$$\frac{1}{x}$$

Ενοτητα 9

Τοποθετω στη θεση του x

κατι πολυ μικρο κοντα στο 0.

$$\frac{1}{0,1} = 10$$

$$\frac{1}{0,01} = 100$$

$$\frac{1}{0,0000001} = 1.000.000$$

$$\boxed{\frac{1}{0^+} = +\infty}$$

Συνηθης στοιχομυθαια

γ' Λυκαων.

$$\frac{1}{0} = \infty$$

Τι απειρο; ($-\infty$ ή $+\infty$)

Τα παντα εξαρτωνται απο

τον παρονομαστη.

κανονα.

$$(+\infty) + (+\infty) = +\infty$$

$$(-\infty) + (-\infty) = -\infty$$

$$(+\infty) + \alpha = +\infty$$

$$(+\infty) \cdot (-\infty) = (-\infty)$$

$$(-\infty) \cdot (-\infty) = (+\infty)$$

$$(+\infty) \cdot (+\infty) = (+\infty)$$

Απροσδιοριστα

$$(+\infty) + (-\infty) = \;;$$

$$(+\infty) \cdot 0 = \;;$$

1. π.χ. $\lim\limits_{x\to 1} \dfrac{x-2}{(x-1)^2} \overset{\left(\frac{a}{0}\right)}{=\!=}$ Χωριζω καλους-κακους

$$= \lim_{x\to 1} (x-2)\frac{1}{(x-1)^2} = (-1)\cdot(+\infty) = -\infty$$

$\downarrow$

Θετικη ποσοτητα

2. π.χ. $\lim\limits_{x\to 0} \dfrac{x-1}{x^3} \overset{\left(\frac{a}{0}\right)}{=\!=} \lim\limits_{x\to 0} (x-1)\cdot\dfrac{1}{x^3}$

$$= (-1)\cdot\infty$$

x		0	
x^3	$-$		$+$

$\rightarrow \lim\limits_{x\to 0^-} (x-1)\dfrac{1}{x^3} = (-1)(-\infty) = +\infty$

$\rightarrow \lim\limits_{x\to 0^+} (x-1)\dfrac{1}{x^3} = (-1)(+\infty) = -\infty$

Το οριο δεν υπαρχει.

7. (δ) $\lim\limits_{x \to -1} \dfrac{x^2 - x}{x^3 + 3x^2 + 3x + 1} \overset{\left(\frac{a}{0}\right)}{=}$

Ενότητα 9

1	3	3	1	-1
↓	-1	-2	-1	
1	2	1	0	

$$= \lim_{x \to -1} \frac{x^2 - x}{(x+1)(x^2+2x+1)} = \lim_{x \to -1} \frac{x(x-1)}{(x+1)(x+1)^2}$$

$$= \lim_{x \to -1} \frac{x(x-1)}{(x+1)^3} = \lim_{x \to -1} x(x-1) \frac{1}{(x+1)^3}$$

$$= 2 \cdot \infty \ ;$$

x	-1
$x+1$	$-\ 0\ +$

$$\rightarrow \lim_{x \to -1^-} x(x-1) \frac{1}{(x+1)^3} = 2 \cdot (-\infty) = -\infty$$

$$\rightarrow \lim_{x \to -1^+} x(x-1) \frac{1}{(x+1)^3} = 2(+\infty) = +\infty$$

Το όριο δεν υπάρχει

6. (ε) $$\lim_{x\to 0} \frac{2x-1}{1-\sigma\upsilon\nu x} =$$

$$= \lim_{x\to 0} (2x-1)\,\frac{1}{\underset{(+)}{1-\sigma\upsilon\nu x}} = (-1)\cdot(+\infty) = -\infty$$

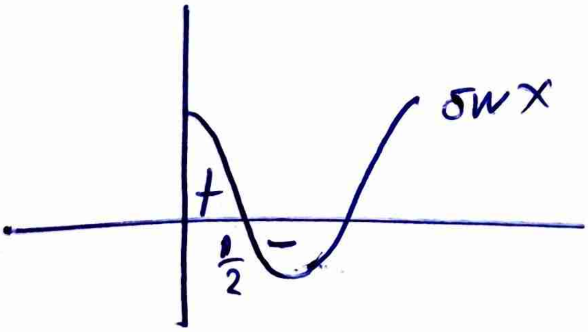
$$-1 \le \sigma\upsilon\nu x \le 1$$

$$1 \ge -\sigma\upsilon\nu x \ge -1$$

$$\boxed{2 \ge 1-\sigma\upsilon\nu x \ge 0}$$

7. (στ) $$\lim_{x\to \frac{\pi}{2}} \varepsilon\varphi x = \lim_{x\to \frac{\pi}{2}} \frac{\eta\mu x}{\sigma\upsilon\nu x} =$$

$$= \lim_{x\to \frac{\pi}{2}} \eta\mu x\,\frac{1}{\sigma\upsilon\nu x} = 1 \cdot \infty$$



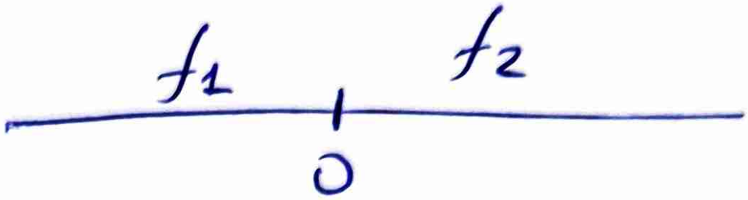
• $\lim_{x\to \frac{\pi}{2}^-} \eta\mu x\,\frac{1}{\sigma\upsilon\nu x} = 1\cdot(+\infty) = +\infty$

• $\lim_{x\to \frac{\pi}{2}^+} \eta\mu x\,\frac{1}{\sigma\upsilon\nu x} = 1\cdot(-\infty) = -\infty$

Το όριο δεν υπάρχει

8. (Β) $f(x) = \begin{cases} \dfrac{x-1}{\eta\mu x}, & -\pi < x < 0 \\[2ex] \dfrac{x+1}{\varepsilon\varphi x}, & 0 < x < \dfrac{1}{2} \end{cases}$

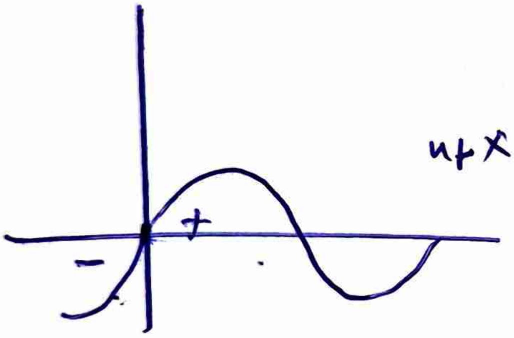
$\lim\limits_{x \to 0} f(x) =$



$\rightarrow \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \dfrac{x-1}{\eta\mu x} = \lim\limits_{x \to 0^-} (x-1) \dfrac{1}{\eta\mu x} =$

$= -1 \cdot (-\infty) =$

$= \underline{\underline{+\infty}}$



$\rightarrow \lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \dfrac{x-1}{\eta\mu x} = \lim\limits_{x \to 0^+} (x-1) \dfrac{1}{\eta\mu x}$

$= -1 \cdot (+\infty) =$

$= \underline{\underline{-\infty}}$

Το όριο δεν υπάρχει.

12. ⓐ $\lim_{x\to -1}\left(\frac{1}{x+1}+\frac{2}{x^2-1}\right)$

Ενότητα 8

$$= \lim_{x\to -1} \frac{1}{x+1}+\frac{2}{(x-1)(x+1)}$$

$$= \lim_{x\to -1} \frac{x-1}{(x-1)(x+1)}+\frac{2}{(x-1)(x+1)}$$

$$= \lim_{x\to -1} \frac{\cancel{x+1}}{(x-1)\cancel{(x+1)}} = \lim_{x\to -1}\frac{1}{(x-1)} = -\frac{1}{2}.$$

11. ⓑ $\lim_{x\to -1}\frac{x^3+1}{x^3+x+2} = \lim_{x\to -1}\frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}(x^2-x+2)}$

1	0	0	1	-1
↓	-1	1	-1	
1	-1	1	0	

1	0	1	2	-1
↓	-1	1	-2	
1	-1	2	0	

$$= \frac{3}{4}$$

δ $\lim_{x\to 1} \dfrac{\sqrt{x+3}-2}{x^2-1} = \lim_{x\to 1} \dfrac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(x+1)(\sqrt{x+3}+2)}$

$= \lim_{x\to 1} \dfrac{\sqrt{x+3}^{\,2}-2^2}{(x-1)(x+1)(\sqrt{x+3}+2)} =$

$= \lim_{x\to 1} \dfrac{\cancel{x+3-4}}{\cancel{(x-1)}(x+1)(\sqrt{x+3}+2)} = \dfrac{1}{2\cdot 4}$

$= \dfrac{1}{8}.$

14. (B) $\lim\limits_{x \to -1} \dfrac{\sqrt{x^2+3} - 2}{\sqrt{x+2} - \sqrt{2x+3}}$

$= \lim\limits_{x \to -1} \dfrac{(\sqrt{x^2+3}-2)(\sqrt{x^2+3}+2)(\sqrt{x+2}+\sqrt{2x+3})}{(\sqrt{x+2}-\sqrt{2x+3})(\sqrt{x+2}+\sqrt{2x+3})(\sqrt{x^2+3}+2)}$

$= \lim\limits_{x \to -1} \dfrac{(\sqrt{x^2+3}^2-2^2)(\sqrt{x+2}+\sqrt{2x+3})}{(\sqrt{x+2}^2-\sqrt{2x+3}^2)(\sqrt{x^2+3}+2)}$

$= \lim\limits_{x \to -1} \dfrac{(x^2-1)(\sqrt{x+2}+\sqrt{2x+3})}{(-x-1)(\sqrt{x^2+3}+2)}$

$= \lim\limits_{x \to -1} \dfrac{(x-1)\cancel{(x+1)}(\sqrt{x+2}+\sqrt{2x+3})}{-\cancel{(x+1)}(\sqrt{x^2+3}+2)} = \dfrac{-2 \cdot 2}{-4}$ 1.

13. B $$\lim_{x \to 5} \frac{\sqrt{x} - \sqrt{5}}{5 - x} = \lim_{x \to 5} \frac{(\sqrt{x} - \sqrt{5})(\sqrt{x} + \sqrt{5})}{(5 - x)(\sqrt{x} + \sqrt{5})}$$

$$= \lim_{x \to 5} \frac{\sqrt{x}^2 - \sqrt{5}^2}{-(x-5)(\sqrt{x} + \sqrt{5})} = \lim_{x \to 5} \frac{\cancel{x-5}}{-\cancel{(x-5)}(\sqrt{x} + \sqrt{5})}$$

$$= \frac{1}{-2\sqrt{5}} = -\frac{\sqrt{5}}{10}$$

Γ $$\lim_{x \to 2} \frac{x - 2}{\sqrt{x+2} - 2} = \lim_{x \to 2} \frac{(x-2)(\sqrt{x+2} + 2)}{(\sqrt{x+2} - 2)(\sqrt{x+2} + 2)}$$

$$= \lim_{x \to 2} \frac{(x-2)(\sqrt{x+2} + 2)}{\sqrt{x+2}^2 - 2^2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(\sqrt{x+2} + 2)}{\cancel{x-2}}$$

$$= \lim_{x \to 2} \frac{\sqrt{x+2} + 2}{1} = 4$$

Απολυτα

$$\lim_{x\to 1} \frac{|x-1|-|x+2|}{|x^2-4|} = \lim_{x\to 1} \frac{|x-1|-(x+2)}{-x^2+4}$$

$$= \lim_{x\to 1} \frac{|x-1|-x-2}{-(x^2-4)}$$

x		1	
$x-1$	$-$	0	$+$

$$\rightarrow \lim_{x\to 1^-} \frac{|x-1|-x-2}{-x^2+4} = \lim_{x\to 1^-} \frac{-x+1-x-2}{-x^2+4} =$$

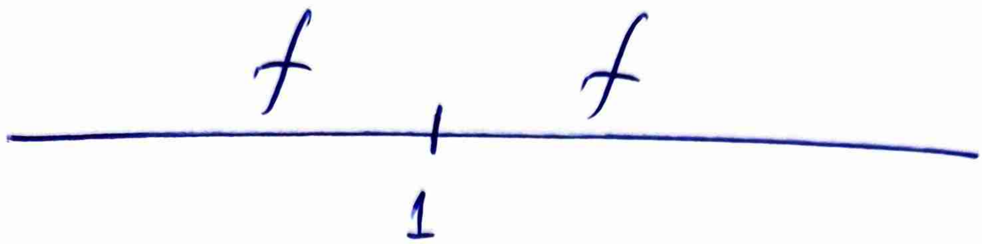
$$= \lim_{x\to 1^-} \frac{-2x-1}{-x^2+4} = \frac{-3}{3} = -1$$

$$\rightarrow \lim_{x\to 1^+} \frac{|x-1|-x-2}{-x^2+4} = \lim_{x\to 1^+} \frac{\cancel{x}-1-\cancel{x}-2}{-x^2+4}$$

$$= \frac{-3}{3} = -1$$

Το οριο υπαρχει και ειναι -1

15. B $f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$



$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = 2$$

20. (α) $\underbrace{x^7+2x-3}_{h(x)}<0$

Ενότητα 4

$h(x)<0$

$h(x)<h(1)$

$h \nearrow$

$x<1$

Μονοτονία $h(x)$

$\cdot\ x_1<x_2 \Rightarrow x_1^7<x_2^7$

$x_1<x_2 \Rightarrow 2x_1-3<2x_2-3$

$\oplus$ $h(x_1)<h(x_2)$

(Β) $e^x<1-2x$

$\underbrace{e^x+2x-1}_{\varphi(x)}<0$

$\varphi(x)<0$

$\varphi(x)<\varphi(0)$

$\varphi \nearrow$

$x<0$

$\cdot\ x_1<x_2 \Rightarrow 2x_1<2x_2$

$\cdot\ x_1<x_2 \Rightarrow e^{x_1}-1<e^{x_2}-1$

$\oplus$

$\varphi \nearrow$

21. $f:(0,+\infty)\to \mathbb{R}$ $\qquad f(1)=1$

$f\downarrow$

(a) $f(x)-\ln x>1$

$$\underbrace{f(x)-\ln x-1}_{g(x)}>0$$

$g(x)>0$

$g(x)>g(1)$

$g\downarrow$

$0<x<1$

Μονοτονία $g(x)$

- $x_1<x_2 \Rightarrow f(x_1)>f(x_2)$ (+)
- $x_1<x_2 \Rightarrow \ln x_1<\ln x_2 \Rightarrow -\ln x_1-1>-\ln x_2-1$

$g(x_1)>g(x_2)$

$g\downarrow$

(B) $x f(x) + 1 > 2x$

$$\underbrace{x f(x) + 1 - 2x}_{g(x)} > 0$$

$$g(x) > 0$$

$$g(x) > g(1)$$

- $x_1 < x_2$
- $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

$\odot$;

$$\frac{x f(x)}{x} + \frac{1}{x} - \frac{2x}{x} > 0$$

$$\underbrace{f(x) + \frac{1}{x} - 2}_{g(x)} > 0$$

$$g(x) > 0$$

$$g(x) > g(1)$$

$$\left.\begin{array}{l}\bullet\ x_1 < x_2 \Rightarrow f(x_1) > f(x_2) \\[1em] \bullet\ x_1 < x_2 \Rightarrow \dfrac{1}{x_1} - 2 > \dfrac{1}{x_2} - 2\end{array}\right\} +$$

$$g(x_1) > g(x_2)$$

$$g \downarrow$$

Apa $g(x) > g(1)$

$$g \downarrow$$

$$0 < x < 1$$

18.

$\boxed{\begin{array}{l} f \text{ γνησίως μονότονη} \\ \Rightarrow f \nearrow \text{ ή } f \searrow \end{array}}$

Ενότητα 7

$A(1,2) \Rightarrow f(1)=2$

$B(3,-2) \Rightarrow f(3)=-2$

(α) Αφού η f γν. μονότονη $\Rightarrow f$ 1-1

άρα f αντιστρέφεται

(β) $\left.\begin{array}{c} 1 < 3 \\ \Leftrightarrow \\ \underset{2}{\underset{\parallel}{f(1)}} > \underset{-2}{\underset{\parallel}{f(3)}} \end{array}\right\} f \searrow$

$$f\left(-2+f^{-1}(x+2)\right)=2$$

$$f\left(-2+f^{-1}(x+2)\right)=f(1)$$

$$f \text{ 1-1}$$

$$-2+f^{-1}(x+2)=1$$

$$f^{-1}(x+2)=3.$$

$$f^{-1}(x+2) = 3$$

$$f\left(f^{-1}(x+2)\right) = f(3)$$

$$x+2 = -2$$

$$\underline{\underline{x = -4}}$$

(8) $$f^{-1}\left(f(e^x-1) - 4\right) < 3$$

$$f\downarrow$$

$$f\left(f^{-1}\left(f(e^x-1)-4\right)\right) > f(3)$$

$$f(e^x-1) - 4 > -2$$

$$f(e^x-1) > 2$$

$$f(e^x-1) > f(1)$$

$$f\downarrow$$

$$e^x-1 < 1$$

$$e^x < 2$$

$$\underline{\underline{x < \ln 2}}$$

19. $f(x) = x^3 + x$

(α) $x_1 < x_2 \Rightarrow x_1^3 < x_2^3$

$x_1 < x_2$ ——— (+) $f(x_1) < f(x_2)$

$f \nearrow$

$\Rightarrow f$ γν. αύξουσα

$\Rightarrow f$ 1-1

$\Rightarrow f$ αντιστρέψιμη

(β) $f^{-1}(10)$

$f(\text{ποσο;}) = 10$

$$\boxed{f(2) = 10 \iff f^{-1}(10) = 2}$$

(γ) i) $f(1 + f^{-1}(x^2 - 3x)) = 0$

$f(1 + f^{-1}(x^2 - 3x)) = f(0)$

f 1-1

$1 + f^{-1}(x^2 - 3x) = 0$

$f^{-1}(x^2 - 3x) = -1$

$$f\left(f^{-1}(x^2-3x)\right) = f(-1)$$

$$x^2-3x = -2$$

$$x^2-3x+2=0$$

$x=1$ $\quad$ $x=2$

11) $f^{-1}\left(f(x^2-1)+4f^{-1}(-10)\right)<1$

$f(-2)=-10$

$f^{-1}(-10)=-2$

$$f^{-1}\left(f(x^2-1)+4\cdot(-2)\right)<1$$

$$f\left(f^{-1}\left(f(x^2-1)-8\right)\right)<f(1)$$

$$f(x^2-1)-8<2$$

$$f(x^2-1)<10$$

$$f(x^2-1)<f(2)$$

$f\nearrow$

$$x^2-1<2$$

$$x^2<3$$

$x\in(-\sqrt{3},\sqrt{3})$

13. (β) $f(e^x) = x^2 - 3x + 1$

Ενότητα 3

Ψάχνω τύπο $f(x)$

$e^x = t$

$\ln e^x = \ln t$

$x = \ln t$

$f(t) = (\ln t)^2 - 3\ln t + 1$

ή $f(x) = (\ln x)^2 - 3\ln x + 1$

Επόμενο Μάθημα

Νέες

Ενότητα 9

5 όλη

6 α β γ δ

7 α β γ ε

8 α.

10 α β

Παλιές

Ενότητα 8

15 α

19 α β

20 α.

Ξεκινάμε την παράδοση με αποστήθιση και Θεωρία

Σ-Π.