Ενότητα 21

2. (α) $f(x) = x^3+x$ $\qquad \Delta=[0,1]$

Η f συνεχής $[0,1]$ ως π.σ.υ

Η f παρ/μη $(0,1)$ ως π.π.υ.

Από ΘΜΤ $\exists \xi \in (0,1)$ τ.ω $f'(\xi) = \dfrac{f(1)-f(0)}{1-0} = \dfrac{2-0}{1}$

$f'(\xi)=2$

$f'(x) = 3x^2+1$

$f'(\xi) = 3\xi^2+1 = 2$

$3\xi^2 = 1$

$\xi^2 = \dfrac{1}{3}$ $\qquad \xi = \pm\dfrac{\sqrt{3}}{3} \longrightarrow \xi = \dfrac{\sqrt{3}}{3}$

(γ) $f(x) = \begin{cases} x^2+x, & x<0 \\ x^3+x, & x\geq 0 \end{cases}$ $\qquad \Delta=[-1,2]$

$\left.\begin{array}{l} \lim\limits_{x\to 0^-} f(x) = 0 \\ \lim\limits_{x\to 0^+} f(x) = 0 \end{array}\right\} \lim\limits_{x\to 0} f(x) = 0$ και $f(0)=0$.

f συνεχής στο 0 γιατί $f(0) = \lim\limits_{x\to 0} f(x)$

Η f συνεχής $[-1,0)$ και $(0,2]$ ως π.σ.σ.

Άρα f συνεχής $[-1,2]$ ✓

$$\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0^-}\frac{x^2+x-0}{x-0}=\lim_{x\to 0^-}\frac{2x+1}{1}=1$$

$$\lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0^+}\frac{x^3+x}{x}=\lim_{x\to 0}\frac{3x^2+1}{1}=1$$

$f'(0)=1$.

Η f παρ/μη $(-1,0)$ και $(0,2)$ ως π.π.σ.

Η f παρ/μη $(-1,2)$. ✓

ΘΜΤ $\exists\, \xi\in(-1,2)$ τ.ω

$$f'(\xi)=\frac{f(2)-f(-1)}{2-(-1)}=\frac{10-0}{3}=\frac{10}{3}.$$

$\underline{x<0}$

$f'(\xi)=\frac{10}{3}$

$2\xi+1=\frac{10}{3}$

$6\xi+3=10$

$6\xi=7$

$\cancel{\xi=\frac{7}{6}}$

$\underline{x>0}$

$f'(\xi)=\frac{10}{3}$

$3\xi^2+1=\frac{10}{3}$

$9\xi^2+3=10$

$9\xi^2=7$

$\xi^2=\frac{7}{9}$

$\xi=\frac{\sqrt{7}}{3}$

3. $f: \mathbb{R} \to \mathbb{R}$ παρ/γιμη

$f(4) - f(1) = 9$

(α) $f'(\xi) = \dfrac{f(4) - f(1)}{4-1} = \dfrac{9}{3} = 3$ <u>ΘΜΤ</u>

<u>Β' τροπο/</u>

$f'(x) = 3$

$f'(x) - 3 = 0$

$\underbrace{\left(f(x) - 3x\right)}_{g(x)}{}' = 0$

$g(1) = f(1) - 3$

$g(4) = f(4) - 12 = 9 + f(1) - 12 = f(1) - 3$

$\Rightarrow g(1) = g(4)$ Rolle $\exists \xi \in (1,4)$ τ.ω $g'(\xi) = 0$

$f'(\xi) - 3 = 0$

$f'(\xi) = 3$.

(β) Αρκει νδο $\exists \xi$ τ.ω $f'(\xi) = 3$

Το εδειξα στο α.

4. $f(1)<f(0)$

f συν+παρ

f' συν+παρ.

f''

(α) $f'(\xi)=\dfrac{f(1)-f(0)}{1-0}<0$

(β) Αφού $f'(x)\neq 0$ και συνεχής

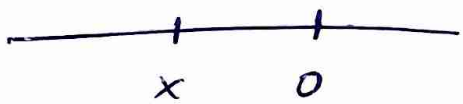
$f'(x)<0$ ή $f'(x)>0$ $\forall x\in\mathbb{R}$.

και αφού $f'(\xi)<0 \Rightarrow f'(x)<0$

$\Rightarrow f\downarrow$

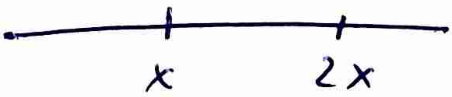
5. (α) $f'(\xi)=\dfrac{f(x)-f(1)}{x-1}=\dfrac{f(x)}{x-1}$

(β) $f'(\xi)=\dfrac{f(0)-f(x)}{0-x}=\dfrac{f(x)}{x}$



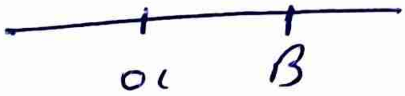
$x f'(\xi)=f(x)$

(γ) $f'(\xi)=\dfrac{f(2x)-f(x)}{2x-x}$



$x f'(\xi)+f(x)=f(2x)$

6. (a) $e^{\alpha} < \dfrac{e^{\beta} - e^{\alpha}}{\beta - \alpha} < e^{\beta}$ $\qquad \alpha < \beta$



$f(x) = e^{x}$

$f'(x) = e^{x}$

$f''(x) = e^{x}$

$\Rightarrow f' \nearrow$

$$f'(\xi) = \frac{f(\beta) - f(\alpha)}{\beta - \alpha} = \frac{e^{\beta} - e^{\alpha}}{\beta - \alpha}$$

$$\alpha < \xi < \beta$$

$$f' \nearrow$$

$$f'(\alpha) < f'(\xi) < f'(\beta)$$

$$e^{\alpha} < \frac{e^{\beta} - e^{\alpha}}{\beta - \alpha} < e^{\beta}.$$

$0 < \alpha < \beta < \frac{\pi}{2}$

B) $(\beta - \alpha)\,\sigma\upsilon\nu\beta < \eta\mu\beta - \eta\mu\alpha < (\beta - \alpha)\,\sigma\upsilon\nu\alpha$

$$\boxed{\sigma\upsilon\nu\beta < \frac{\eta\mu\beta - \eta\mu\alpha}{\beta - \alpha} < \sigma\upsilon\nu\alpha}$$

$f(x) = \eta\mu x$

$f'(x) = \sigma\upsilon\nu x$

$f''(x) = -\eta\mu x < 0$

$f' \downarrow$

$\alpha < \xi < \beta$

$f' \downarrow$

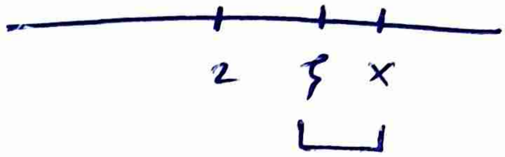
$f'(\alpha) > f'(\xi) > f'(\beta)$

$$\sigma\upsilon\nu\alpha > \frac{\eta\mu\beta - \eta\mu\alpha}{\beta - \alpha} > \sigma\upsilon\nu\beta.$$

8. ⓐ $f'\uparrow$

i) νδο $f'(x) > \dfrac{f(x)}{x-2}$ $\forall x > 2$ $f(2) = 0$

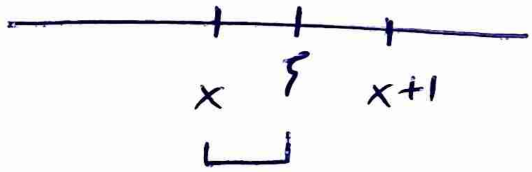
$$f'(\xi) = \frac{f(x) - f(2)}{x-2} = \frac{f(x)}{x-2}$$



$$\xi < x \Rightarrow f'(\xi) < f'(x) \Rightarrow \underline{\underline{\frac{f(x)}{x-2} < f'(x)}}$$

ii) νδο $f'(x) < f(x+1) - f(x)$ $\forall x \in \mathbb{R}$

$$f'(\xi) = \frac{f(x+1) - f(x)}{x+1-x}$$

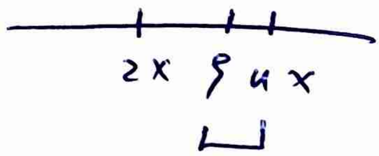


$$\xi > x \Rightarrow f'(\xi) > f'(x)$$

$$f(x+1) - f(x) > f'(x) \checkmark$$

iii) νδο $f(4x) - f(2x) < 2x\, f'(4x)$

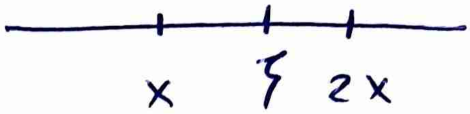
$$f'(\xi) = \frac{f(4x) - f(2x)}{4x - 2x} = \frac{f(4x) - f(2x)}{2x}$$



$$\xi < 4x \Rightarrow f'(\xi) < f'(4x) \Rightarrow \frac{f(4x) - f(2x)}{2x} < f'(4x)$$

$$f(4x) - f(2x) < 2x\, f'(4x).$$

(B) $\eta\mu 2x - \eta\mu x > x\,\sigma\upsilon\nu 2x \qquad x \in \left(0, \frac{\pi}{2}\right)$



$$\left.\begin{aligned} f(x) &= \eta\mu x \\ f'(x) &= \sigma\upsilon\nu x \\ f''(x) &= -\eta\mu x < 0 \end{aligned}\right\} f' \downarrow$$

$$f'(\xi) = \frac{f(2x) - f(x)}{2x - x} = \frac{\eta\mu 2x - \eta\mu x}{x}$$

$$\xi < 2x$$

$$f' \downarrow$$

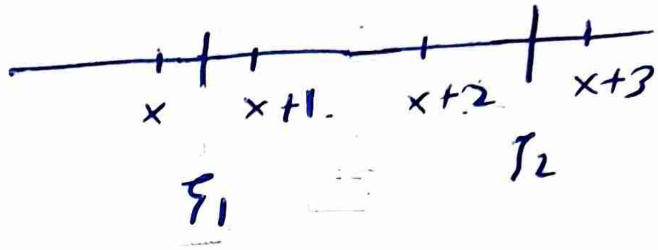
$$f'(\xi) > f'(2x)$$

$$\frac{\eta\mu 2x - \eta\mu x}{x} > \sigma\upsilon\nu 2x$$

$$\eta\mu 2x - \eta\mu x > x\,\sigma\upsilon\nu 2x$$

9. $f' \nearrow$

ⓐ $f(x+1) + f(x+2) < f(x) + f(x+3)$



$$f'(\xi_1) = \frac{f(x+1) - f(x)}{x+1-x}$$

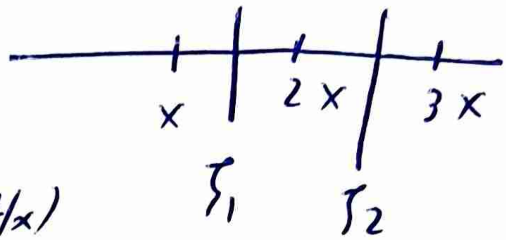
$$f'(\xi_2) = \frac{f(x+3) - f(x+2)}{x+3-x-2}$$

$$\xi_1 < \xi_2 \xrightarrow{f' \nearrow} f'(\xi_1) < f'(\xi_2)$$

$$\frac{f(x+1) - f(x)}{1} < \frac{f(x+3) - f(x+2)}{1}$$

$$\underline{f(x+1) + f(x+2) < f(x+3) + f(x)}$$

(B) $f(x) + f(3x) > 2f(2x) \quad \forall x > 0$



$$f'(\xi_1) = \frac{f(2x)-f(x)}{2x-x} = \frac{f(2x)-f(x)}{x}$$

$$f'(\xi_2) = \frac{f(3x)-f(2x)}{3x-2x} = \frac{f(3x)-f(2x)}{x}$$

$$\xi_1 < \xi_2 \Rightarrow f'(\xi_1) < f'(\xi_2)$$

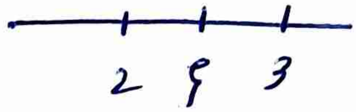
$$\frac{f(2x)-f(x)}{x} < \frac{f(3x)-f(2x)}{x}$$

$$2f(2x) < f(3x) + f(x)$$

10. $f' \downarrow$

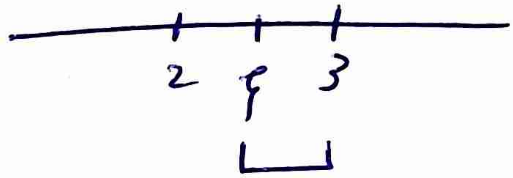
(α) $f'(3) < f(3)$ $\qquad f(2) = 0$

$$f'(\xi) = \frac{f(3)-f(2)}{3-2} = f(3)$$



$$\xi < 3 \Rightarrow f'(\xi) > f'(3) \Rightarrow \underline{\underline{f(3) > f'(3)}}$$

(β) $f'(3) + f(2) < f(3)$

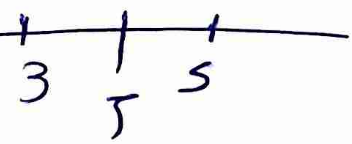


$$f'(\xi) = \frac{f(3)-f(2)}{3-2} = f(3)-f(2)$$

$$\xi < 3 \Rightarrow f'(\xi) > f'(3) \Rightarrow f(3)-f(2) > f'(3)$$

$$f(3) > f'(3)+f(2).$$

(γ) $f(5) < f(3) + 2f'(3)$

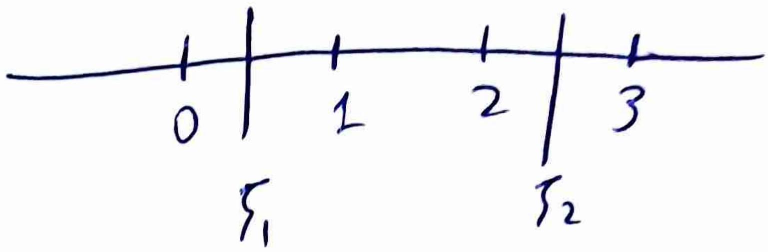


$$f'(\xi) = \frac{f(5)-f(3)}{5-3} = \frac{f(5)-f(3)}{2}$$

$$3 < \xi \Rightarrow f'(3) > f'(\xi) \Rightarrow f'(3) > \frac{f(5)-f(3)}{2}$$

$$2f'(3) > f(5) - f(3) \Rightarrow 2f'(3)+f(3) > f(5)$$

⑧ $f(1)+f(2) > f(0)+f(3)$



$$f'(\xi_1) = \frac{f(1)-f(0)}{1-0} = f(1)-f(0)$$

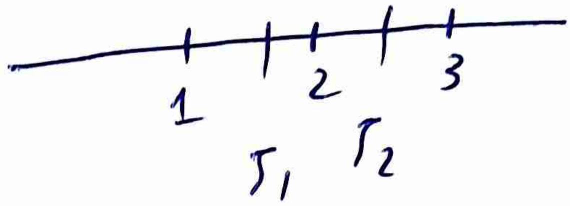
$$f'(\xi_2) = \frac{f(3)-f(2)}{3-2} = f(3)-f(2)$$

$\xi_1 < \xi_2 \Rightarrow f'(\xi_1) > f'(\xi_2)$

$f(1)-f(0) > f(3)-f(2)$

$f(1)+f(2) > f(3)+f(0)$.

ε $f(1)+f(3) < 2f(2)$



$$f'(\xi_1) = \frac{f(2)-f(1)}{2-1} = f(2)-f(1)$$

$$f'(\xi_2) = \frac{f(3)-f(2)}{3-2} = f(3)-f(2)$$

$$\xi_1 < \xi_2 \implies f'(\xi_1) > f'(\xi_2)$$

$$f(2)-f(1) > f(3)-f(2).$$

Ενοτητα 22

$f(1)=0$

2. $f:(0,+\infty)\to\mathbb{R}$ παραγωγίσιμη

$$f'(x)=\frac{1-xf(x)}{x^2} \quad \forall x>0$$

$$x^2f'(x)+xf(x)-1=0$$

(α) $g(x)=xf(x)-\ln x$

$$g'(x)=f(x)+xf'(x)-\frac{1}{x}=\frac{xf(x)+x^2f'(x)-1}{x}=0$$

άρα $g(x)=c$

(β) Αφού $g(x)=c$

$$xf(x)-\ln x=c$$

$$f(1)-0=c$$

$$f(1)=c$$

$$c=0$$

$$f(x)=\frac{\ln x}{x}$$

3. $f: \mathbb{R} \to \mathbb{R}$ παρ/μη

$f(x) \neq 0 \quad \forall x \in \mathbb{R}$. $\longrightarrow$ $f'(x) + 2x f^2(x) = 0$

$f'(x) = -2x f^2(x) \quad \forall x \in \mathbb{R}$.

(α) $g(x) = \dfrac{1}{f(x)} - x^2$

$$g'(x) = \frac{-f'(x)}{f^2(x)} - 2x = \frac{-f'(x) - 2x f^2(x)}{f^2(x)}$$

$$g'(x) = -\frac{f'(x) + 2x f^2(x)}{f^2(x)} = 0$$

$g(x) = c$

(β) Αν $f(0) = 1$ τοτε $\dfrac{1}{f(x)} - x^2 = c$

$x = 0$

$\dfrac{1}{f(0)} - 0^2 = c$

$c = 1$

$f(x) = \dfrac{1}{x2}$

5 $f: [0,\pi] \longrightarrow R$ $\qquad f(0)=0 \qquad \cos x$

$$f'(x) = \sin x \cdot x$$

Nδo $\qquad f(x) = \sin x - x\cos x$

$$\underbrace{f(x) - \sin x + x\cos x}_{g(x)} = 0$$

$$g'(x) = f'(x) - \cancel{\cos x} + \cancel{\cos x} - x\sin x$$

$$g'(x) = f'(x) - x\sin x \quad = 0$$

$$g(x) = c$$

$$f(x) - \sin x + x\cos x = c$$

$$x = 0$$

$$f(0) = c = 0$$

$$\underline{f(x) = \sin x - x\cos x}$$

6. $x\left(2f(x) + x f'(x)\right) = 1 - f'(x)$

$$\boxed{2x f(x) + x^2 f'(x) + f'(x) = 1}$$

$f(0) = 0$

Ndo $f(x) = \dfrac{x}{x^2+1}$

$g(x) = (x^2+1) f(x) - x$

$g'(x) = 2x f(x) + (x^2+1) f'(x) - 1$

$g'(x) = 2x f(x) + x^2 f'(x) + f'(x) - 1$

$g'(x) = 1 - 1$

$g'(x) = 0$

$g(x) = c$

$(x^2+1) f(x) - x = c$

$x = 0$

$c = 0$

$f(x) = \dfrac{x}{x^2+1}.$

8. $f(0) = 1$

$x\left(f(x) + x f'(x)\right) = -f'(x)$ $\longrightarrow$ $x f(x) + x^2 f'(x) + f'(x) = 0$

$x f(x) + (x^2+1) f'(x) = 0$

Nvo $f(x) = \dfrac{1}{\sqrt{x^2+1}}$

$$\boxed{g(x) = f(x)\sqrt{x^2+1} - 1.}$$

$$g'(x) = f'(x)\sqrt{x^2+1} + f(x)\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}$$

$$g'(x) = \frac{f'(x)(x^2+1) + x f(x)}{\sqrt{x^2+1}} = 0$$

$g(x) = C$

$f(x)\sqrt{x^2+1} - 1 = C$

$x = 0$

$C = 0$ $\qquad f(x) = \dfrac{1}{\sqrt{x^2+1}}$

10. $f'(x) + f(x)\,\text{εφ}x = 0$

$f'(x) + f(x)\dfrac{\text{ημ}x}{\text{συν}x} = 0$

Νδο $f(x) = \text{συν}x$

$\text{συν}x\, f'(x) + f(x)\,\text{ημ}x = 0$

$$\boxed{\begin{aligned} &\underbrace{f(x) - \text{συν}x}_{g(x)} = 0 \\ &g'(x) = f'(x) + \text{ημ}x \end{aligned}}$$

$\dfrac{f(x)}{\text{συν}x} = 1 \Rightarrow \underbrace{\dfrac{f(x)}{\text{συν}x} - 1}_{g(x)} = 0$

$$g'(x) = \frac{f'(x)\,\text{συν}x + f(x)\,\text{ημ}x}{\text{συν}^2x} = 0$$

$g(x) = c$

$\dfrac{f(x)}{\text{συν}x} - 1 = c$

$\dfrac{x=0}{c=0}$

$f(x) = \text{συν}x$

Ενοτητα 24

48. (α) ΝΔο $e^x > 1+x+\frac{1}{2}x^2$, $x>0$

$$\underbrace{e^x - 1 - x - \frac{1}{2}x^2}_{f(x)} > 0$$

$f'(x) = e^x - 1 - x \geq 0$ $\quad f \nearrow$

αφου $e^x \geq x+1$ αρα $e^x - 1 - x \geq 0$

Αρα αν $x>0$ τοτε $f(x) > f(0)$

$$e^x - 1 - x - \frac{1}{2}x^2 > 0$$

$$\boxed{e^x > 1 + x + \frac{1}{2}x^2}$$

(Β) $\begin{cases} h'(x) = \varphi'(x) \\ h(x) = \varphi(x) \end{cases} \Leftrightarrow \begin{cases} e^x - 1 = x \Rightarrow \boxed{x=0} \\ e^x - x = \frac{x^2}{2} + 1 \end{cases}$

$h(x) = e^x - x$ $\quad \varphi(x) = \frac{x^2}{2} + 1$ $\quad 1 - 0 = 1$

$h'(x) = e^x - 1$ $\quad 1 = 1$ ✓

$\varphi'(x) = x$

41. $\forall x>0$ νδο $3\eta\mu x < x(2+\sigma\upsilon\nu x)$

$$3\eta\mu x < 2x + x\sigma\upsilon\nu x$$

$$\underbrace{3\eta\mu x - 2x - x\sigma\upsilon\nu x}_{f(x)} < 0$$

$$f'(x) = 3\sigma\upsilon\nu x - 2 - \sigma\upsilon\nu x + x\eta\mu x$$

$$f'(x) = 2\sigma\upsilon\nu x - 2 + x\eta\mu x$$

Πρόβλημα

$$f'(x) = 2(\sigma\upsilon\nu x - 1) + x\eta\mu x$$

$\ominus$;

$$3\eta\mu x < x(2+\sigma\upsilon\nu x)$$

$\oplus$

$$\frac{3\eta\mu x}{2+\sigma\upsilon\nu x} - x < 0.$$

$$f(x) = \frac{3\eta\mu x}{2+\sigma\upsilon\nu x} - x$$

$$f'(x) = \frac{3\text{συν}x(2+\text{συν}x) + 3\text{ημ}^2x}{(2+\text{συν}x)^2} - 1$$

$$f'(x) = \frac{6\text{συν}x + 3\text{συν}^2x + 3\text{ημ}^2x - (2+\text{συν}x)^2}{(2+\text{συν}x)^2}$$

$$f'(x) = \frac{6\text{συν}x + 3(\text{συν}^2x + \text{ημ}^2x) - (2+\text{συν}x)^2}{(2+\text{συν}x)^2}$$

$$f'(x) = \frac{6\text{συν}x + 3 - (4 + \text{συν}^2x + 4\text{συν}x)}{(2+\text{συν}x)^2}$$

$$f'(x) = \frac{6\text{συν}x + 3 - 4 - \text{συν}^2x - 4\text{συν}x}{(2+\text{συν}x)^2}$$

$$f'(x) = \frac{-\text{συν}^2x + 2\text{συν}x - 1}{(2+\text{συν}x)^2} = -\frac{(\text{συν}x - 1)^2}{(2+\text{συν}x)^2} \leq 0$$

$f \downarrow$

Αν $x>0$

$f(x)<f(0)$

$$\frac{3ημx}{2+συνx}-x<0$$

$$3ημx<x(2+συνx).$$

52. $f(x) = \begin{cases} \dfrac{1-\text{συν}x}{x}, & x>0 \\ 0, & x=0 \end{cases}$

$y - f(0) = f'(0)(x-0)$

$y - 0 = \frac{1}{2}x$ $\qquad$ $y = \frac{1}{2}x$

$$f'(0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} \frac{\frac{1-\text{συν}x}{x} - 0}{x} = \lim_{x\to 0} \frac{1-\text{συν}x}{x^2}$$

$$= \lim_{x\to 0} \frac{\text{ημ}x}{2x} = \lim_{x\to 0} \frac{\text{συν}x}{2} = \frac{1}{2}$$

Αρκεί νδο $\quad \frac{1}{2}x > \frac{1-\text{συν}x}{x} \quad \forall x>0$

$\underbrace{\frac{1}{2}x^2 - 1 + \text{συν}x}_{g(x)} > 0$

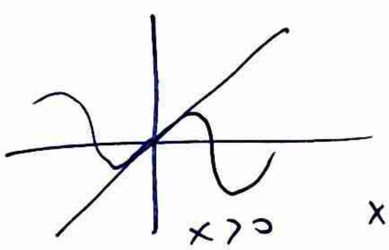
Αν $x>0$

$g \nearrow$

$g(x) > g(0)$

$\frac{1}{2}x^2 - 1 + \text{συν}x > 0$

$g'(x) = x - \text{ημ}x > 0$ $\qquad g \nearrow$



$x>0$ $\qquad x > \text{ημ}x$

56. $f(x) = \dfrac{\ln x}{\sqrt{x}}, \quad x > 0$

ⓐ Η f συνεχής $[4,16]$ ως π.σ.σ

Η f παρ/μη $(4,16)$ ως π.π.σ

$$f(4) = \frac{\ln 4}{\sqrt{4}} = \frac{\ln 2^2}{2} = \frac{2\ln 2}{2} = \ln 2$$

$$f(16) = \frac{\ln 16}{\sqrt{16}} = \frac{\ln 2^4}{4} = \frac{4\ln 2}{4} = \ln 2$$

✓

$f(4) = f(16)$

Ⓑ $$f'(x) = \frac{\frac{1}{x}\sqrt{x} - \ln x \frac{1}{2\sqrt{x}}}{x}$$

$$f'(x) = \frac{2\sqrt{x}^2 - x\ln x}{2x^2\sqrt{x}} = \frac{x(2-\ln x)}{2x^2\sqrt{x}}$$

$$f'(x) = \frac{2-\ln x}{2x\sqrt{x}}$$

x	0	e^2	$+\infty$
f'	$+$	$-$	
f	↗	↘	

$\rightarrow f'(x) = 0 \Rightarrow 2 - \ln x = 0 \Rightarrow x = e^2$

$f(x) \le f(e^2)$

$f(x) \le \frac{2}{e}$

(γ) $x^2 = 2^{2\sqrt{x}}$, $x > 0$

$$\ln x^2 = \ln 2^{2\sqrt{x}}$$

$$2\ln|x| = 2\sqrt{x}\,\ln 2$$

$$2\ln x = 2\sqrt{x}\,\ln 2$$

$$\frac{\ln x}{\sqrt{x}} = \ln 2$$

$$f(x) = \ln 2$$

Εχει δυο προφανείς ρίζα $x=4$ $x=16$.

Η f εχει δυο διαστηματα μονοτονίας αρα η εξισωση $f(x)=\ln 2$ εχει το πολυ 2 λυσεις αρα 2.

57. $f(x)=\dfrac{e^x}{x}\ ,\ x>0$

(a) Η f συνεχής $[\ln 2, \ln 4]$ ως σ.σ.

Η f παρ/μη $(\ln 2, \ln 4)$ ως π.σ.

$$f(\ln 2)=\frac{e^{\ln 2}}{\ln 2}=\frac{2}{\ln 2}$$

$$f(\ln 4)=\frac{e^{\ln 4}}{\ln 4}=\frac{4}{2\ln 2}=\frac{2}{\ln 2}$$

$f(\ln 2)=f(\ln 4)$ ✓

(β) $$f'(x)=\frac{e^x x-e^x}{x^2}=\frac{e^x(x-1)}{x^2}$$

x		1	
f'	$-$		$+$
f	↘		↗

(γ) $$2^{e^x}=e^{2x} \Rightarrow e^x\ln 2=2x$$

$$\frac{e^x}{x}=\frac{2}{\ln 2} \Leftrightarrow f(x)=\frac{2}{\ln 2}$$

$$f(x) = \frac{2}{\ln x}$$

Προφανείς ρίζα

$x = \ln 2$ $\qquad$ $x = \ln 4$

Έχει δυο διαστήματα

μονοτονίας άρα έχει

το πολύ 2 λύσεις.

έχω ήδη βρει τις 2.

άρα έχει 2.

58. $f(x) = (x-1)\ln x \qquad x>0$

(α) $f'(x) = \ln x + \frac{x-1}{x} = \ln x + 1 - \frac{1}{x} \qquad f'(1)=0$

$$f''(x) = \frac{1}{x} + \frac{1}{x^2} > 0$$

> • $\ln x \le x-1$
>
> $\ln\frac{1}{x} \le \frac{1}{x} - 1$
>
> $-\ln x \le \frac{1}{x} - 1$
>
> Δεν βγήκε.

x		1	
f''	+		+
f'	↗	0	+ ↗
f	↘		↗

$f(x) \ge f(1)$

$f(x) \ge 0$

(β) Νδο $(a+1)^{a} > a^{a-1} \qquad \forall a>1$

$a\ln(a+1) > (a-1)\ln a$

$f(a+1) > f(a)$

$\forall x>1$ η f ↗

$a+1 > a$

①

$$\frac{(\eta\mu x)^{\eta\mu x}}{(\sigma\upsilon\nu x)^{\sigma\upsilon\nu x}} = \varepsilon\varphi x \qquad x\in\left(0,\frac{\pi}{2}\right)$$

$$\ln \frac{(\eta\mu x)^{\eta\mu x}}{(\sigma\upsilon\nu x)^{\sigma\upsilon\nu x}} = \ln \varepsilon\varphi x$$

$$\ln (\eta\mu x)^{\eta\mu x} - \ln(\sigma\upsilon\nu x)^{\sigma\upsilon\nu x} = \ln \frac{\eta\mu x}{\sigma\upsilon\nu x}$$

$$\eta\mu x \ln \eta\mu x - \sigma\upsilon\nu x \ln \sigma\upsilon\nu x = \ln \eta\mu x - \ln \sigma\upsilon\nu x$$

$$\eta\mu x \ln \eta\mu x - \ln \eta\mu x = \sigma\upsilon\nu x \ln \sigma\upsilon\nu x - \ln \sigma\upsilon\nu x$$

$$(\eta\mu x - 1)\ln \eta\mu x = (\sigma\upsilon\nu x - 1)\ln \sigma\upsilon\nu x$$

$$f(\eta\mu x) = f(\sigma\upsilon\nu x)$$

Όταν $x\in\left(0,\frac{\pi}{2}\right)$ το $\eta\mu x, \sigma\upsilon\nu x \in (0,1)$

$$f \downarrow$$

$$\eta\mu x = \sigma\upsilon\nu x$$

$$x = \frac{\pi}{4}.$$

59. $f(x) = x^3 - 3\ln x$

(א) $f'(x) = 3x^2 - \frac{3}{x} = \frac{3x^3 - 3}{x} = 3\,\frac{x^3 - 1}{x}$

x	0		1	
f'		$-$	0	$+$
f		$\searrow$		$\nearrow$

$f(x) \geq f(1)$

$\underline{\underline{f(x) \geq 1}}$

(ב) $\ln \cot^3 x < \cos^3 x - \sin^3 x$

$\ln \frac{\cos^3 x}{\sin^3 x} < \cos^3 x - \sin^3 x$

$\sin^3 x - 3\ln \sin x < \cos^3 x - 3\ln \cos x$

$f(\sin x) < f(\cos x)$

כאשר $x \in (0, \frac{\pi}{2})$ אז $\sin x, \cos x \in (0,1)$

$f \downarrow$

$\sin x > \cos x$

$\frac{\sin x}{\cos x} > 1 \qquad \tan x > 1$

$\frac{\pi}{2} > x > \frac{\pi}{4}$

(γ) $f(x^2+3) + 3\ln(x^4+1) = (x^4+1)^3$

$$f(x^2+3) = (x^4+1)^3 - 3\ln(x^4+1)$$

$$f(x^2+3) = f(x^4+1)$$

f 1-1 $\forall x > 1$

$$x^2+3 = x^4+1$$

$$x^4 - x^2 - 2 = 0$$

$x^2 = 2$ $\qquad$ $x^2 = -1$ Αδύ

$$x = \pm\sqrt{2}$$

60. $f(x) = \begin{cases} e^{x^2} - 1 & , x \leq 0 \\ x^2 e^{-x} & , x > 0 \end{cases}$

$$\left.\begin{array}{l} \lim\limits_{x \to 0^-} f(x) = 0 \\ \lim\limits_{x \to 0^+} f(x) = 0 \end{array}\right\} \lim_{x \to 0} f(x) = f(0) = 0 .$$

<u>$x < 0$</u>

$f_1(x) = e^{x^2} - 1$

$f_1'(x) = 2x e^{x^2} < 0$

<u>$x > 0$</u>

$f_2(x) = x^2 e^{-x}$

$f_2'(x) = 2x e^{-x} - x^2 e^{-x}$

$f_2'(x) = e^{-x}(2x - x^2)$

$x = 0$ $x = 2$

x		0		2	
f_1'	$-$		$////$		$////$
f_2'	$////$	0	$+$	0	$-$
f'	$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$

(γ) $f\left(2e^{2-x}\right) = f(6-2x)$ $\qquad [0,2]$

- $0 \leq x \leq 2$

$0 \geq -x \geq -2$

$2 \geq 2-x \geq 0$

$e^2 \geq e^{2-x} \geq 1$

$2e^2 \geq 2e^{2-x} \geq 2$

$0 \leq x \leq 2$

$0 \geq -2x \geq -4$

$6 \geq 6-2x \geq 2$

$\forall x > 2$ η $f \downarrow$

$$2e^{2-x} = 6-2x$$

$$e^{2-x} = 3-x$$

- $e^x \geq x+1$ "=" $\underline{\underline{x=0}}$

$e^{2-x} \geq 2-x+1$

$e^{2-x} \geq 3-x$ $\qquad \underline{\underline{x=2}}$

8 Νδο $f\left(\frac{4}{x}\right) > f\left(\frac{8}{x^2}\right)$ $\forall x > 2$

$x > 2$

$\frac{1}{x} < \frac{1}{2}$

$\underline{\underline{0 < \frac{4}{x} < 2}}$

$x > 2$

$x^2 > 4$

$\frac{1}{x^2} < \frac{1}{4}$

$\underline{\underline{< \frac{8}{x^2} < 2}}$

$\forall x \in (0,2)$ η $f \nearrow$

$$\frac{4}{x} > \frac{8}{x^2}$$

$$4x^2 > 8x$$

$\underline{\underline{x > 2}}$ που ισχύει!

Επομενο Μαθημα

Δευτερα 5/1/26 9-11:30

1. Διαβασμα το pdf "Επαναληψη Γ Λυκειου"
2. Στο επομενο Μαθημα ξεκιναω με ρυθμο μεταβολης
3. Αναλυουμε και επεξηγουμε τις αποδειξεις της υλης
4. Δουλευουμε πανω στο pdf "Επαναληψη Γ' Λυκειου" να λυσουμε τυχον αποριες.