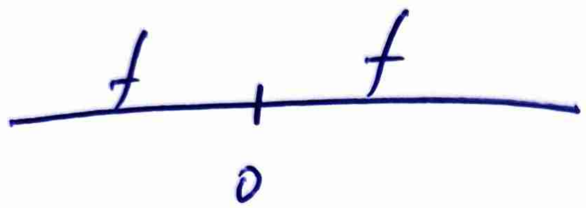
20. ① $f(x) = \begin{cases} x^2 ημ\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Ενοτητα 11



$$\lim_{x \to 0} f(x) = \lim_{x \to 0} x^2 ημ\frac{1}{x} \xrightarrow[\text{χιασμ}]{ημ\infty} 0$$

$$-1 \le ημ\frac{1}{x} \le 1$$

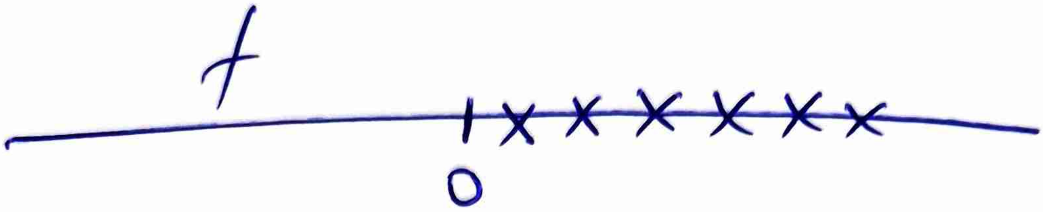
$$\boxed{-x^2 \le x^2 ημ\frac{1}{x} \le x^2}$$

$$\left.\begin{aligned} &\lim_{x \to 0} -x^2 = 0 \\ &\lim_{x \to 0} x^2 = 0 \end{aligned}\right\} \text{Από Κ.Π.} \quad \lim_{x \to 0} x^2 ημ\frac{1}{x} = 0$$

$$f(0) = 0$$

Η $f(x)$ συνεχής στο 0.

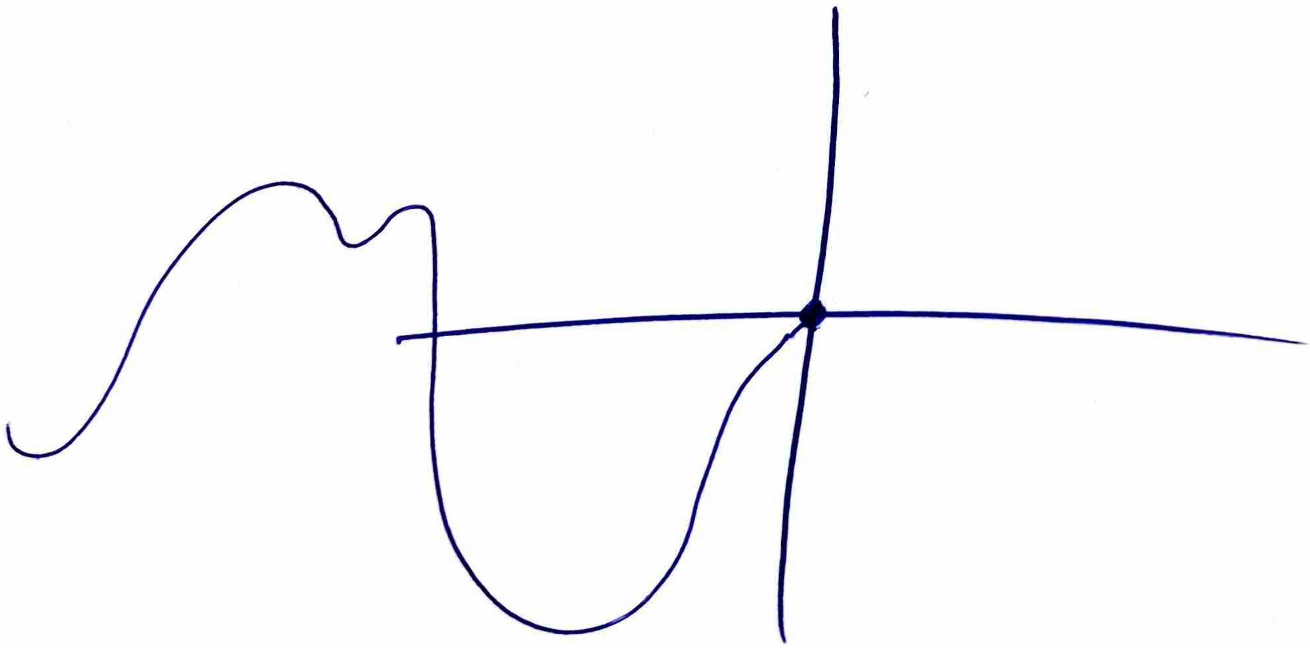
21. (a) $f(x) = \begin{cases} x e^{\frac{1}{x}}, & x < 0 \\ 0, & x = 0 \end{cases}$



$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x e^{\frac{1}{x}} = 0 \cdot e^{-\infty} = 0 \cdot 0 = 0$$

$f(0) = 0$

Συνεχής στο 0!



(B) $f(x) = \begin{cases} xe^{1-\frac{1}{x}}, & x>0 \\ \ln(x^2+1), & x \le 0 \end{cases}$

$f_1 \quad f_2$ (0)

• $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \ln(x^2+1) = 0$

• $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(xe^{1-\frac{1}{x}}\right) = 0\cdot e^{-\infty}$
$= 0\cdot 0 = 0$

$\boxed{\lim_{x\to 0} f(x) = 0}$

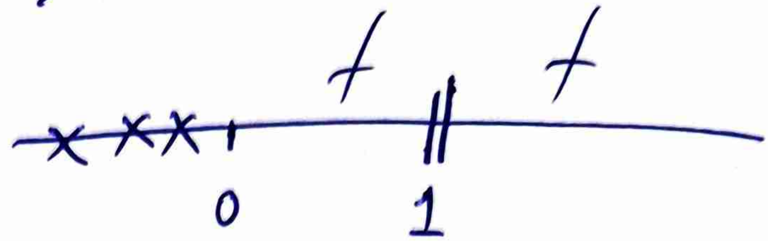
$f(0) = \ln(0^2+1) =$
$= \ln 1 = 0$

$f(0) = 0$

Αφού $f(0) = \lim_{x\to 0} f(x) = 0$

Συνεχής στο 0.

(γ) $f(x) = \begin{cases} \dfrac{x}{\ln x}, & 0 < x \neq 1 \\ 0, & x = 0 \end{cases}$



$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x}{\ln x} = \lim_{x \to 0^+} x \cdot \frac{1}{\ln x}$$

$$= 0 \cdot \frac{1}{-\infty} = 0 \cdot 0 = 0$$

$f(0) = 0$

Συνεχής στο 0!

Όρια

1. $$\lim_{x\to+\infty}\left[\sqrt{x^2-4}-x\right] =$$

$$\lim_{x\to+\infty}\frac{\left[\sqrt{x^2-4}-x\right]\left[\sqrt{x^2-4}+x\right]}{\sqrt{x^2-4}+x} =$$

$$=\lim_{x\to+\infty}\frac{\cancel{\sqrt{x^2-4}}^{\cancel{2}}-x^2}{\sqrt{x^2-4}+x}=\lim_{x\to+\infty}\frac{\cancel{x^2}-4-\cancel{x^2}}{\sqrt{x^2-4}+x}$$

$$=\lim_{x\to+\infty}\frac{-4}{\sqrt{x^2-4}+x} = \frac{-4}{+\infty}\ 0.$$

2. $\lim\limits_{x\to 1} \dfrac{x^2-4x+3}{x^2-4x+4} = \dfrac{1^2-4\cdot 1+3}{1^2-4\cdot 1+4} = \dfrac{0}{1} = 0$

3. $\lim\limits_{x\to -\infty} \dfrac{x^2-4x+3}{x^2-4x+4} = \lim\limits_{x\to -\infty} \dfrac{\cancel{x^2}}{\cancel{x^2}} = 1$

4. $\lim\limits_{x\to 2} \dfrac{x^2-4x+3}{x^2-4x+4} =$

$= \lim\limits_{x\to 2} (x^2-4x+3) \dfrac{1}{(x-2)^2} = (-1)(+\infty)$

$= -\infty$

5. $\lim\limits_{x\to -1} \dfrac{x^2+5x+4}{x^3+1} = \lim\limits_{x\to -1} \dfrac{(x+4)\cancel{(x+1)}}{\cancel{(x+1)}(x^2-x+1)}$

$= \dfrac{3}{3} = 1.$

6. $$\lim_{x\to-\infty} \frac{x^2+1}{\sqrt{x^2-1}} = \lim_{x\to-\infty} \frac{x^2\left(1+\frac{1}{x^2}\right)}{\sqrt{x^2\left(1-\frac{1}{x^2}\right)}}$$

$$= \lim_{x\to-\infty} \frac{x^{\cancel{2}}\left(1+\frac{1}{x^2}\right)}{-\cancel{x}\sqrt{1-\frac{1}{x^2}}} = \frac{1(-\infty)}{-1} = +\infty$$

7. $$\lim_{x\to 0} \frac{\ln x}{x} = \lim_{x\to 0^+} \frac{\ln x}{x} =$$

$$= \lim_{x\to 0^+} \ln x \, \frac{1}{x} = \ln 0 \cdot (+\infty)$$

$$= -\infty \cdot +\infty$$

$$= -\infty$$

Tip

$$\frac{\infty}{0} = \infty \cdot \frac{1}{0} = \infty \cdot \infty = \infty$$

$$\frac{0}{\infty} = 0.$$

8. $\lim\limits_{x\to 0} (1-\sigma\upsilon\nu x)\,\eta\mu\frac{1}{x}$ ημ∞ / χασιμο

$$-1 \le \eta\mu\frac{1}{x} \le 1$$

$$\boxed{-(1-\sigma\upsilon\nu x) \le (1-\sigma\upsilon\nu x)\,\eta\mu\frac{1}{x} \le 1-\sigma\upsilon\nu x}$$

$$-1 \le \sigma\upsilon\nu x \le 1$$

$$1 \ge -\sigma\upsilon\nu x \ge -1$$

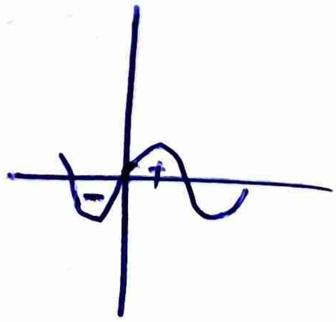
$$\boxed{2 \ge 1-\sigma\upsilon\nu x \ge 0}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 0} -(1-\sigma\upsilon\nu x) = 0 \\ \lim\limits_{x\to 0} 1-\sigma\upsilon\nu x = 0 \end{array}\right\}$$

Από Κ.Π

$$\lim\limits_{x\to 0} (1-\sigma\upsilon\nu x)\,\eta\mu\frac{1}{x} = 0.$$

9. $\lim\limits_{x\to 0} \left(\eta\mu x\ \sigma\upsilon\nu\frac{1}{x}\right)$ συν∞ / χασιμο



$$-1 \le \sigma\upsilon\nu\frac{1}{x} \le 1$$

$$\left|\sigma\upsilon\nu\frac{1}{x}\right| \le 1$$

$$|\eta\mu x|\left|\sigma\upsilon\nu\frac{1}{x}\right| \le 1\cdot|\eta\mu x|$$

$$\left|\eta\mu x\ \sigma\upsilon\nu\frac{1}{x}\right| \le |\eta\mu x|.$$

$$\boxed{-|\text{ημ}x| \le \text{ημ}x\,\text{συν}\frac{1}{x} \le |\text{ημ}x|}$$

$$\left.\begin{array}{l} \bullet\ \lim\limits_{x\to 0} -|\text{ημ}x| = 0 \\[2ex] \bullet\ \lim\limits_{x\to 0} |\text{ημ}x| = 0 \end{array}\right\} \lim_{x\to 0} \text{ημ}x\,\text{συν}\frac{1}{x} = 0,$$