

1.

$$P(x) = (x^2 - 1)x^2 + (x^2 + 1)x - x - 1$$

$$\bullet x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$$\underline{x=1} \text{ u' } \underline{x=-1}$$

1. Av $x=1$ tot: $P(x) = 2x - 2$

1o Bando

Seu exo Bando

2. Av $x=-1$ tot: $P(x) = 0$

3. Av $x \neq 1, x \neq -1$ tot: 2o Bando.

2.

$$\begin{array}{r|l} \textcircled{a} & x^2 + x \\ 3x^3 - 2x^2 - 1 & \\ - (3x^3 + 3x^2) & 3x - 5 \\ \hline -5x^2 - 1 & \\ - (-5x^2 - 5x) & \\ \hline 5x - 1 & \end{array}$$

$$3x^3 - 2x^2 - 1 = (x^2 + x)(3x - 5) + 5x - 1$$

$$\begin{array}{r|l} \textcircled{b} & x - 1 \\ x^4 - x + 2 & \\ - (x^4 - x^3) & x^3 + x^2 + x \\ \hline x^3 - x + 2 & \end{array}$$

$$x^3 - x + 2$$

$$- (x^3 - x^2)$$

$$+ x^2 - x + 2$$

$$- (x^2 - x)$$

$$(x^4 - x + 2)(x - 1)(x^3 + x^2 + x) + 2$$

3. (a) $(x^4 - x + 2) : (x - 1)$

$$\begin{array}{r}
 1 \quad 0 \quad 0 \quad -1 \quad 2 \quad \textcircled{1} \\
 \downarrow \\
 1 \quad 1 \quad 1 \quad 0 \quad \textcircled{2}
 \end{array}$$

$$x^4 - x + 2 = (x - 1)(x^3 + x^2 + x) + 2$$

(b) $(x^3 + 2x^2 + 4x - 1) : (x - \frac{1}{2})$

$$\begin{array}{r}
 1 \quad 2 \quad 4 \quad -1 \quad \textcircled{\frac{1}{2}} \\
 \downarrow \\
 1 \quad \frac{5}{2} \quad \frac{21}{4} \quad \frac{13}{8}
 \end{array}$$

$$x^3 + 2x^2 + 4x - 1 = \left(x - \frac{1}{2}\right)\left(x^2 + \frac{5}{2}x + \frac{21}{4}\right) + \frac{13}{8}$$

4. a) $P(x) = x^4 + ax^3 + bx^2 - 22x + 6a$

Εχουμε παραγοντα $\Rightarrow x^2 + 2x - 3$

$$\rightarrow x^2 + 2x - 3 = (x+3)(x-1)$$

$$P(-3) = 0 \Rightarrow 81 - 27a + 9b + 66 + 6a = 0$$

$$P(1) = 0 \Rightarrow 1 + a + b - 22 + 6a = 0$$

$$\boxed{7a + b = 21}$$

$$\boxed{-21a + 9b = -147}$$

$$\begin{cases} 7a + b = 21 \\ -21a + 9b = -147 \end{cases}$$

$$\Rightarrow \dots \Rightarrow \underline{\underline{a=4}}$$

$$b = -7.$$

(B) $P(x) \geq 0 \Rightarrow \boxed{x^4 + 4x^3 - 7x^2 - 22x + 24 \geq 0}$

1	4	-7	-22	24	(1)
↓	1	5	-2	-24	
1	5	-2	-24	0	

$$(x-1)(x^3 + 5x^2 - 2x - 24) = 0.$$

$$(x-1)(x^3+5x^2-2x-24) \geq 0$$

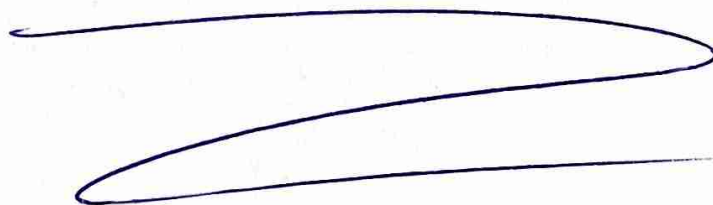
$$\begin{array}{rrrrr} 1 & 5 & -2 & -24 & (-3) \\ \downarrow & -3 & -6 & 24 & \\ 1 & 2 & -8 & 0 & \end{array}$$

$$(x-1)(x+3)(x^2+2x-8) \geq 0$$

$$\textcircled{1} \quad \textcircled{-3} \quad \textcircled{-4} \quad \textcircled{2}$$

x		-4	-3	1	2
x-1	-	-	-	+	+
x+3	-	-	+	+	+
x ² +2x-8	+	+	-	-	+
P(x)	+	-	+	-	+

$$x \in (-\infty, -4] \cup [-3, 1] \cup [2, +\infty)$$



5. (a) $P(x) = x^4 + 3x^3 + \alpha x^2 + \beta x - 6$ έχ

παράγοντα $x^2 + 2x + 1 = (x+1)^2$.

$$\begin{array}{cccccc}
 1 & 3 & \alpha & \beta & -6 & (-1) \\
 \downarrow & & & & & \\
 1 & -1 & -2 & 2-\alpha & \alpha-\beta-2 & \\
 1 & 2 & \alpha-2 & 2-\alpha+\beta & \boxed{\alpha-\beta-8} & \rightarrow 0
 \end{array}$$

$$\begin{array}{cccccc}
 1 & 2 & \alpha-2 & 2-\alpha+\beta & (-1) & \\
 \downarrow & & & & & \\
 1 & -1 & -1 & 3-\alpha & & \\
 1 & 1 & \alpha-3 & \boxed{\beta-2\alpha+5} & \rightarrow 0 &
 \end{array}$$

$$\begin{cases}
 \alpha - \beta = 8 \\
 \beta - 2\alpha = -5
 \end{cases} \quad (+)$$

$$-\alpha = 3$$

$$\alpha = -3$$

$$\beta = -11$$

③ $P(x) = 0$

$$x^4 + 3x^3 - 3x^2 - 11x - 6 = 0$$

$$\begin{array}{cccccc} 1 & 3 & -3 & -11 & -6 & (-1) \\ \downarrow & & & & & \\ 1 & 2 & -5 & -6 & 0 & \end{array}$$

$$(x+1)(x^3 + 2x^2 - 5x - 6) = 0$$

$$\begin{array}{cccc} 1 & 2 & -5 & -6 & (-1) \\ \downarrow & & & & \\ 1 & 1 & -6 & 0 & \end{array}$$

$$(x+1)(x+1)(x^2 + x - 6) = 0$$

$$(x+1)^2(x^2 + x - 6) = 0$$

$$(x+1)^2 = 0$$

$$x = -1$$

$$\vee \quad x^2 + x - 6 = 0$$

$$x = -3$$

$$x = 2$$

⑧ $P(x) > 0$

$$(x+1)^2 (x^2+x-6) > 0$$

X	-3	-1	2
$(x+1)^2$	+	+	+
x^2+x-6	+	0	-
$P(x)$	+	-	+

$$x \in (-\infty, -3) \cup (2, +\infty)$$

6. a) $P(x) = 12x^3 - 8x^2 - 10x + \alpha$ EX4

παράγονται το $3x+1$

$$P\left(-\frac{1}{3}\right) = 0$$

12	-8	-10	α	$\left(-\frac{1}{3}\right)$
		4	2	
$\downarrow$	-4			
12	-12	-6	$\alpha+2$	αλυστω $\alpha+2=0$
				$\alpha = -2$

b) $P(x) \leq 3x+1$

$$12x^3 - 8x^2 - 10x - 2 \leq 3x+1$$

$$\left(x + \frac{1}{3}\right) (12x^2 - 12x - 6) \leq 3x+1$$

$$(3x+1) (4x^2 - 4x - 2) \leq 3x+1$$

$$(3x+1) (4x^2 - 4x - 2 - 1) \leq 0$$

$$(3x+1) (4x^2 - 4x - 3) \leq 0$$

$$\left(-\frac{1}{3}\right)$$

$$\Delta = 16 + 48 = 64$$

$$x = \frac{4 \pm 8}{8}$$

$$\frac{12}{8} = \left(\frac{3}{2}\right)$$

$$-\frac{4}{8} = \left(-\frac{1}{2}\right)$$

$$(3x+1)(4x^2-4x-3) \leq 0$$

x	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{3}{2}$	
$3x+1$	$-$	$- \bigcirc +$	$+$	
$4x^2-4x-3$	$+$	$\bigcirc -$	$- \bigcirc +$	
$P(x)$	$-$	$+$	$-$	$+$

$$x \in (-\infty, -\frac{1}{2}] \cup [-\frac{1}{3}, \frac{3}{2}]$$

7. (a) $P(x) = ax^3 + (b-1)x^2 - 3x - 2b + 6$

• $P(1) = 0 \Rightarrow a + b - 1 - 3 - 2b + 6 = 0$

• $P(-1) = 2 \Rightarrow -a + b - 1 + 3 - 2b + 6 = 2$

$$\begin{cases} a - b = -2 \\ -a - b = -6 \end{cases}$$

(+) $-2b = -8$
 $\underline{\underline{b = 4}}$

$\underline{\underline{a = 2}}$

$P(x) = 2x^3 + 3x^2 - 3x - 2$

(b) $P(x) = 0 \Rightarrow 2x^3 + 3x^2 - 3x - 2 = 0$

2	3	-3	-2	(2)
↓	2	5	2	
2	5	2	0	

$(x-1)(2x^2 + 5x + 2) = 0$

(x=1)

$\Delta = 25 - 16 = 9$

$x = \frac{-5 \pm 3}{4}$

$-\frac{1}{2}$

-2

1 8. $p(x) : (x^2+2) \xrightarrow{\text{undivide}} 3x-2$

$$p(x) = x^4 - 2x^3 + x^2 + ax + b$$

$$\begin{array}{r|l}
 x^4 - 2x^3 + x^2 + ax + b & x^2 + 2 \\
 \hline
 -(x^4 + 2x^2) & \\
 \hline
 2x^3 - x^2 + ax + b & \\
 \hline
 -(2x^3 + 4x) & \\
 \hline
 -x^2 + (a-4)x + b & \\
 \hline
 -(-x^2 - 2) & \\
 \hline
 (a-4)x + b + 2 &
 \end{array}$$

ananta ka diva $3x-2$

$$\begin{cases}
 a-4 = 3 \\
 b+2 = -2
 \end{cases}$$

$$a = 7$$

$$b = -4$$

③ $P(x) = 3x - 2$

$$x^4 - 2x^3 + x^2 + 7x - 4 = 3x - 2$$

$$(x^2 + 2)(x^2 + 2x - 1) + \cancel{3x - 2} = \cancel{3x - 2}$$

$$(x^2 + 2)(x^2 + 2x - 1) = 0$$

$$x^2 + 2 = 0$$

∧

$$x^2 + 2x - 1 = 0$$

Δ sama

$$\Delta = 4 + 4 = 8$$

$$x = \frac{-2 \pm \sqrt{8}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$x = -1 \pm \sqrt{2}$$