

ΕΥΟΤΥΤΑ 13

14. (a) $\sigma \omega^2 x - 7 \eta \nu^2 x + 1 = 0$

$$(1 - \eta \nu^2 x) - 7 \eta \nu^2 x + 1 = 0$$

$$1 - \eta \nu^2 x - 7 \eta \nu^2 x + 1 = 0$$

$$- 8 \eta \nu^2 x + 2 = 0$$

$$8 \eta \nu^2 x = 2 \Rightarrow \eta \nu^2 x = \frac{1}{4}$$

$$\eta \nu x = \frac{1}{2}$$

$$\eta \nu x = -\frac{1}{2}$$

$$\eta \nu x = -\eta \nu \frac{\eta}{6}$$

$$\eta \nu x = \eta \nu \frac{\eta}{6}$$

$$\eta \nu x = \eta \nu / -\frac{\eta}{6}$$

$$x = 2k\eta + \frac{\eta}{6} \quad \vee \quad x = 2k\eta + \eta - \frac{\eta}{6}$$

$$x = 2k\eta + \frac{5\eta}{6}$$

$$x = 2k\eta - \frac{\eta}{6}$$

$$x = 2k\eta + \eta + \frac{\eta}{6}$$
$$x = 2k\eta + \frac{7\eta}{6}$$

$$(B) \quad 3\sigma\omega^2x - \eta\mu^2x - 3 = 0$$

$$3\sigma\omega^2x - (1 - \sigma\omega^2x) - 3 = 0$$

$$3\sigma\omega^2x - 1 + \sigma\omega^2x - 3 = 0$$

$$4\sigma\omega^2x - 4 = 0$$

$$\sigma\omega^2x = 1$$

$$\sigma\omega x = 1$$

or

$$\sigma\omega x = -1$$

$$\sigma\omega x = \sigma\omega \quad 0$$

$$\sigma\omega x = \sigma\omega \pi$$

$$x = 2/\omega + 0$$

$$x = 2/\omega + \pi$$

$$x = 2/\omega - 0$$

$$x = 2/\omega - \pi$$

$$x = 2/\omega$$

$$18. \textcircled{a} \quad \mu_P 2X = \sigma_W X$$

$$\mu_P 2X = \mu_P \left(\frac{\eta}{2} - X \right)$$

$$2X = 2K\eta + \frac{\eta}{2} - X \quad \vee \quad 2X = 2K\eta + \eta - \frac{\eta}{2} + X$$

$$3X = 2K\eta + \frac{\eta}{2}$$

$$X = 2K\eta + \frac{\eta}{2}$$

$$X = \frac{2K\eta}{3} + \frac{\eta}{6}$$

$$\textcircled{B} \quad \sigma_W 3X = \mu_P \left(X - \frac{\eta}{3} \right)$$

$$\mu_P \left(\frac{\eta}{2} - 3X \right) = \mu_P \left(X - \frac{\eta}{3} \right)$$

$$\frac{\eta}{2} - 3X = 2K\eta + X - \frac{\eta}{3}$$

$$\vee \quad \frac{\eta}{2} - 3X = 2K\eta + \eta - X$$

$$-4X = 2K\eta - \frac{\eta}{2} - \frac{\eta}{3}$$

$$-2X = 2K\eta + \eta + \frac{\eta}{3} - \frac{\eta}{2}$$

$$-4X = 2K\eta - \frac{5\eta}{6}$$

$$X = -\frac{K\eta}{2} + \frac{5\eta}{24}$$

$$X = -K\eta - \frac{\eta}{2} - \frac{\eta}{6} + \frac{\eta}{4}$$

19. (a) $\eta kx = \sqrt{3} \sigma \omega x$

$$\frac{\eta kx}{\sigma \omega x} = \sqrt{3}$$

$$\varepsilon \varphi x = \sqrt{3}$$

$$\varepsilon \varphi x = \varepsilon \varphi \frac{n}{3}$$

$$x = kn + \frac{n}{3}$$

Εστω $\sigma \omega x = 0$

Τότε $\eta kx = 0$

οπω

$$\eta v^2 x + \sigma \omega^2 x = 1$$

ΑΤΩ!

ΑΡΗ

$$\sigma \omega x \neq 0$$

(8) $\eta kx = -\sigma \omega x$

$$-\eta kx = \sigma \omega x$$

$$\eta k(-x) = \sigma \omega x$$

$$\eta k(-x) = \eta k\left(\frac{n}{2} - x\right)$$

$$-x = 2kn + \frac{n}{2} - x$$

Αδωκ

$$x = -kn - \frac{n}{4}$$

$$-x = 2kn + n - \frac{n}{2} + k$$

$$-2x = 2kn + \frac{n}{2}$$

1

$$24. \textcircled{a} \text{ NS } \frac{urx}{1-\sigma wx} + \frac{urx}{1+\sigma wx} = \frac{2}{urx}$$

$$\in K \cap : (1-\sigma wx)(1+\sigma wx)/urx$$

$$ur^2x(1+\sigma wx) + ur^2x(1-\sigma wx) = 2(1-\sigma wx)(1+\sigma wx)$$

$$ur^2x(1+\cancel{\sigma wx} + 1-\cancel{\sigma wx}) = 2(1-\sigma wx^2x)$$

$$\underline{\underline{2ur^2x = 2ur^2x}}$$

$$(B) \quad \frac{\sin x}{1 - \cos x} + \frac{\sin x}{1 + \cos x} = \frac{4}{\sqrt{3}}$$

$$\frac{2}{\sin x} = \frac{4}{\sqrt{3}}$$

$$2\sqrt{3} = 4 \sin x$$

$$\sqrt{3} = 2 \sin x$$

$$\boxed{\sin x = \frac{\sqrt{3}}{2}, \quad x \in [0, 2\pi]}$$

$$\sin x = \sin \frac{\pi}{6}$$

$$x = 2k\pi + \frac{\pi}{6}$$

$$x = 2k\pi + \pi - \frac{\pi}{6}$$

$$x = 2k\pi + \frac{5\pi}{6}$$

α' τροχός

$$0 \leq x \leq 2\pi$$

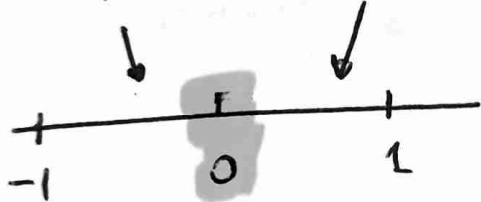
$$0 \leq 2\pi + \frac{\pi}{6} \leq 2\pi$$

$$0 \leq 2\pi + \frac{1}{6} \leq 2$$

$$0 \leq 12\pi + 1 \leq 12$$

$$-1 \leq 12\pi \leq 11$$

$$-\frac{1}{12} \leq \pi \leq \frac{11}{12}$$



for $\pi = 0$

$$x = \frac{\pi}{6}$$

$$0 \leq x \leq 2\pi$$

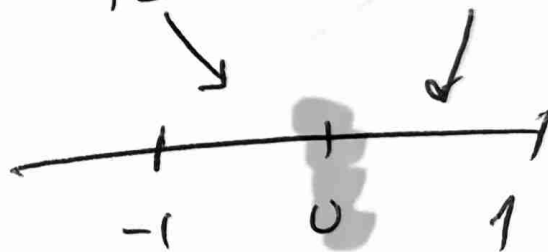
$$0 \leq 2\pi + \frac{5\pi}{6} \leq 2\pi$$

$$0 \leq 2\pi + \frac{5}{6} \leq 2$$

$$0 \leq 12\pi + 5 \leq 12$$

$$-5 \leq 12\pi \leq 7$$

$$-\frac{5}{12} \leq \pi \leq \frac{7}{12}$$



$$x = \frac{5\pi}{6}$$

Β' τροποί

$$\underline{\underline{0 \leq x \leq 2\pi}}$$

Γενική λύση

$$x = 2k\pi + \frac{\pi}{6}$$

$$k=0 \Rightarrow x = \frac{\pi}{6} \checkmark$$

$$k=1 \Rightarrow x = 2\pi + \frac{\pi}{6} \quad \text{---}$$

$$k=-1 \Rightarrow x = -2\pi + \frac{\pi}{6} \quad \text{---}$$

$$x = 2k\pi + \frac{5\pi}{6}$$

$$k=0$$

$$x = \frac{5\pi}{6} \checkmark$$

$$k=1$$

$$x = 2\pi + \frac{5\pi}{6} \quad \text{---}$$

$$k=-1$$

$$x = -2\pi + \frac{5\pi}{6} \quad \text{---}$$

ΕΥΟΤΥΤΑ 18

$$2. \textcircled{a} \quad 2x^4 + 5x = 2x^3 + 5$$

$$2x^4 + 5x - 2x^3 - 5 = 0$$

$$2x^3(x-1) + 5(x-1) = 0$$

$$(x-1)(2x^3 + 5) = 0$$

$$x-1=0$$

$$\textcircled{x=1}$$

$$\hookrightarrow 2x^3 + 5 = 0$$

$$x^3 = -\frac{5}{2}$$

$$\textcircled{x = -\sqrt[3]{\frac{5}{2}}}$$

$$\textcircled{52} \quad x^3 - 8 = x^2 - 5x + 6$$

$$(x-2)(x^2+2x+4) = (x-2)(x-3)$$

$$(x-2) \left(x^2+2x+4 - x+3 \right) = 0$$

$$(x-2) \left(x^2+x+7 \right) = 0$$

$$x-2=0$$

$$\vee \quad x^2+x+7=0$$

$$\textcircled{x=2}$$

$$\Delta < 0$$

6. (B) $2x^3 - x - 1 = 0$, $\frac{-1}{\pm 1}$

Horner

$$2 \quad 0 \quad -1 \quad -1 \quad \textcircled{1}$$

$$\downarrow \quad 2 \quad 2 \quad 1$$

$$2 \quad 2 \quad 1 \quad 0$$

$$(x-1)(2x^2+2x+1) = 0$$

$$x-1=0$$

$$\vee 2x^2+2x+1=0$$

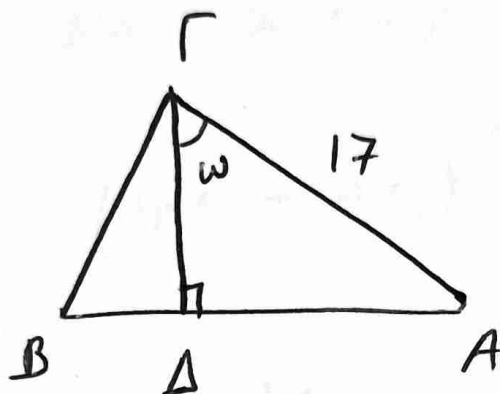
$$\Delta < 0$$

$$\textcircled{x=1}$$

Επορω Μαθημα

Ασκηση

1.



$$\text{Ar } \eta\psi\omega = \frac{15}{17} \text{ και } \sin\omega = \frac{15}{17}$$

$$\epsilon\psi\beta = \frac{4}{3}$$

Να βρω $AD, \Gamma D, AB, \rho\Gamma$

2. Να βρω η ακριβή τιμή παραστάσεως

$$A = \sin^2 \frac{61\pi}{3} + \sin^2 \frac{73\pi}{6} - \epsilon\psi \frac{9\pi}{4}.$$

3. Έστω γωνία x ώστε $\pi < x < \frac{3\pi}{2}$

$$3\sqrt{5} \eta\psi^2 x - \eta\psi x - \sqrt{5} = 0.$$

$$A = \frac{3\eta\psi^2 x + \eta\psi x \cdot \sin x}{\epsilon\psi x - 2\epsilon\psi x.}$$

4. Έστω ότι $\pi < x < \frac{3\pi}{2}$ και

$$\varepsilon\varphi^2 x - \frac{2}{\sigma\varphi x} = 8$$

Βρείτε $\eta\rho x$, $\sigma\omega x$, $\varepsilon\varphi x$, $\sigma\varphi x$.

5. Να αποδείξετε τα ισχύοντα.

$$\textcircled{a} (\eta\rho x - \sigma\omega x)^2 + (\eta\rho x + \sigma\omega x)^2 = 2$$

$$\textcircled{b} \frac{1 + \eta\rho^2 x}{\sigma\omega^2 x} - 2\varepsilon\varphi^2 x = 1$$

$$\textcircled{g} \frac{1}{\eta\rho x} - \frac{\eta\rho x}{1 + \sigma\omega x} = \sigma\varphi x$$

6. Να υπολογιστεί η παρασταση

$$\frac{\eta\rho\left(\frac{41\pi}{2} - \theta\right) \cdot \sigma\omega\left(\frac{35\pi}{2} + \theta\right) \varepsilon\varphi(21\pi - \theta)}{\sigma\omega\left(\frac{25\pi}{2} + \theta\right) \sigma\varphi\left(\frac{5\pi}{2} - \theta\right) \sigma\omega(33\pi + \theta)}$$

7. Να γίνει η γραφική παράσταση
αφού πρώτα βρούτε $T, \text{Max}, \text{Min}$

(α) $f(x) = 2 \sin 3x + 2$

(β) $f(x) = 2 \sin 2x - 3$

8. Να λύσετε οι εξισώσεις.

(α) $\sin\left(2x + \frac{\pi}{6}\right) + \sin\left(x + \frac{\pi}{3}\right) = 0$

(β) $\sin\left(\frac{\pi}{3} - x\right) - \sin\left(2x + \frac{\pi}{4}\right) = 0$

(γ) $2\sin\left(2x + \frac{\pi}{6}\right) + \sqrt{3} = 0 \quad \omega_1(-\pi, \pi)$