \frac{f(x)}{x} = 1$$

(β) $\lim\limits_{x\to 0} \dfrac{f(x)}{\eta\mu x} = \lim\limits_{x\to 0} \dfrac{\dfrac{f(x)}{x}}{\dfrac{\eta\mu x}{x}} = \dfrac{1}{1}$

$= 1.$

(γ) $\lim\limits_{x\to 0} \dfrac{x+f(x)}{2x-f(x)} = \lim\limits_{x\to 0} \dfrac{1+\dfrac{f(x)}{x}}{2-\dfrac{f(x)}{x}}$

$= \dfrac{1+1}{2-1} = \dfrac{2}{1} = 2$

(δ) $\lim\limits_{x\to 0} \dfrac{x f(x) + \sqrt{x^2+1} - 1}{\eta\mu^2 x}$

$= \lim\limits_{x\to 0} \dfrac{\dfrac{x}{x}\cdot\dfrac{f(x)}{x} + \dfrac{\sqrt{x^2+1}-1}{x^2}}{\dfrac{\eta\mu^2 x}{x^2}}$

$= \lim\limits_{x\to 0} \dfrac{\dfrac{f(x)}{x} + \dfrac{\sqrt{x^2+1}-1}{x^2}}{\dfrac{\eta\mu^2 x}{x^2}} = \dfrac{1+\frac{1}{2}}{1} = \dfrac{3}{2}$

$$\lim_{x \to 0} \frac{\sqrt{x^2+1} - 1}{x^2} =$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{x^2+1} - 1\right)\left(\sqrt{x^2+1} + 1\right)}{x^2 \left(\sqrt{x^2+1} + 1\right)}$$

$$= \lim_{x \to 0} \frac{\cancel{\sqrt{x^2+1}^{\,2}} - 1^2}{x^2 \left(\sqrt{x^2+1} + 1\right)}$$

$$= \lim_{x \to 0} \frac{\cancel{x^2}}{\cancel{x^2}\left(\sqrt{x^2+1} + 1\right)} = \frac{1}{2}$$

22. B $|xf(x)-x^3| \leq x^2$

Βρεί $\lim_{x \to 0} f(x)$

$$-x^2 \leq xf(x)-x^3 \leq x^2$$

$$x^3-x^2 \leq xf(x) \leq x^3+x^2$$

Για $x<0$

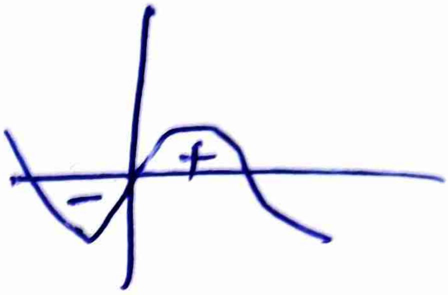
$$x^2-x \geq f(x) \geq x^2+x$$

$$\left.\begin{aligned} &\lim_{x\to 0^-} x^2-x = 0 \\ &\lim_{x\to 0^-} x^2+x = 0 \end{aligned}\right\}$$ Από Κ.Π.

$$\boxed{\lim_{x \to 0^-} f(x) = 0}$$

33. $f(x) = \text{ημ}x \cdot \text{ημ}\frac{1}{x}$

(α) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \text{ημ}x \; \text{ημ}\frac{1}{x} \xrightarrow[\text{χαοτικό}]{\text{ημ}\infty}$



$$-1 \le \text{ημ}\frac{1}{x} \le 1$$

$$\left|\text{ημ}\frac{1}{x}\right| \le 1$$

$$|\text{ημ}x| \left|\text{ημ}\frac{1}{x}\right| \le |\text{ημ}x|$$

$$\left|\text{ημ}x \; \text{ημ}\frac{1}{x}\right| \le |\text{ημ}x|$$

$$\boxed{-|\text{ημ}x| \le \text{ημ}x \; \text{ημ}\frac{1}{x} \le |\text{ημ}x|}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 0} -|\text{ημ}x| = 0 \\ \lim\limits_{x\to 0} |\text{ημ}x| = 0 \end{array}\right\} \text{Από Κ.Π. } \lim\limits_{x\to 0} f(x) = 0$$

(B) $\lim\limits_{x \to 0} \dfrac{f(x)\, \eta\mu f(x)}{\sqrt{1+f^2(x)} - 1}$ $\qquad$ $\dfrac{f(x) = t}{t \to 0}$

$$= \lim_{t \to 0} \frac{t\, \eta\mu\, t}{\sqrt{1+t^2} - 1} =$$

$$= \lim_{t \to 0} \frac{t\, \eta\mu t \left(\sqrt{1+t^2} + 1\right)}{\cancel{1} + t^2 - \cancel{1}}$$

$$= \lim_{t \to 0} \frac{\cancel{t}\, \eta\mu t \left(\sqrt{1+t^2} + 1\right)}{t^{\cancel{2}}} = 1 \cdot 2$$

$$= 2,$$

(γ) $\displaystyle \lim_{x\to 0} \frac{|f^3(x)+f(x)-1|-1}{\sigmaυν\, 2f(x) - 1 + f(x)}$ $\quad f(x)=t$

$$\lim_{t\to 0} \frac{|\overset{\ominus}{t^3+t-1}|-1}{\sigmaυν\, 2t - 1 + t} =$$

$$= \lim_{t\to 0} \frac{-t^3 - t \cancel{+1} \cancel{-1}}{\sigmaυν\, 2t - 1 + t}$$

$$= \lim_{t\to 0} \frac{-t^3 - \cancel{t}}{\sigmaυν\, 2t - 1 + t}$$

$$\lim_{x\to 0} \frac{\sigmaυν\, x - 1}{x} = 0$$

$$= \lim_{t\to 0} \frac{-t^2 - 1}{2\,\frac{\sigmaυν\, 2t - 1}{2t} + 1} = \frac{-1}{2\cdot 0 + 1}$$

$$= -1$$

Επόμενο μάθημα

Ημερομηνία μαθήματος : 24/7

Ώρα μαθήματος : 10:30 - 12

Ασκήσεις

Ενοτητα 8

21 α

22 α

23

24 α

43

παρατηρήσεις