Ενότητα 30

6. $f(x) = (x-2)e^x + 2$

ⓐ $f'(x) = e^x + (x-2)e^x = e^x(1+x-2) = e^x(x-1)$

$$\boxed{f'(x) = e^x(x-1)}$$

$\to f'(x) = 0 \Rightarrow e^x(x-1) = 0$

$\underline{\underline{x = 1}}$

x		1	
f'	$-$		$+$
f	$\searrow$		$\nearrow$

$f(x) \geq f(1)$

$f(x) \geq 2 - e$

ⓑ $f''(x) = e^x(x-1) + e^x$

$f''(x) = e^x(x-1+1)$

$$\boxed{f''(x) = e^x x}$$

$\to f''(x) = 0 \Rightarrow e^x x = 0 \Rightarrow x = 0$

x		0	
f''	$-$		$+$
f	$\frown$		$\smile$

ⓖ $y - f(0) = f'(0)(x-0)$

$y - 0 = -x$

$y = -x$

SOS

$\forall x > 0$ f κυρτή $\Rightarrow f(x) \geq -x$

$\forall x < 0$ f κοίλη $\Rightarrow f(x) \leq -x$

(γ) $f(x^2+1) = 2 - e$

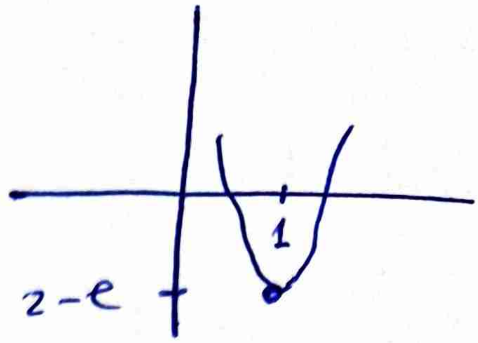
$f(x^2+1) = f(1)$

1

$x^2+1 = 1$

$x^2 = 0$

$x = 0$



Μονο το $f(1) = 2 - e$

(δ) $f(x) > -x$

$\forall x > 0$ f ωρται $f(x) > -x$

$x \in (0, +\infty)$

7. $f(x) = \dfrac{x+2}{e^x}$

ⓐ $f'(x) = \dfrac{e^x - (x+2)e^x}{(e^x)^2} = \dfrac{1-x-2}{e^x} = \dfrac{-x-1}{e^x}$

$f''(x) = \dfrac{-e^x - (-x-1)e^x}{(e^x)^2} = \dfrac{-1-(-x-1)}{e^x}$

$f''(x) = \dfrac{-1+x+1}{e^x} = \dfrac{x}{e^x}$

$f''(x) = \dfrac{x}{e^x}$

x		0	
f''	$-$		$+$
f	$\frown$		$\smile$

$y - f(0) = f'(0)(x-0)$

$y - 2 = -x$

$y = -x + 2$

$\forall x \geq 0 \quad f(x) \geq -x+2$

$\forall x \leq 0 \quad f(x) \leq -x+2$

Ⓑ $f'(x^2) \leq f'(x^4)$

x		0	
f''	$-$		$+$
f'	$\searrow$		$\nearrow$

- $x^2 \geq 0$
- $x^4 \geq 0$

$\Big\}\ \forall x \geq 0 \ \text{u}\ f' \nearrow$

$x^2 \leq x^4$

$x^2 - x^4 \leq 0$

$x^2(1-x^2) \leq 0$

x		-1		0		1	
x^2	$+$		$+$		$+$		$+$
$1-x^2$	$-$		$+$		$+$		$-$
$x^2(1-x^2)$	$-$		$+$		$+$		$-$

$x \in (-\infty, -1] \cup [?$

γ) $f(x) \le 2 - x \qquad \forall x \le 0$

$\forall x \le 0$ η f κοίλη $\quad f(x) \le 2 - x$

δ) i) $x + f(x) = 2$

$f(x) = 2 - x$

$\forall x \le 0$ η $f(x) \le 2 - x$ Το "=" για $\underline{\underline{x = 0}}$

$\forall x \ge 0$ η $f(x) \ge 2 - x$ Το "=" για $\underline{x = 0}$

$$\underline{\underline{x = 0}}$$

ii) $2 - f(x) > x$

$f(x) < 2 - x$

$\forall x < 0 \quad f(x) < 2 - x \qquad \underline{\underline{x \in (-\infty, 0)}}$

$\forall x \ge 0 \quad f(x) \ge 2 - x$

Δεν έχω λύσει εδώ

(ε) Ν.δ.ο $f(e^{x-1}-x)+e^{x-1} \geq x+2$

- $e^{x} \geq x+1$

$e^{x-1} \geq x-1+1$

$e^{x-1} \geq x$

$e^{x-1}-x \geq 0$

$\forall x \geq 0$ η f ισχύει $f(x) \geq 2-x$

$f(e^{x-1}-x) \geq 2-(e^{x-1}-x)$

$f(e^{x-1}-x) \geq 2-e^{x-1}+x$

$f(e^{x-1}-x)+e^{x-1} \geq x+2$

52 Νδο $f(e^{-x}-1)+e^{-x} \le 3 \quad \forall x \geq 0$

- $x \geq 0 \Rightarrow -x \le 0 \Rightarrow e^{-x} \le e^{0}$

$$e^{-x} \le 1$$

$$e^{-x}-1 \le 0$$

$\forall x \le 0 \quad f(x) \le 2-x$

$$f(e^{-x}-1) \le 2-(e^{-x}-1)$$

$$f(e^{-x}-1) \le 2-e^{-x}+1$$

$$f(e^{-x}-1)+e^{-x} \le 3$$

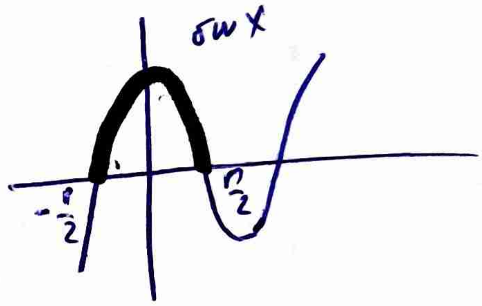
8. $f(x) = \varepsilon\varphi x + \eta\mu x \qquad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(α) $f'(x) = \dfrac{1}{\sigma\upsilon\nu^2 x} + \sigma\upsilon\nu x$

$f''(x) = \dfrac{-2\sigma\upsilon\nu x\,(-\eta\mu x)}{\sigma\upsilon\nu^4 x} - \eta\mu x$

$f''(x) = \dfrac{2\eta\mu x}{\sigma\upsilon\nu 3 x} - \eta\mu x = \dfrac{2\eta\mu x}{\sigma\upsilon\nu^3 x} - \dfrac{\eta\mu x\,\sigma\upsilon\nu^3 x}{\sigma\upsilon\nu^3 x}$

$$\boxed{f''(x) = \frac{\eta\mu x\,(\overset{+}{2 - \sigma\upsilon\nu^3 x})}{\sigma\upsilon\nu^3 x \ (+)}}$$

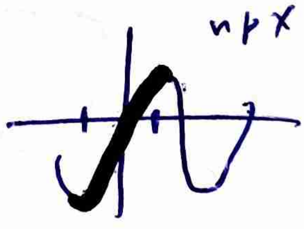


- $-1 \le \sigma\upsilon\nu x \le 1$

$-1 \le \sigma\upsilon\nu^3 x \le 1$

$1 \geq -\sigma\upsilon\nu^3 x \geq -1$

$3 \geq 2 - \sigma\upsilon\nu^3 x \geq 1$



x	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$
f''	$-$	$+$	
f	$\frown$	$\smile$	

$y - f(0) = f'(0)(x-0)$

$y - 0 = 2x \qquad y = 2x$

$\forall x \geq 0 \quad f(x) \geq 2x$

$\forall x \leq 0 \quad f(x) \leq 2x$

$f(x) = 2x$

$x = 0$

(β) Ν.δ.ο $\varepsilon\varphi x > 2x - \eta\mu x \quad \forall x \in \left(0, \frac{\pi}{2}\right)$.

$\varepsilon\varphi x + \eta\mu x > 2x$

$f(x) > 2x$

$\forall x > 0$ η f ισχύει $f(x) > 2x$

$x \in (0, +\infty)$.

(γ) $\lim_{x \to 0} \frac{\ln x}{f(x) - 2x} = \lim_{x \to 0^+} \ln x \cdot \frac{1}{\underset{(+)}{f(x) - 2x}}$

$= -\infty \cdot (+\infty) = -\infty$.

$$\boxed{\begin{array}{l} \forall x > 0 \text{ η } f(x) > 2x \\ \qquad f(x) - 2x > 0 \end{array}}$$

(δ) $\varepsilon\varphi x + \eta\mu x + (x-1)^2 = 1$

$f(x) + x^2 - 2x + \cancel{1} = \cancel{1}$ $\qquad \left[0, \frac{\pi}{2}\right)$

$\boxed{f(x) - 2x + x^2 = 0}$

- $f(x) \geq 2x \quad \forall x \geq 0$ Το "=" για $x = 0$ $\qquad x = 0$
- $x^2 \geq 0$ Το "=" $x = 0$

10. $f: \mathbb{R} \to \mathbb{R}$ κυρτή $\Rightarrow f' \nearrow$

$y = x$ εφαπτεται στο $x_0 = 0$

$f'(0) = 1$ $\quad$ $f(0) = 0$

$y - f(0) = f'(0)(x - 0)$

$y - 0 = x$

$y = x$

$g(x) = f(x) - x$

$g'(x) = f'(x) - 1$ $\quad$ $g'(0) = 0$

Αφου f κυρτη

$f(x) \geq x$

$\to g'(x) = 0$

$f'(x) - 1 = 0$

$f'(x) = 1$

$f'(x) = f'(0)$

f' 1-1

$x = 0$

$x_1 < x_2$

$f'(x_1) < f'(x_2)$

$f'(x_1) - 1 < f'(x_2) - 1$

$g'(x_1) < g'(x_2)$

$g' \nearrow$

$x < 0 \Rightarrow g'(x) < g'(0)$

$g'(x) < 0$

x		0	
g'	$\nearrow$ $-$	0	$+$ $\nearrow$
g	$\searrow$		$\nearrow$

11. $f(0)=2$ $\qquad f(x)\geq 2$

f κυρτή $\Rightarrow f'\nearrow$

(α) Αφού $f(x)\geq 2$.

$f(x)\geq f(0)$

Ακρότατο

Fermat $f'(0)=0$

x		0	
f'	$\nearrow$ $-$	0	$+$ $\nearrow$
f	$\searrow$		$\nearrow$

f

$x<0 \Rightarrow f'(x)<f'(0) \Rightarrow f'(x)<0$

$x>0 \Rightarrow f'(x)>f'(0) \Rightarrow f'(x)>0$

(β) $\left.\begin{array}{l} f(1)=3 \\ f'(1)=1 \end{array}\right\}$ $y-f(1)=f'(1)(x-1)$

$y-3=x-1$ $\qquad y=x+2$

f κυρτή

$f(x)\geq x+2$

$f(x)+f(x+1)=x+5$

$f(x+1)\geq x+1+2$

$f(x)+f(x+1)=x+2+3$

$f(x+1)\geq x+3$

$$\boxed{f(x)-2 \quad + f(x+1)-(x-3)=0}$$

- $f(x)\geq 2 \Rightarrow f(x)-2\geq 0$ $\qquad x=0$
- $f(x+1)\geq x+3 \Rightarrow f(x+1)-(x+3)\geq 0$

$x+1=1$

$x=0$

(γ) $\lim_{x \to 1} \dfrac{1}{f(f(x)) - f(x+2)} = +\infty$.

$f(x) \geq x+2$

$f \nearrow \quad \forall x > 0$

$f(f(x)) \geq f(x+2)$

$f(f(x)) - f(x+2) \geq 0$

9. $f: \mathbb{R} \to \mathbb{R}$ δύο φορές παρ/μη

$f''(x) \neq 0$ $\qquad f''(0) = -1$

f συν + παρ

f' συν + παρ

f'' συνεχής

ⓐ Αφού $f''(x) \neq 0$ και f'' συνεχής

$\Rightarrow f''(x) > 0$ ή $f''(x) < 0$

$f''(0) = -1$

$f''(x) < 0$ $\qquad$ f κοίλη, $\Rightarrow f' \downarrow$

ⓑ Αν $f(1) = f'(1) = 2$

$y - f(1) = f'(1)(x-1)$

$y - 1 = x - 1$

$\underline{\underline{y = x}}$

Αφού f κοίλη

$f(x) \leq x$

$\forall x \in \mathbb{R}$

i) $\lim\limits_{x \to 1} \dfrac{\ln(x-1)}{f'(x^2) - f'(x)}$

$= \lim\limits_{x \to 1^+} \ln(x-1) \dfrac{1}{f'(x^2) - f'(x)} = (-\infty) \cdot (-\infty) = +\infty$

• $\forall x > 1 \quad x^2 > x \Rightarrow f'(x^2) < f'(x) \Rightarrow f'(x^2) - f'(x) < 0$

ii) $\lim\limits_{x \to -\infty} f(x) = -\infty$

$$f(x) \leq x$$

$$\lim_{x \to -\infty} f(x) \leq \lim_{x \to -\infty} x$$

$$\lim_{x \to -\infty} f(x) \leq -\infty$$

iii) $\lim\limits_{x \to 1} \dfrac{\ln x}{(x-1)(f(x)-2x)} = \lim\limits_{x \to 1} \dfrac{\ln x}{x-1} \dfrac{1}{f(x)-2x} = -\infty.$

$\rightarrow \lim\limits_{x \to 1} \dfrac{\ln x}{x-1} = \lim\limits_{x \to 1} \dfrac{1}{x} = 1$

$\rightarrow \lim\limits_{x \to 1} \dfrac{1}{f(x)-2x} = -\infty$

① $f(x)+f'(e^{x-1}) = f'(x)+2x$

$f(x)-2x + f'(e^{x-1}) - f'(x) = 0$

- $f(x) \le 2x$ Το "=" $\underline{x=1}$

$f(x)-2x \le 0$

- $e^{x} \ge x+1$ Το "=" $x=0$

$e^{x-1} \ge x-1+1$

$e^{x-1} \ge x$

$f'(e^{x-1}) \le f'(x)$

$f'(e^{x-1}) - f'(x) \le 0$

Το "=" $x-1=0$

$x=1$

$\underbrace{f(x)-2x}_{\ominus} + \underbrace{f'(e^{x-1}) - f'(x)}_{\ominus} \le 0$

Το "=" $\underline{x=1}$

13. f κυρτή $\Rightarrow f' \nearrow$

f συν+παρ

f' συν+παρ

f''

$$f(x) = f(2-x)$$

$\underline{x=2}$

$f(2) = f(0)$

Rolle $\exists \xi \in (0,2)$ τ.ω $f'(\xi) = 0$.

x		1	
f'	$\nearrow$ $-$		$+$ $\nearrow$
f	$\searrow$		$\nearrow$

$f'(x) = -f'(2-x)$

$\underline{x=1}$

$f'(1) = -f'(1)$

$2f'(1) = 0$

$f'(1) = 0$

$g(x) = f(x) - f(2-x)$

$g'(x) = f'(x) + f'(2-x)$

12. f κυρτή $\Rightarrow f' \nearrow$

$f(0)=1$

$f(x) \geq e^x + x - \ln(x^2+1)$

$$\underbrace{f(x) - e^x - x + \ln(x^2+1)}_{g(x)} \geq 0$$

$g(x) \geq 0$

$g(x) \geq g(0)$

Ακρότατο στο 0.

Fermat $g'(0)=0$.

$$g'(x) = f'(x) - e^x - 1 + \frac{2x}{x^2+1}$$

$g'(0) = f'(0) - 1 - 1 = 0$

$f'(0) = 2$

$y - f(0) = f'(0)(x-0)$

$y - 1 = 2x$ $\quad$ $y = 2x-1$

Αφού f κυρτή

$f(x) \geq 2x-1$

Το

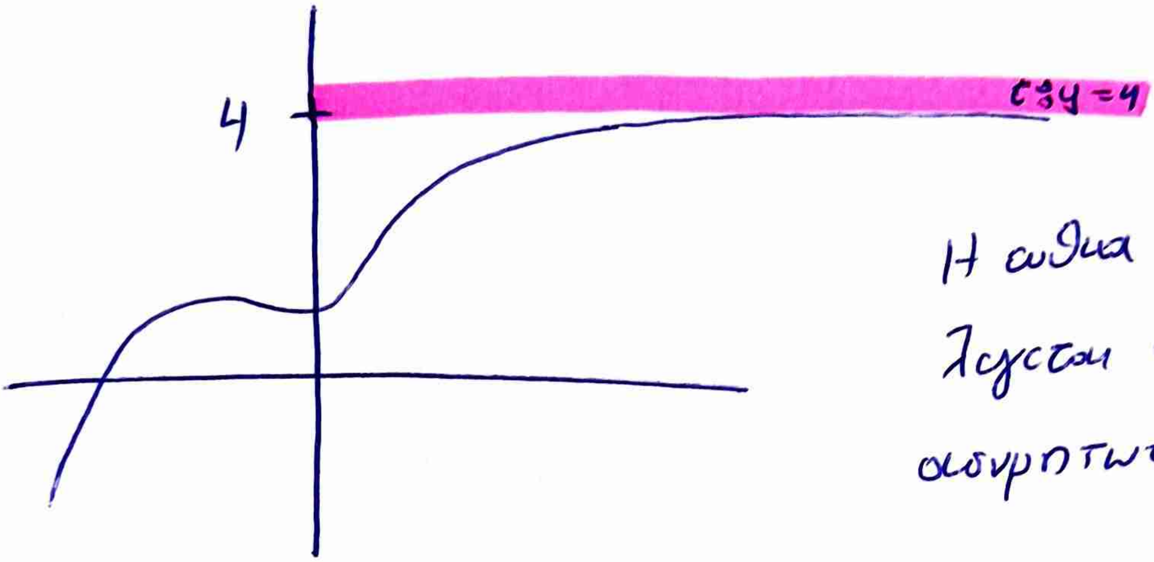
$'' = ''$

$x=0$

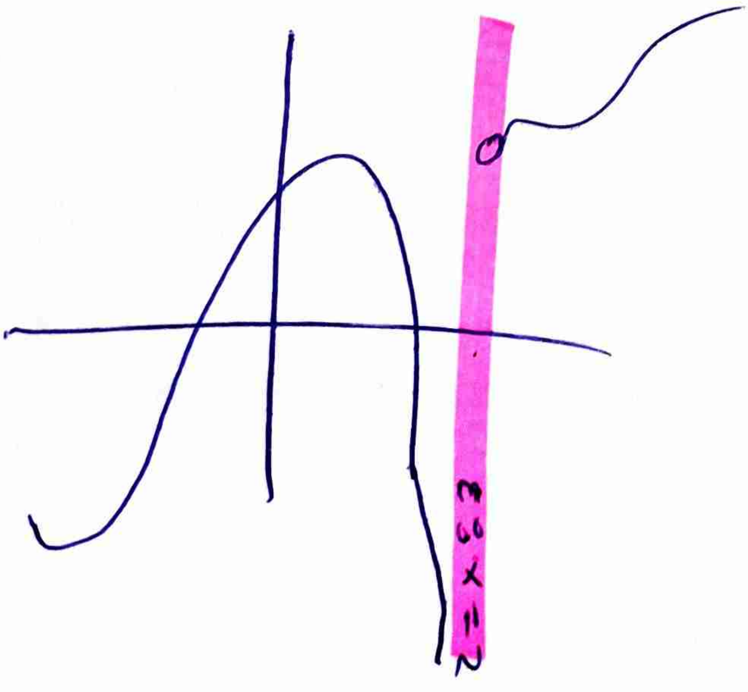
Ασυμπτωτα

Τι ειναι αυτα;

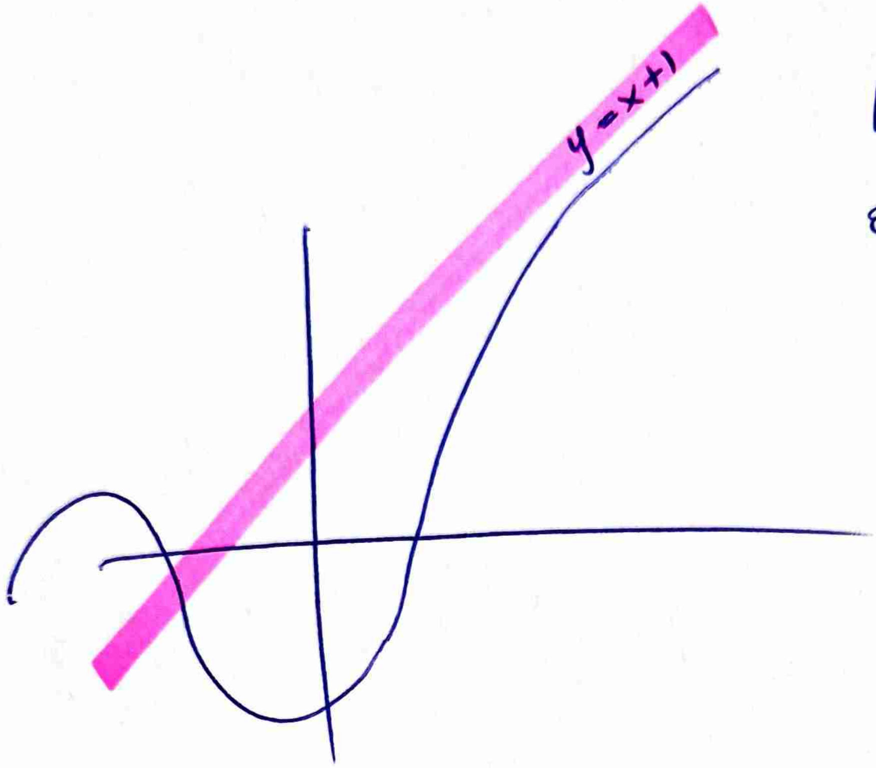
Ευθειες! Ψαχνω ευθειες!



Η ευθεια ε: $y=4$ λεγεται οριζοντια ασυμπτωτη στο $+\infty$



Η ευθεια ε: $x=2$ λεγεται κατακορυφη ασυμπτωτη.



Η $y=x+1$ ειναι πλαγια ασυμπτωτη στο $+\infty$.

κατακορυφη ασυμπτωτη

Τις ψαχνω σε σημειο x_0 οσον το πεδιο ορισμου ειναι κατι τετοιο

π.χ $(x_0, +\infty)$ $(-\infty, x_0)$ $(-\infty, x_0) \cup (x_0, +\infty)$

(x_0, α)

$\lim\limits_{x \to x_0^-} f(x) = \infty$ ή $\lim\limits_{x \to x_0^+} f(x) = \infty$

Τοτε $\varepsilon: x = x_0$ κατακορυφη ασυμπτωτη.

Οριζοντια ασυμπτωτη

Αν $\lim_{x \to \infty} f(x) = l \in R$ τοτε

η ευθεια $\varepsilon: y = l$ ειναι οριζοντια

ασυμπτωτη στο ∞

Πλαγια ασυμπτωτη

Αν εχει οριζοντια ασυμπτωτη στο ∞

δεν εχει πλαγια.

Αν ομως δεν εχει οριζοντια ψαχνω

για πλαγια.

$$\lim_{x \to \infty} \frac{f(x)}{x} = \alpha \qquad \alpha \in R^*$$

$$\lim_{x \to +\infty} f(x) - \alpha x = b \in R$$

Τοτε $\varepsilon: y = \alpha x + b$ πλαγια ασυμπτωτη στο ∞

7. (δ) $f(x) = x^2 \eta\mu \frac{1}{x}$

$\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} x^2 \eta\mu \frac{1}{x}$

$-1 \leq \eta\mu \frac{1}{x} \leq 1$

$-x^2 \leq x^2 \eta\mu \frac{1}{x} \leq x^2$

$\left.\begin{array}{l} \lim\limits_{x\to 0} -x^2 = 0 \\ \lim\limits_{x\to 0} x^2 = 0 \end{array}\right\}$ Απο Κ.Π $\lim\limits_{x\to 0} x^2 \eta\mu \frac{1}{x} = 0$

Δεν εχω κατακορυφη ασυμπτωτη.

$\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} x^{\cancel{2}} \frac{\eta\mu \frac{1}{x}}{\frac{1}{x}} \cancel{\frac{1}{x}} = +\infty \cdot 1 = +\infty$

$\lim\limits_{x\to -\infty} f(x) = \lim\limits_{x\to -\infty} x^2 \frac{\eta\mu \frac{1}{x}}{\frac{1}{x}} \frac{1}{x} = -\infty$

Δεν εχω οριζοντια!

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{x^2\,ημ\frac{1}{x}}{x} = \lim_{x\to+\infty} x\,ημ\frac{1}{x} =$$

$$= \lim_{x\to+\infty} \cancel{x}\,\frac{ημ\frac{1}{x}}{\frac{1}{x}}\,\frac{1}{\cancel{x}} = 1$$

$$\lim_{x\to+\infty} f(x) - x = \lim_{x\to+\infty} x^2\,ημ\frac{1}{x} - x$$

$$= \lim_{x\to+\infty} x\left(x\,ημ\frac{1}{x} - 1\right)$$

$$= \lim_{x\to+\infty} x\left(\frac{ημ\frac{1}{x}}{\frac{1}{x}} - 1\right)$$

$$\to \lim_{x\to+\infty} x^2\,ημ\frac{1}{x} - x \;\xrightarrow[\substack{x\to+\infty\\ u\to0^+}]{\frac{1}{x}=u}\; \lim_{u\to0^+} \frac{1}{u^2}\,ημ u - \frac{1}{u}$$

$$= \lim_{u\to0} \frac{ημ u - u}{u^2} = \lim_{u\to0} \frac{συν u - 1}{2u} = \lim_{u\to0} \frac{-ημ u}{2} = 0$$

$$y = x \quad +\infty$$

7. (B) $f(x) = \sqrt{x^2+x+1}$ πρέπει $x^2+x+1 \geq 0$ $\Delta<0$

$D_f = \mathbb{R}$

(A) Αφού $D_f = \mathbb{R}$ δεν έχει κατακόρυφη.

$$\left.\begin{aligned} \lim_{x\to-\infty} f(x) &= +\infty \\ \lim_{x\to+\infty} f(x) &= +\infty \end{aligned}\right\}$$ Δεν έχει οριζόντιες

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\sqrt{x^2+x+1}}{x} = \lim_{x\to+\infty} \frac{\cancel{x}\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{\cancel{x}} = 1$$

$$\lim_{x\to+\infty} (f(x) - 1\cdot x) = \lim_{x\to+\infty} \sqrt{x^2+x+1} - x$$

$$= \lim_{x\to+\infty} \frac{(\sqrt{x^2+x+1} - x)(\sqrt{x^2+x+1} + x)}{\sqrt{x^2+x+1} + x} =$$

$$= \lim_{x \to +\infty} \frac{\left(\sqrt{x^2+x+1}\right)^2 - x^2}{\sqrt{x^2+x+1} + x} = \lim_{x \to +\infty} \frac{x+1}{\sqrt{x^2+x+1} + x}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x}\left(1+\frac{1}{x}\right)}{\cancel{x}\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} + \cancel{x}} = \frac{1}{2}$$

$$\varepsilon : y = x + \frac{1}{2} \qquad +\infty$$

$$\lim_{x \to -\infty} \frac{f(x)}{x} = -1$$

$$\lim_{x \to -\infty} \left(f(x) + x\right) = \lim_{x \to -\infty} \frac{x+1}{-x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} - x} = -\frac{1}{2}$$

$$y = -x - \frac{1}{2} \qquad -\infty$$

15. (B) $f(x) = \dfrac{\ln x}{\sqrt{x}}$, $x > 0$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \ln x \,\frac{1}{\sqrt{x}} \quad ``(-\infty)(+\infty)" = -\infty$$

$\varepsilon_1 : x = 0$

κατακόρυφη

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{\ln x}{\sqrt{x}} = \lim_{x\to+\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}$$

$$= \lim_{x\to+\infty} \frac{2\sqrt{x}}{x} = \lim_{x\to+\infty} 2\frac{1}{2\sqrt{x}} = 0$$

$\varepsilon_2 : y = 0$

οριζόντια $+\infty$,

$x+1>0 \quad x>-1$

$x>0$

⑥ $f(x) = \ln(x+1) - \ln x$

$D_f = (0, +\infty)$

$f(x) = \ln\left(\frac{x+1}{x}\right)$

$\lim_{x \to 0^+} f(x) = \ln(+\infty) = +\infty$

$\varepsilon_1 : x = 0$

κατακόρυφη

$\lim_{x \to +\infty} f(x) = \ln 1 = 0$

$\varepsilon_2 : y = 0$

οριζόντια $+\infty$.

4. (i) $f(x) = e^{-x^2}$ $\quad x \in \mathbb{R}$

$$\lim_{x\to-\infty} f(x) = e^{-\infty} = 0$$

$\varepsilon_1 : y = 0$ οριζόντια

$\mp\infty$

$$\lim_{x\to+\infty} f(x) = 0$$

(ε) $f(x) = x\,ημ\frac{1}{x}$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x\,ημ\frac{1}{x} = \lim_{x\to+\infty} \frac{ημ\frac{1}{x}}{\frac{1}{x}} = 1$$

$\varepsilon : y = 1$

$\mp\infty$

$$\lim_{x\to-\infty} f(x) = 1$$

20 $f(x) = \dfrac{1+\ln x}{e^x} \quad , x > 0$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\frac{1}{x}}{e^x} = \lim_{x \to +\infty} \frac{1}{xe^x} = 0$$

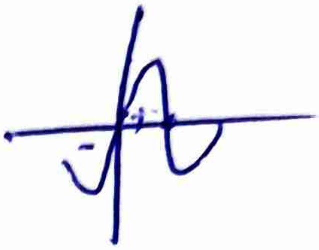
$y = 0$

$+\infty$

3. (β) $f(x) = \dfrac{x}{x-1}$ $\quad D_f = \mathbb{R} - \{1\}$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x \cdot \frac{1}{x-1} = 1 \cdot (-\infty) = -\infty$$

$\boxed{\varepsilon : x = 1 \quad \text{κατακόρυφη}}$



(γ) $f(x) = \sigma\varphi x = \dfrac{\sigma\upsilon\nu x}{\eta\mu x}$ $\quad x \in (0, \pi)$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sigma\upsilon\nu x}{\eta\mu x} = \lim_{x\to 0^+} \sigma\upsilon\nu x \, \frac{1}{\eta\mu x} = 1 \cdot (+\infty) = +\infty$$

$$\lim_{x\to \pi^-} f(x) = \lim_{x\to \pi^-} \sigma\upsilon\nu x \, \frac{1}{\eta\mu x} = -1 \cdot (+\infty) = -\infty$$

$\varepsilon_1 : x = 0$

κατακόρυφες

$\varepsilon_2 : x = \pi$

(ε) $f(x) = e^{\frac{1}{x}} \ln x \quad , x > 0$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} e^{\frac{1}{x}} \ln x = (+\infty)(-\infty) = -\infty$$

$\varepsilon : x = 0$

(στ) $f(x) = \dfrac{\ln x}{x-2} \qquad x \in (0,2) \cup (2, +\infty)$

$$\lim_{x \to 0^+} f(x) = \frac{-\infty}{-2} = +\infty$$

$\varepsilon : x = 0$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \ln x \frac{1}{x-2} = \ln 2 \cdot (-\infty)$$

$$= -\infty$$

$\varepsilon_1 : x = 2$

Επομενο Μαθημα

24	26	27
61	11	1 γδ
62	12	2
64	13	4
65	20	5
	23	7 αβ
	25	8
		11
		12