

Ενότητα 14

1.

$$x^2 - 3x - 2 = 0$$

$$x_1 + x_2 = 3$$

$$x_1 \cdot x_2 = -2$$

$$\textcircled{a} \quad x_1 x_2 - x_1 - x_2 = -2 - (x_1 + x_2) = -2 - 3 = -5$$

$$\textcircled{b} \quad \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_2 + x_1}{x_1 x_2} = \frac{3}{-2}$$

$$\textcircled{c} \quad x_1^2 x_2 + x_1 x_2^2 = x_1 x_2 (x_1 + x_2) = -2 \cdot (3) = -6 \dots$$

2.

$$2x^2 - x - 5 = 0$$

$$x_1 + x_2 = \frac{1}{2}$$

$$x_1 x_2 = -\frac{5}{2}$$

$$\textcircled{a} \quad (x_1 - 1)(x_2 - 1) = x_1 x_2 - x_1 - x_2 + 1 = x_1 x_2 - (x_1 + x_2) + 1 \\ = -\frac{5}{2} - \frac{1}{2} + 1 = -\frac{6}{2} + 1 = -3 + 1 = -2$$

$$\textcircled{b} \quad \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 x_2} = \frac{\frac{1}{2}}{-\frac{5}{2}} = \frac{-2}{10} = -\frac{1}{5}$$

$$\textcircled{c} \quad x_1^2 x_2 + x_1 x_2^2 = x_1 x_2 (x_1 + x_2) = -\frac{5}{2} \cdot \frac{1}{2} = -\frac{5}{4}$$

$$\textcircled{d} \quad x_1^2 x_2 - x_1 + x_1^2 x_2 - x_2 = -\frac{5}{4} - (x_1 + x_2) = -\frac{5}{4} - \frac{1}{2} = -\frac{7}{4}$$

6. (a) $x^2 - Sx + P = 0$

$$\begin{cases} S = 3 + (-2) = 1 \\ P = 3(-2) = -6 \end{cases}$$

$$\underline{\underline{x^2 - x - 6 = 0}}$$

(b) $x^2 - Sx + P = 0$

$$\begin{cases} S = 2 + (-\frac{1}{3}) = 2 - \frac{1}{3} = \frac{5}{3} \\ P = 2(-\frac{1}{3}) = -\frac{2}{3} \end{cases}$$

$$P = 2(-\frac{1}{3}) = -\frac{2}{3}$$

$$x^2 - \frac{5}{3}x - \frac{2}{3} = 0$$

$$3x^2 - 5x - 2 = 0$$

10. $x^2 - (\lambda - 1)x - \lambda = 0$

$$\textcircled{a} \Delta = (\lambda - 1)^2 - 4(-\lambda) = \lambda^2 - 2\lambda + 1 + 4\lambda = \lambda^2 + 2\lambda + 1 = (\lambda + 1)^2 \geq 0 \quad \checkmark$$

β) i) Για να έχω δύο ρίζες αντιστοίχως

$$\Delta > 0$$

$$(\lambda + 1)^2 > 0$$

$$\underline{\underline{\lambda \neq -1}}$$

$$S = 0$$

$$-\frac{B}{a} = 0$$

$$\lambda - 1 = 0$$

$$\underline{\underline{\lambda = 1}}$$

ii) Δύο ρίζες αντιστοίχως

$$\Delta \geq 0$$

$$(\lambda + 1)^2 \geq 0$$

✓

$$\text{και } P = 1$$

$$\frac{\gamma}{a} = 1$$

$$-\lambda = 1$$

$$\underline{\underline{\lambda = -1}}$$

3.

$$x^2 - 2x - 1 = 0$$

$$x_1 + x_2 = 2$$

$$x_1 x_2 = -1$$

$$\textcircled{a} \quad x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2 = 2^2 - 2(-1) = 4 + 2 = 6$$

$$\textcircled{b} \quad \frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{x_1^2 + x_2^2}{x_1 x_2} = \frac{6}{-1} = -6$$

$$\textcircled{c} \quad x_1^3 x_2 + x_2^3 x_1 = x_1 x_2 (x_1^2 + x_2^2) = -1 \cdot (6) = -6$$

$$\textcircled{d} \quad (x_1 - x_2)^2 = x_1^2 - 2x_1 x_2 + x_2^2 = 6 - 2 \cdot (-1) = 8$$