

1.

(a) Av $\alpha < -3$ kai $B < 2$

vfo $\alpha B - 6 > 2\alpha - 3B$.

$$\alpha B - 6 - 2\alpha + 3B > 0$$

$$\alpha(B-2) + 3(B-2) > 0$$

$$\underbrace{(B-2)}_{(-)} \underbrace{(\alpha+3)}_{(+) > 0}$$

✓

• $\alpha < -3 \Rightarrow \alpha + 3 < 0$

• $B < 2 \Rightarrow B - 2 < 0$

(b) Nfo $B^2 \geq \frac{\alpha B}{2} - \frac{(\alpha - B)^2}{8}$

$$8B^2 \geq 4\alpha B - (\alpha - B)^2$$

$$8B^2 \geq 4\alpha B - (\alpha^2 - 2\alpha B + B^2)$$

$$8B^2 \geq 4\alpha B - \alpha^2 + 2\alpha B - B^2$$

$$9B^2 - 6\alpha B + \alpha^2 \geq 0 \quad (3B - \alpha)^2 \geq 0 \quad \checkmark$$

⑦ NĐD $x^2 + 6x + 2026 > 0$

$$x^2 + 6x + 9 + 2017 > 0$$

$$(x+3)^2 + 2017 > 0 \quad \checkmark$$

⑧. NĐD ĐV $a+b \geq 0$ thì $a^3 + b^3 \geq ab(a+b)$

$$(a+b)(a^2 - ab + b^2) - ab(a+b) \geq 0$$

$$(a+b)(a^2 - ab + b^2 - ab) \geq 0$$

$$(a+b)(a^2 - 2ab + b^2) \geq 0$$

$$(a+b) \quad (a-b)^2 \geq 0$$

⊕ ⊕

✓

$$\textcircled{\varepsilon} (a^2 + b^2)(x^2 + y^2) \geq (ax + by)^2$$

$$a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2 \geq a^2x^2 + 2axby + b^2y^2$$

$$a^2y^2 - 2axby + b^2x^2 \geq 0$$

$$(ay - bx)^2 \geq 0$$

⑦ Av $x \geq -1$ vdo $x^3 - x^2 \geq x - 1$

$$x^3 - x^2 - x + 1 \geq 0$$

$$x^2(x-1) - (x-1) \geq 0$$

$$(x-1)(x^2-1) \geq 0$$

$$(x-1)(x-1)(x+1) \geq 0$$

$$\boxed{\begin{array}{c} (x-1)^2 (x+1) \geq 0 \\ \oplus \quad \oplus \end{array}} \quad \checkmark$$

• $x \geq -1 \rightarrow x+1 \geq 0$

$$\textcircled{n} \quad A \vee a < b < -4 \quad \text{vs} \quad 4b - a(4-b) < a^2$$

$$4b - a(4-b) - a^2 < 0$$

$$4b - 4a + a(4-b) - a^2 < 0$$

$$4(b-a) + a(b-a) < 0$$

$$\boxed{\begin{array}{cc} (b-a) & (4+a) < 0 \\ (+) & (-) \end{array}} \quad \checkmark$$

$$\bullet \quad a < b \Rightarrow 0 < b-a$$

$$\bullet \quad a < -4 \Rightarrow a+4 < 0$$

⑨ $\forall a, b$ ομοσημι $\text{και } a \leq b$.

$$\text{νλο } a - \frac{1}{a} \leq b - \frac{1}{b}$$

- $a \leq b$

- $a \leq b \rightarrow \frac{1}{a} \geq \frac{1}{b} \Rightarrow -\frac{1}{a} \leq -\frac{1}{b}$

$$a - \frac{1}{a} \leq b - \frac{1}{b}$$

Β' τροπή

Αφω a, b ομοσημι

$$\Rightarrow a \cdot b > 0$$

$$a^2 b - a b \frac{1}{a} \leq a b^2 - a b \frac{1}{b}$$

$$a^2 b - b \leq a b^2 - a$$

$$a^2 b - b - a b^2 + a \leq 0$$

$$a b (\underbrace{a - b}_{\ominus}) + \underbrace{a - b}_{\oplus} \leq 0$$

$$| (\underbrace{a - b}_{\ominus}) (\underbrace{a b + 1}_{\oplus}) \leq 0 \checkmark$$

- $a \leq b \Rightarrow a - b \leq 0$

- $a b > 0$

$$a b + 1 > 1$$

2.

(a) N/A

$$x^2 + x + 1 > 0$$

$$2x^2 + 2x + 2 > 0$$

$$x^2 + x^2 + 2x + 1 + 1 > 0$$

$$x^2 + (x+1)^2 + 1 > 0$$



(b)

N/A

$$x^2 + \frac{1}{x^2} \geq x + \frac{1}{x}$$

$$x^4 + 1 \geq x^3 + x$$

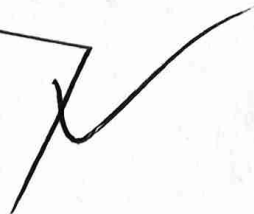
$$x^4 - x^3 - x + 1 \geq 0$$

$$x^3(x-1) - (x-1) \geq 0$$

$$(x-1)(x^3-1) \geq 0$$

$$(x-1)(x-1)(x^2+x+1) \geq 0$$

$$\boxed{\begin{matrix} (x-1)^2 (x^2+x+1) \geq 0 \\ (+) \quad (+) \end{matrix}}$$



$$3. \quad a^2 + b^2 + c^2 \geq ab + bc + ca.$$

$$2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca \geq 0$$

$$a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + a^2 - 2ac + c^2 \geq 0$$

$$(a-b)^2 + (b-c)^2 + (a-c)^2 \geq 0$$

✓

$$4. \quad (a) \quad \forall x, y > 0 \quad \text{vdo} \quad \frac{x}{y} + \frac{y}{x} \geq 2$$

$$\Rightarrow xy \frac{x}{y} + xy \frac{y}{x} \geq 2xy$$

$$x^2 + y^2 \geq 2xy$$

$$x^2 + y^2 - 2xy \geq 0$$

$$(x-y)^2 \geq 0$$

✓

$$\textcircled{B} \text{ v } \int \int \frac{a+b}{r} + \frac{b+r}{a} + \frac{r+a}{b} > 6$$

$$\frac{a}{r} + \frac{b}{r} + \frac{b}{a} + \frac{r}{a} + \frac{r}{b} + \frac{a}{b} \geq 6$$

$$\frac{a}{r} + \frac{r}{a} + \frac{b}{r} + \frac{r}{b} + \frac{b}{a} + \frac{a}{b} \geq 6$$

Ανάλυση πριν ότι $\frac{x}{y} + \frac{y}{x} \geq 2$

ομοίως ισχύει ότι

$$\frac{a}{r} + \frac{r}{a} \geq 2$$

$$\frac{b}{r} + \frac{r}{b} \geq 2$$

$$\frac{b}{a} + \frac{a}{b} \geq 2$$

$$\left. \begin{array}{l} \frac{a}{r} + \frac{r}{a} \geq 2 \\ \frac{b}{r} + \frac{r}{b} \geq 2 \\ \frac{b}{a} + \frac{a}{b} \geq 2 \end{array} \right\} \textcircled{1} \frac{a+b}{r} + \frac{b+r}{a} + \frac{r+a}{b} > 6$$

5. $\neg \sigma \tau w \quad 0 < x < y$

$\sum \chi \kappa \rho \iota \sigma \eta \ni A = (x - 2y)^2$

$B = y(4y - 3x)$

$\neg \sigma \tau w \quad A < B$

$(x - 2y)^2 < y(4y - 3x)$

$x^2 - 4xy + \cancel{4y^2} < \cancel{4y^2} - 3xy$

$x^2 - xy < 0$

$x(x - y) < 0$	
$\oplus$	$\ominus$
✓	

• $x > 0$

• $x < y \Rightarrow x - y < 0$

$\sigma \tau w$

$A < B$

$$6. \textcircled{a} \quad x^2 + y^2 + 10 = 2(x - 3y)$$

$$x^2 + y^2 + 10 = 2x - 6y$$

$$x^2 + y^2 + 10 - 2x + 6y = 0$$

$$x^2 - 2x + 1 + y^2 + 6y + 9 = 0.$$

$$(x-1)^2 + (y+3)^2 = 0$$

$$x-1=0$$

$$\textcircled{x=1}$$

$$\text{or } y+3=0$$

$$\textcircled{y=-3}$$

$$\textcircled{B} \quad (x+5)^2 + (y-2)^2 = 4(x+y+1)$$

$$x^2 + 10x + 25 + y^2 - 4y + 4 = 4x + 4y + 4$$

$$x^2 + 6x + 9 + y^2 - 8y + 16 = 0$$

$$(x+3)^2 + (y-4)^2 = 0$$

$$\textcircled{x=-3} \quad \textcircled{y=4}$$

7. COTU $1 < X < 2$ KUN $2 < Y < 3$

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(c) $2x - 4y$

• $1 < x < 2 \Rightarrow 2 < 2x < 4 \Rightarrow 4 > 2x > 2$

- $2 < y < 3 \Rightarrow -8 > -4y > -12 \rightarrow \textcircled{7}$

$$-4 > 2x - 4y > -10$$

(B) $x^2 - \frac{1}{x}$

• $1 < x < 2 \Rightarrow 1 < x^2 < 4$

• $1 < x < 2 \Rightarrow 1 > \frac{1}{x} > \frac{1}{2} \Rightarrow -1 < -\frac{1}{x} < -\frac{1}{2}$

~~$$0 < x^2 - \frac{1}{x} < 4 - \frac{1}{2}$$~~

$$0 < x^2 - \frac{1}{x} < \frac{7}{2}$$

⑧ $y^2 - \frac{x}{y}$

• $2 < y < 3 \Rightarrow 4 < y^2 < 9$

④

• $1 < x < 2 \Rightarrow 2 > x > 1$

$\left. \begin{array}{l} \text{• } 1 < x < 2 \\ \text{• } 2 < y < 3 \end{array} \right\} \text{④ } 1 > \frac{x}{y} > \frac{1}{3}$

• $2 < y < 3 \Rightarrow \frac{1}{2} > \frac{1}{y} > \frac{1}{3}$

$-1 < -\frac{x}{y} < -\frac{1}{3}$

$3 < y^2 - \frac{x}{y} < 9 - \frac{1}{3}$

$3 < y^2 - \frac{x}{y} < \frac{26}{3}$

⑧ $(x+y)^2 - 2x$

• $1 < x < 2$
• $2 < y < 3$ } $3 < x+y < 5 \Rightarrow 9 < (x+y)^2 < 25$

• $1 < x < 2 \Rightarrow -2 > -2x > -4$
 $-4 < -2x < -2$ } $(+)$

$5 < (x+y)^2 - 2x < 23$

$$\textcircled{E} \cdot \frac{x-y}{x+y}$$

$$\begin{aligned} \bullet 1 < x < 2 & \Rightarrow 2 > x > 1 \\ \bullet 2 < y < 3 & \Rightarrow -2 > -y > -3 \end{aligned} \quad \left. \vphantom{\begin{aligned} \bullet 1 < x < 2 \\ \bullet 2 < y < 3 \end{aligned}} \right\} \textcircled{+}$$

$$0 > x - y > -2$$

$$-2 < x - y < 0$$

$$\begin{aligned} \bullet 1 < x < 2 \\ \bullet 2 < y < 3 \end{aligned} \quad \left. \vphantom{\begin{aligned} \bullet 1 < x < 2 \\ \bullet 2 < y < 3 \end{aligned}} \right\} \textcircled{+}$$

$$3 < x + y < 5$$

$$\frac{1}{3} > \frac{1}{x+y} > \frac{1}{5}$$

$$-\frac{1}{3} < -\frac{1}{x+y} < -\frac{1}{5}$$

$$\frac{2}{3} > \frac{x-y}{x+y} > 0$$

$$-\frac{2}{3} < \frac{x-y}{x+y} < 0$$

7. ⑦

Ερωτη
8

$$3 < x < 5$$

$$\Gamma = |x-2|^{\oplus} + |x-6|^{\ominus} + |x|^{\oplus}$$

$$\bullet \quad 3 < x < 5 \quad \Rightarrow \quad 1 < x-2 < 3$$

$$\bullet \quad 3 < x < 5 \quad \Rightarrow \quad -3 < x-6 < -1$$

$$\bullet \quad 3 < x < 5$$

$$\Gamma = \cancel{x-2} - \cancel{x+6} + x$$

$$\Gamma = x+4$$

8. ⑧ $x > 3$

$$A = |x-2|^{\oplus} - |x-3|^{\oplus} = x-2 - (x-3) = 1$$

$$\bullet \quad x > 3 \quad \Rightarrow \quad x-2 > 3-2 \quad \Rightarrow \quad x-2 > 1$$

$$\bullet \quad x > 3 \quad \Rightarrow \quad x-3 > 0$$

$$9. \textcircled{B} B = |6 - |x - 5||$$

$$\bullet -1 < x < 3$$

$$-1 - 5 < x - 5 < 3 - 5$$

$$-6 < x - 5 < -2$$

$$B = |6 - (-x + 5)|$$

$$B = |6 + x - 5|$$

$$B = |1 + x| \stackrel{\textcircled{+}}{=} x + 1$$

$$\bullet -1 < x < 3$$

$$0 < x + 1 < 4$$

Σχολικά Σελ 66

ιδιωτικά

$$\left| \frac{x}{y} \right| = \frac{|x|}{|y|}$$

$$|x| = |-x|$$

4. $\forall a \neq b$

$$A = \left| \frac{a-b}{b-a} \right| = \frac{|a-b|}{|b-a|} = \frac{|a-b|}{|a-b|} = 1,$$

5. $\forall x \neq 0, y \neq 0$

$$A = \frac{|x|}{x} + \frac{|y|}{y}.$$

1. $\forall x > 0, y > 0 \Rightarrow A = \frac{x}{x} + \frac{y}{y} = 1 + 1 = 2$

2. $\forall x > 0, y < 0 \Rightarrow A = \frac{x}{x} - \frac{y}{y} = 1 - 1 = 0$

3. $\forall x < 0, y > 0 \Rightarrow A = -\frac{x}{x} + \frac{y}{y} = -1 + 1 = 0$

4. $\forall x < 0, y < 0 \Rightarrow A = -\frac{x}{x} - \frac{y}{y} = -1 - 1 = -2$

$$10. \textcircled{B} \quad B = \left| |a - b| + |b - c| \right|$$

$$\bullet \quad a < b < c$$

$$a < b \Rightarrow a - b < 0$$

$$b < c \Rightarrow b - c < 0$$

$$B = \left| -a + b - b + c \right|$$

$$B = \left| c - a \right| = c - a.$$

$$c > a \Rightarrow c - a > 0$$

Μεγαλύτερη Προσέγγιση

1. $|x| = \theta \Rightarrow x = \theta \text{ ή } x = -\theta$

2. $|x| = |y| \Rightarrow x = y \text{ ή } x = -y$

3. $|x| \leq \theta \Rightarrow -\theta \leq x \leq \theta$
 $x \in [-\theta, \theta]$

4. $|x| \geq \theta \Rightarrow x \geq \theta \text{ ή } x \leq -\theta$
 $x \in (-\infty, -\theta] \cup [\theta, +\infty)$