Ενοτητα 12

2. α) Η f συνεχής στο $[0,1]$ ως

π.σ.σ.

$$\left.\begin{array}{l} f(0)=1 \\ f(1)=-1 \end{array}\right\} f(0)f(1)=-1<0$$

Ικανοποιούνται οι προυποθέσεις του Bolzano.

β) Από πριν $\exists\, \xi \in (0,1)$ τ.ω $f(\xi)=0$

$$\xi^3-3\xi+1=0$$

4. (α) Νδο η εξίσωση $2x^3+3x-1=0$ έχει ρίζα στο $(0,1)$

$$\boxed{f(x)=2x^3+3x-1}$$

Η f συνεχής στο $[0,1]$ ως π.σ.σ

$$\left.\begin{array}{l} f(0)=-1 \\ f(1)=4 \end{array}\right\} \begin{array}{l} f(0)f(1)<0 \quad \text{Bolzano} \\ \exists\, \xi\in(0,1) \text{ τ.ω } f(\xi)=0 \end{array}$$

$$2\xi^3+3\xi-1=0.$$

5. Νδο $\exists x_0 \in(0,\pi)$ τ.ω $\sigma\upsilon\nu x_0 = x_0-4$

$$\sigma\upsilon\nu x = x-4$$

$$\underbrace{\sigma\upsilon\nu x - x+4}_{f(x)} = 0$$

Η $f(x)$ συνεχής στο $[0,\pi]$ ως π.σ.σ

$$\left.\begin{array}{l} f(0)=5>0 \\ f(\pi)=-1-\pi+4=3-\pi<0 \end{array}\right\} f(0)f(\pi)<0$$

Bolzano $\exists x_0 \in(0,\pi)$ τ.ω $f(x_0)=0$

$$\sigma\upsilon\nu x_0 = x_0-4$$

6. ⓐ Νδο $\exists x_0 \in (0,1)$ τ.ω $\ln(x_0+1) = 1 - x_0$

$$\ln(x+1) = 1 - x$$

$$\underbrace{\ln(x+1) - 1 + x}_{f(x)} = 0$$

Η f συνεχής $[0,1]$ ως π.σ.σ

$$\left.\begin{aligned} f(0) &= \ln 1 - 1 = -1 < 0 \\ f(1) &= \ln 2 > 0 \end{aligned}\right\} f(0)f(1) < 0$$

Bolzano $\exists x_0 \in (0,1)$ τ.ω $f(x_0) = 0$

$$\ln(x_0+1) = 1 - x_0.$$

8. (α) $\dfrac{x^4+1}{x-1} + \dfrac{x^6+1}{x-2} = 0$

$$\underbrace{(x-2)(x^4+1) + (x^6+1)(x-1)}_{f(x)\ \text{συνεχής στο } [1,2] \text{ ως π.σ.σ}} = 0$$

$$\left.\begin{aligned} f(1) &= -2 < 0 \\ f(2) &= 65 > 0 \end{aligned}\right\} f(1)f(2) < 0$$

Bolzano $\exists\, \xi \in (1,2)$ τ.ω $f(\xi) = 0$

$$\frac{\xi^4+1}{\xi-1} + \frac{\xi^6+1}{\xi-2} = 0.$$

③ $\dfrac{x^2+1}{x} + \dfrac{x^4+1}{x-1} = 1$

$$\underbrace{(x^2+1)(x-1) + x(x^4+1) - x(x-1)}_{f(x)} = 0$$

$f(x)$ συνεχής $[0,1]$ ως π.δ.σ

$f(0) = -1$

$f(1) = 2$

$\Big\}\ f(0)f(1) < 0$

Bolzano $\exists\, x_0 \in (0,1)$

τ.ω $f(x_0) = 0$.

$$\frac{x_0^2+1}{x_0} + \frac{x_0^4+1}{x_0-1} = 1$$

8 $x = \dfrac{2x+1}{x^2-1}$

$$\underbrace{x(x^2-1) - 2x - 1}_{f(x)} = 0$$

$f(x)$ συνεχής $[-1,1]$ ως π.δ.σ

$$\left.\begin{array}{l} f(-1) = 1 \\ f(1) = -3 \end{array}\right\} f(-1)f(1) < 0$$

Bolzano $\exists\, x_0 \in (-1,1)$

τ.ω $f(x_0) = 0$,

$$x_0 = \frac{2x_0+1}{x_0^2-1}$$

9. Νδο η εξίσωση $f(x) = \dfrac{2e^x - 3}{x^2 - x}$

έχει ρίζα $(0,1)$.

$$\underbrace{f(x)(x^2-x) - 2e^x + 3}_{\varphi(x)} = 0$$

$\varphi(x)$ συνεχής $[0,1]$ ως π.δ.σ.

$$\left.\begin{aligned} \varphi(0) &= 1 > 0 \\ \varphi(1) &= 3 - 2e < 0 \end{aligned}\right\} \varphi(0)\varphi(1) < 0$$

Bolzano $\exists\, \alpha \in (0,1)$ τ.ω $\varphi(\alpha) = 0$.

$$f(\alpha) = \frac{2e^{\alpha} - 3}{\alpha^2 - \alpha}.$$

10. $f(x)=2x^3+x-2$

(α) Έστω $x_1<x_2 \Rightarrow 2x_1^3<2x_2^3$

(+)

Έστω $x_1<x_2 \Rightarrow x_1-2<x_2-2$

$f \nearrow$

(β) $x=\dfrac{2}{2x^2+1}$

$2x^3+x=2$

$\underbrace{2x^3+x-2}=0$

$f(x)$ συνεχής $[0,1]$ ως π.δ.σ.

$f(0)=-2$

$f(1)=1$ $\Big\}$ $f(0)f(1)<0$

Bolzano $\exists!\,\xi\in(0,1)$ τ.ω. $f(\xi)=0$

λόγω μονοτονίας.

11. (α) $\underbrace{x^3+x-1}_{\varphi(x)}=0$

- $x_1<x_2 \Rightarrow x_1^3<x_2^3$

(+)

- $x_1<x_2 \Rightarrow x_1-1<x_2-1$

$\varphi \nearrow$

$\left.\begin{aligned}\varphi(0)&=-1\\ \varphi(1)&=1\end{aligned}\right\}\ \varphi(0)\varphi(1)<0$

Bolzano $\exists\, \xi\in(0,1)$ τ.ω. $\varphi(\xi)=0$.

$\xi^3+\xi-1=0$

(β) $e^x+3x=2$

$\underbrace{e^x+3x-2}_{h(x)}=0$

$h(x)$ συνεχής $[0,1]$ ως π.δ.ο.

$\left.\begin{aligned}h(0)&=-1<0\\ h(1)&=e+1>0\end{aligned}\right\}\ h(0)h(1)<0$

Bolzano $\exists\, \xi\in(0,1)$ τ.ω. $h(\xi)=0$

$e^{\xi}+3\xi=2$ Επιπλέον $h\nearrow$

Το ξ μοναδικό!

12. $f(x) = 3x^5 - x - 1$

H $f(x)$ συνεχής $[0,1]$ ως π.δ.σ.

$$\left.\begin{array}{l} f(0) = -1 \\ f(1) = 1 \end{array}\right\} f(0)f(1) < 0$$

Bolzano $\exists x_0 \in (0,1)$ τ.ω $f(x_0) = 0$.

14. $f(x) = e^x \qquad g(x) = \frac{1}{x}$

$\left(\frac{1}{2}, \ln 2\right)$

$f(x) = g(x) \Rightarrow e^x = \frac{1}{x}$

$\underbrace{e^x - \frac{1}{x}}_{h(x)} = 0$

$h\left(\frac{1}{2}\right) = e^{1/2} - 2 = \sqrt{e} - 2 < 0$

$h(\ln 2) = e^{\ln 2} - \frac{1}{\ln 2} = 2 - \frac{1}{\ln 2} > 0$

$h\left(\frac{1}{2}\right) h(\ln 2) < 0$

- $x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2}$ (+)
- $x_1 < x_2 \Rightarrow \frac{1}{x_1} > \frac{1}{x_2} \Rightarrow -\frac{1}{x_1} < -\frac{1}{x_2}$

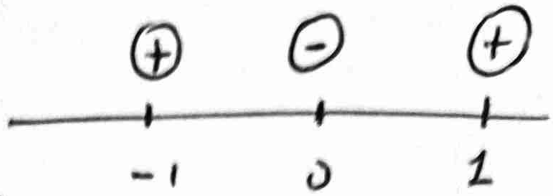
$h \nearrow$

Bolzano

$\exists x_0$ τ.ω $h(x_0) = 0$

$f(x_0) = g(x_0)$.

16. $f(x)=3x^4-x-1$



Η f συνεχής $[-1,1]$ ως π.σ.σ

$f(-1)=3$

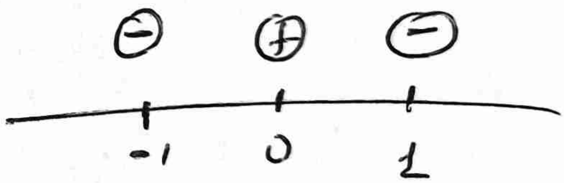
$f(0)=-1$

$f(1)=1$

$f(-1)f(0)<0$ Bolzano $\exists \xi_1 \in (-1,0)$ τ.ω $f(\xi_1)=0$

$f(0)f(1)<0$ Bolzano $\exists \xi_2 \in (0,1)$ τ.ω $f(\xi_2)=0$.

17. B $f(x)=x^3-6x^2+3$



Η f συνεχής $[-1,1]$ ως π.σ.σ.

$f(-1)=-4$

$f(0)=3$

$f(1)=-2$

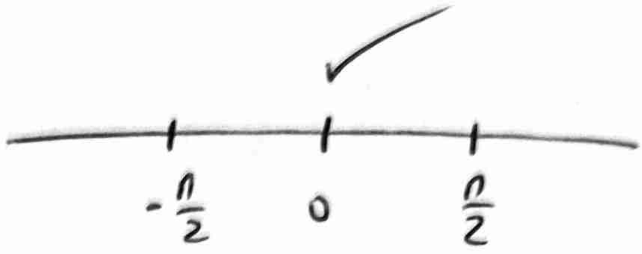
$f(-1)f(0)<0$ Bolzano $\exists x_1 \in (-1,0)$ τ.ω $f(x_1)=0$

$f(0)f(1)<0$ Bolzano $\exists x_2 \in (0,1)$ τ.ω $f(x_2)=0$

18. $f(x) = \begin{cases} x^2 , & x \leq 0 \\ e^x - 1 , & x > 0 \end{cases}$

(α)

$\left.\begin{aligned} \lim_{x\to 0^-} f(x) = 0 \\ \lim_{x\to 0^+} f(x) = 0 \end{aligned}\right\} \lim_{x \to 0} f(x) = 0$



$f(0) = 0$ Συνεχής στο 0!

Η f συνεχής $[-\frac{\pi}{2}, 0)$ και $(0, \frac{\pi}{2}]$ ως π.σ.σ

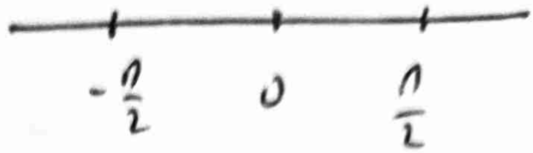
$\Rightarrow$ f συνεχής $[-\frac{\pi}{2}, \frac{\pi}{2}]$ συνεχής π.σ.σ.

$\left.\begin{aligned} f(-\tfrac{\pi}{2}) = \tfrac{\pi^2}{4} > 0 \\ f(\tfrac{\pi}{2}) = e^{\pi/2} - 1 > 0 \end{aligned}\right\}$ Δεν ικανοποιείται το θ. Bolzano.

(β) $f(x) = συνx$

$\underbrace{f(x) - συνx}_{g(x)} = 0$

$x \in [-\frac{\pi}{2}, 0]$



Η $g(x) = f(x) - συνx$

$g(x) = x^2 - συνx$ συνεχής στο $[-\frac{\pi}{2}, 0]$ ως π.σ.σ

$g(-\frac{\pi}{2}) = \frac{\pi^2}{4} > 0$

$g(0) = -1 < 0$

$\Big\}$ $g(-\frac{\pi}{2})g(0) < 0$ Bolzano

$\exists\, \xi_1 \in (-\frac{\pi}{2}, 0)$ τ.ω $g(\xi_1) = 0$

$\Rightarrow f(\xi_1) = συν\xi_1$

$x \in [0, \frac{\pi}{2}]$

$g(x) = f(x) - συνx$

$g(x) = e^x - 1 - συνx$

$g(0) = -1 <$

$g(\frac{\pi}{2}) = e^{\pi/2} - 1 > 0$

$\Big\}$ Bolzano $\exists\, \xi_2 \in (0, \frac{\pi}{2})$

τ.ω $g(\xi_2) = 0$

$f(\xi_2) = συν\xi_2$

Μονοτονία της $g(x)$

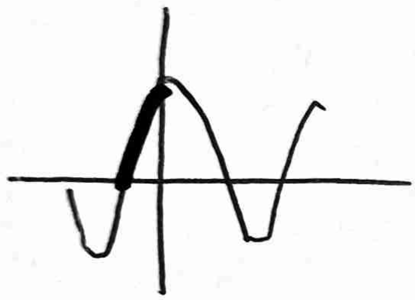
$x \in [-\frac{\pi}{2}, 0]$

$g(x) = x^2 - \text{συν}\, x$

$x_1 < x_2 \Rightarrow x_1^2 > x_2^2$ (+)

$x_1 < x_2 \Rightarrow \text{συν}\, x_1 < \text{συν}\, x_2 \Rightarrow -\text{συν}\, x_1 > -\text{συν}\, x_2$

$g \searrow$ το x_1 μοναδικό



$x \in [0, \frac{\pi}{2}]$

$g(x) = e^x - 1 - \text{συν}\, x$

- $x_1 < x_2 \Rightarrow e^{x_1} - 1 < e^{x_2} - 1$ (+)
- $x_1 < x_2 \Rightarrow \text{συν}\, x_1 > \text{συν}\, x_2 \Rightarrow -\text{συν}\, x_1 < -\text{συν}\, x_2$

$g \nearrow$ το x_2 μοναδικός

20. $f : [1,2] \longrightarrow \mathbb{R}$ συνεχής.

$1 \leq f(x) \leq 2 \quad \forall x \in [1,2]$.

Ν.δ.ο $\exists x_0 \in [1,2]$ τ.ω $f(x_0) = x_0$

$$\underbrace{f(x) - x}_{g(x)} = 0$$

$1 \leq f(1) \leq 2$

$0 \leq f(1) - 1 \leq 1$

$g(1) = f(2) - 1 \geq 0$

$g(2) = f(2) - 2 \leq 0$

$1 \leq f(2) \leq 2$

$-1 \leq f(2) - 2 \leq 0$

$g(1)\, g(2) \leq 0$

1. $g(1)\, g(2) < 0$

Bolzano

$\exists x_0 \in [1,2]$ τ.ω $g(x_0) = 0$.

2. $g(1)\, g(2) = 0$

$g(1) = 0$ ή $g(2) = 0$

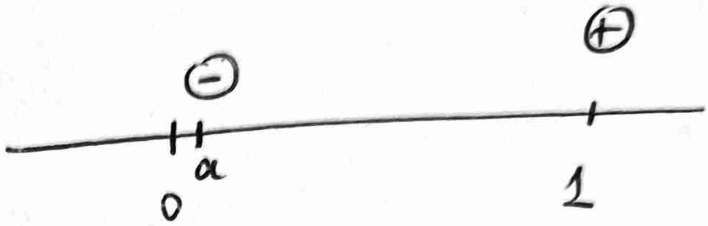
$x_0 = 1$ ή $x_0 = 2$.

$f(x_0) = x_0$

23. ΝΔΟ η εξίσωση

$\ln x = x^2 - 2x$ έχει ρίζα $(0,1)$

$\underbrace{\ln x - x^2 + 2x}_{f(x)} = 0$



$f(1) = 1.$

$\lim\limits_{x \to 0^+} f(x) = -\infty$ $\quad \exists\, \alpha > 0$ κοντά στο 0^+ τ.ω $f(\alpha) < 0$

Η f συνεχής $[\alpha, 1]$ ως π.σ.σ.

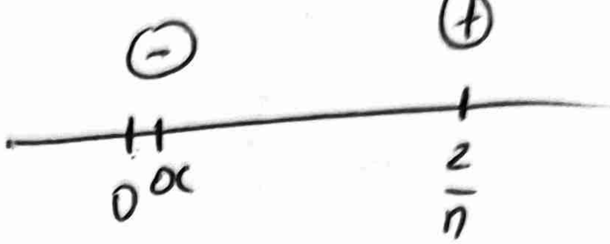
$f(\alpha) f(1) < 0$

Bolzano $\exists\, x_0 \in [\alpha, 1]$ τ.ω

$f(x_0) = 0$

24. Νδο η εξίσωση $2x\eta\mu\frac{1}{x} = 1$

έχει ρίζα στο $\left(0, \frac{2}{\pi}\right)$.



$$\boxed{f(x) = 2x\eta\mu\frac{1}{x} - 1}$$

$$f\left(\frac{2}{\pi}\right) = 2\cdot\frac{2}{\pi}\eta\mu\frac{\pi}{2} - 1 = \frac{4}{\pi} - 1 > 0$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(\cancelto{0}{2x\eta\mu\frac{1}{x}} - 1\right) = -1$$

$$-1 \le \eta\mu\frac{1}{x} \le 1$$

$$-2x \le 2x\eta\mu\frac{1}{x} \le 2x$$

$$\left.\begin{array}{l}\lim\limits_{x\to 0^+} -2x = 0 \\ \lim\limits_{x\to 0^+} 2x = 0\end{array}\right\} \text{Από Κ.Π} \lim_{x\to 0} 2x\eta\mu\frac{1}{x} = 0$$

Αφού $\lim\limits_{x\to 0^+} f(x) = -1$ $\exists\, \alpha > 0$ κοντά στο 0

τ.ω $f(\alpha) < 0$.

Συνεπώς $f(\alpha) f(\frac{2}{\pi}) < 0$

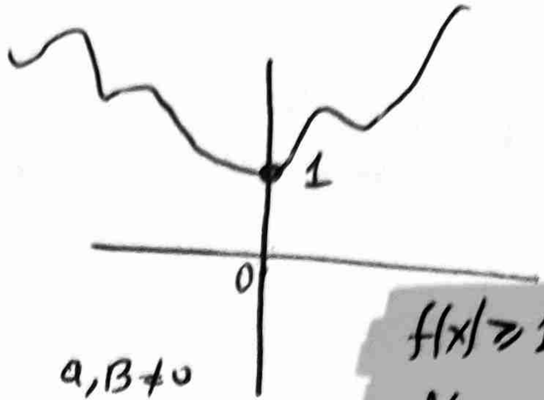
Bolzano $\exists\, \xi \in (\alpha, \frac{2}{\pi})$ τ.ω

$f(\xi) = 0$.

Η f συνεχής $[\alpha, \frac{2}{\pi}]$

ως π.δ.τ.

38.



① $\dfrac{f(\alpha)-1}{x-1} + \dfrac{f(\beta)-1}{x-2} = 5$ $\quad \alpha,\beta \neq 0$

$f(x) \geq 1$
$\forall x \in \mathbb{R}$

$$\boxed{g(x) = (x-2)(f(\alpha)-1) + (x-1)(f(\beta)-1) - 5(x-1)(x-2)}$$

$$\left.\begin{array}{l} g(1) = 1 - f(\alpha) < 0 \\ g(2) = f(\beta) - 1 > 0 \end{array}\right\} g(1)\,g(2) < 0$$

Αφού $f(x) > 1 \quad \forall x \in \mathbb{R}$.

$f(\alpha) > 1$ και $f(\beta) > 1$.

$0 > 1 - f(\alpha)$ $\qquad$ $f(\beta) - 1 > 0$

Bolzano $\exists\, \xi \in (1,2)$ τ.ω

$$g(\xi) = 0 \rightarrow \frac{f(\alpha)-1}{\xi - 1} + \frac{f(\beta)-1}{\xi - 2} = 5.$$

(B) · Νδο η εξισωση $f(g(x))=1$ εχει 1 ριζα μονο $(0,1)$.

$f(g(x))=1$.

$g(x)=e^x+x-2$

Μονο $g(x)=0$

$e^x+x-2=0$.

$\left.\begin{array}{l} g(0)=-1<0 \\ g(1)=e-1>0 \end{array}\right\}$ $g(0)g(1)<0$

Bolzano $\exists \xi \in (0,1)$

τ.ω $g(\xi)=0$.

ξ μοναδικο.

Μονοτονια

$\cdot\ x_1<x_2 \Rightarrow \underbrace{e^{x_1}<e^{x_2}}_{(+)}$

$g \nearrow$

40. $f(x) = e^{-x} - \ln(x+1) - 1$.

$\alpha \in (-1, 0)$

$$\frac{f(\alpha)}{x-1} - \frac{f(\beta)}{x-2} = 1$$

$\beta > 0$,

$$\boxed{\varphi(x) = f(\alpha)(x-2) - f(\beta)(x-1) - (x-1)(x-2)}$$

$\varphi(1) = -f(\alpha)$

$\varphi(2) = -f(\beta)$

$\varphi(1)\varphi(2) = \underset{(+)}{f(\alpha)}\,\underset{(-)}{f(\beta)} < 0$

Bolzano $\exists\, \xi \in (\alpha, \beta)$ τω $\varphi(\xi) = 0$

Μονοτονία $f(x)$

Εύκολα

$f \downarrow$

x	α	0	β
f	$\searrow$ $+$	0	$-$ $\searrow$

$f(\alpha) > 0$ $\quad$ $f(\beta) < 0$

Ενότητα 13

10. $f: \mathbb{R} \to \mathbb{R}$ συνεχής.

$f(\tfrac{1}{2}) > 0$

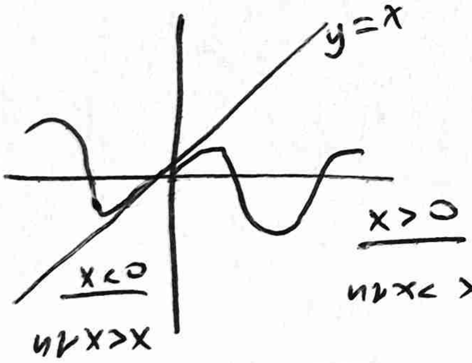
x		0		1	
$f(x)$	;	0	$+$	0	;

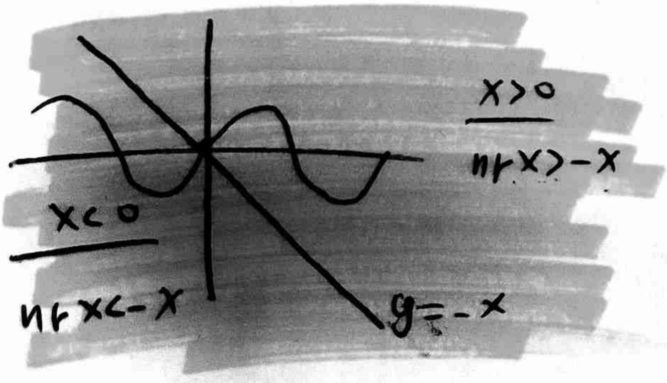
(α) $\lim\limits_{x \to 0^+} \dfrac{1}{f(x)} = +\infty$

Αν $x>0$ τότε $f(x)>0$ κοντά στο 0.

(β) $\lim\limits_{x \to 1^-} e^{-\frac{1}{f(x)}} = e^{-(+\infty)} = e^{-\infty} = 0$.

11. (β) $f(x) = \eta\mu x + x$





Ρίζα $f(x)$

$f(x)=0$

$\eta\mu x + x = 0$

$\eta\mu x = -x$

$x = 0$

x		0	
$f(x)$	$-$	0	$+$

$x>0 \Rightarrow \eta\mu x > -x \Rightarrow \eta\mu x + x > 0$

$f(x) > 0$

II. (γ) $f(x) = \sqrt{x^2+1} + x$

$D_f = \mathbb{R}$.

$f(0) = 1$

Ρίζα $f(x)$

$f(x) = 0$

$\sqrt{x^2+1} + x = 0$

$\sqrt{x^2+1} = -x$

Αν $x \geq 0$ αδύνατη!

Αν $x < 0$ τότε $\cancel{x^2}+1 = \cancel{x^2}$

$1 = 0$ Αδύνατη.

Η εξίσωση $f(x)=0$
Αδύνατη.

$f(x) \neq 0$ και συνεχής.

$f(x) > 0$ ή $f(x) < 0$.

Αφού $f(0) = 1$ $f(x) > 0$

12. (B) $f(x)=\sqrt{2}\cos x+1 \quad , x\in[0,\pi]$

Piru $f(x)$

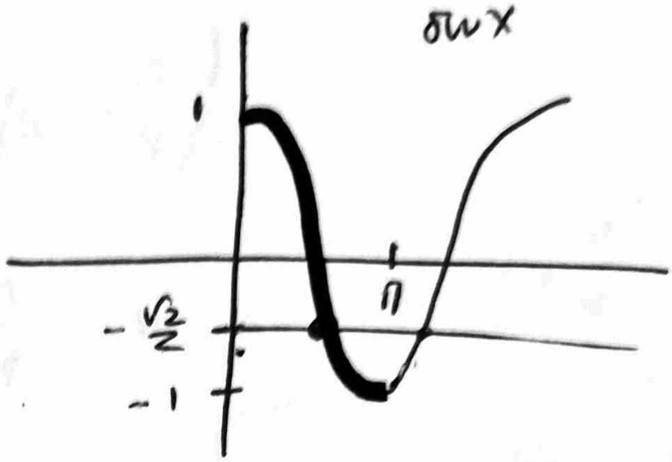
$f(x)=0$

$\sqrt{2}\cos x+1=0$

$\cos x=-\frac{1}{\sqrt{2}}$

$\cos x=-\frac{\sqrt{2}}{2}$

$x=\frac{2\pi}{3}$



$\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$\cos 120=-\frac{\sqrt{2}}{2}$

$\frac{a}{\pi}=\frac{r}{180}$

$\frac{a}{\pi}=\frac{12\cancel{0}}{18\cancel{0}}$

$\frac{a}{\pi}=\frac{2}{3}$

$a=\frac{2\pi}{3}$

x	0	$\frac{2\pi}{3}$	π
$f(x)$	$+$	0	$-$

$f(0)=\sqrt{2}+1$

$f(\pi)=-\sqrt{2}+1$

B' τρόπος $\quad \sigma\upsilon\nu x = -\frac{\sqrt{2}}{2}$ $\quad \sigma\upsilon\nu(\pi - x) = -\sigma\upsilon\nu x$

Παραμετρική

$$\sigma\upsilon\nu x = -\sigma\upsilon\nu\frac{\pi}{4}$$

$$\sigma\upsilon\nu x = \sigma\upsilon\nu\left(\pi - \frac{\pi}{4}\right).$$

$$x = 2\kappa\pi + \pi - \frac{\pi}{4} \quad \text{ή} \quad x = 2\kappa\pi - \pi + \frac{\pi}{4}$$

$$\boxed{x = 2\kappa\pi + \frac{3\pi}{4}} \qquad \boxed{x = 2\kappa\pi - \frac{3\pi}{4}}$$

$k=0 \quad x = \frac{3\pi}{4}$ $\qquad$ $k=0 \quad \cancel{x = -\frac{3\pi}{4}}$

$k=1 \quad x = 2\pi + \frac{3\pi}{4}$ $\qquad$ $k=1 \quad x = 2\pi - \frac{3\pi}{4}$

$\cancel{x = \frac{11\pi}{4}}$ $\qquad$ $\cancel{x = \frac{5\pi}{4}}$

γ' τρόπος

$$\etaμx = -\frac{\sqrt{2}}{2}$$

$$\etaμx = -\etaμ\frac{\pi}{4}$$

$$\etaμx = \etaμ\left(\pi - \frac{\pi}{4}\right)$$

$$\etaμx = \etaμ\frac{3\pi}{4}.$$

$$\boxed{x = 2k\pi + \frac{3\pi}{4}}$$

$$0 \le x \le \pi$$

$$0 \le 2k\pi + \frac{3\pi}{4} \le \pi$$

$$0 \le 2k + \frac{3}{4} \le 1$$

$$0 \le 8k + 3 \le 4$$

$$-3 \le 8k \le 1$$

$$-\frac{3}{8} \le k \le \frac{1}{8}$$

$k \in \mathbb{Z}$. $\underline{\underline{k=0}}$ $x = \frac{3\pi}{4}$

$$\boxed{x = 2k\pi - \frac{3\pi}{4}}$$

$$0 \le x \le \pi$$

$$0 \le 2k\pi - \frac{3\pi}{4} \le \pi$$

$$0 \le 2k - \frac{3}{4} \le 1$$

$$0 \le 8k - 3 \le 4$$

$$3 \le 8k \le 7$$

$$\frac{3}{8} \le k \le \frac{7}{8}$$

$$k \in \mathbb{Z}$$

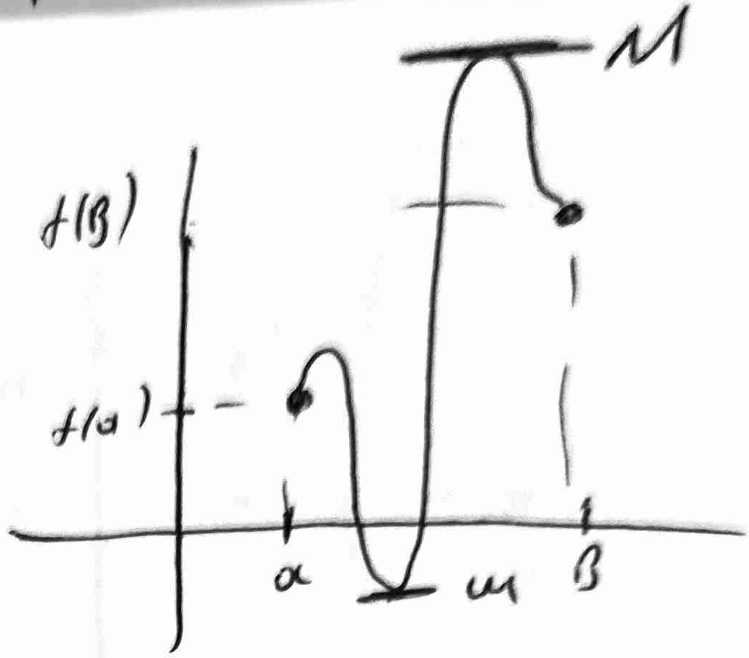
Θεώρημα Μέγιστης Ελάχιστης Τιμής

- f συνεχής $[\alpha, \beta]$

Τότε

$$m \leq f(x) \leq M$$

$\forall x \in [\alpha, \beta]$.



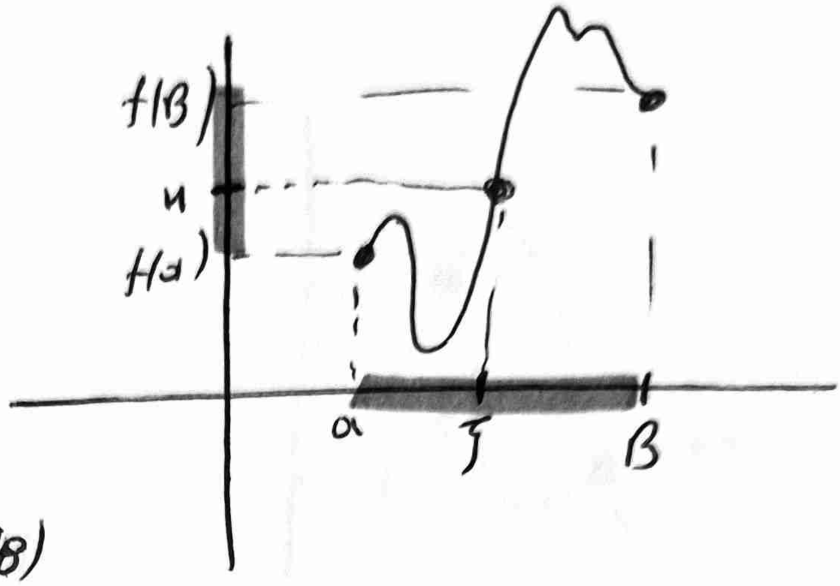
Θεωρημα Ενδιαμεσων Τιμων (ΘΕΤ)

- f συνεχη $[\alpha,\beta]$
- $f(\alpha) \neq f(\beta)$

$\exists\, \xi \in (\alpha,\beta)$

τ.ω $f(\xi)=\eta$

εφοσον $f(\alpha)<\eta<f(\beta)$



Αποδειξη

Αρκει νδο $\exists\, \xi \in (\alpha,\beta)$ τ.ω $f(\xi)=\eta$

$$f(x)=\eta$$

$$\underbrace{f(x)-\eta}_{g(x)}=0$$

$f(\alpha)<\eta$

$f(\alpha)-\eta<0$

$$g(\alpha)=f(\alpha)-\eta<0$$

$f(\beta)>\eta$

$$g(\beta)=f(\beta)-\eta>0$$

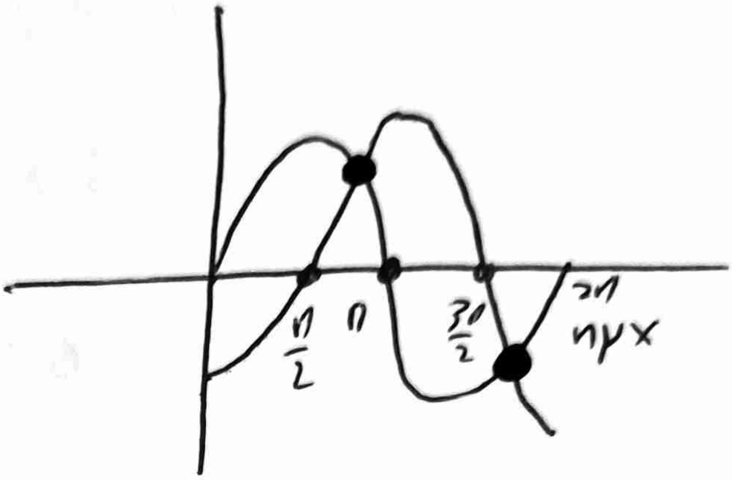
$f(\beta)-\eta>0$

$g(\alpha)g(\beta)<0$ Bolzano $\exists\, \xi \in (\alpha,\beta)$

τ.ω $g(\xi)=0 \Rightarrow f(\xi)=\eta$

12. (8) $f(x) = \sin x + \cos x \qquad x \in [0, 2\pi)$.

P,7d f(x)



$f(x) = 0$

$\sin x + \cos x = 0$

$\sin x = -\cos x$

$\sin x = \cos(\pi - x)$

$\sin x = \sin\left(\frac{\pi}{2} - (\pi - x)\right)$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\sin x = \sin\left(\frac{\pi}{2} - \pi + x\right)$

$\sin x = \sin\left(x - \frac{\pi}{2}\right)$

$\cancel{x} = 2k\pi + \cancel{x} - \frac{\pi}{2}$

Adwa

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$x = 2k\pi + \pi - x + \frac{\pi}{2}$

$2x = 2k\pi + \frac{3\pi}{2}$

$\boxed{x = k\pi + \frac{3\pi}{4}}$

$$0 \le x \le 2\pi$$

$$0 \le k\pi + \frac{3\pi}{4} \le 2\pi$$

$$0 \le k + \frac{3}{4} \le 2$$

$$0 \le 4k + 3 \le 8$$

$$-3 \le 4k \le 5$$

$$-\frac{3}{4} \le k \le \frac{5}{4}$$

$$k \in \mathbb{Z}.$$

$$k = 0 \Rightarrow \boxed{x = \frac{3\pi}{4}}$$

$$k = 1 \Rightarrow x = \pi + \frac{3\pi}{4} = \boxed{\frac{7\pi}{4}}.$$

x	0		$\frac{3\pi}{4}$		$\frac{7\pi}{4}$		2π
$f(x)$		$+$	0	$-$	0	$+$	

$f(\pi) = -1$

$f(0) = 1$

$f(2\pi) = 1$

Επομενο Μαθημα

Δευτερα 9-11:30

13

2

5

7

9

11. α

12 α γ.