16. $f(x) = x^2 + (\lambda - 2)x - 2$

Ενότητα 2.5

$A(2,4)$

(α) $f(2) = 4 \iff 2^2 + (\lambda-2)\cdot 2 - 2 = 4$

$4 + 2\lambda - 4 - 2 = 4$

$2\lambda = 6$ $\qquad$ $\lambda = 3$

$f(x) = x^2 + x - 2$

(β) $B(\alpha-1, -2)$

$f(\alpha-1) = -2$

$(\alpha-1)^2 + \alpha - 1 \cancel{-2} = \cancel{-2}$

$\alpha^2 - 2\alpha + 1 + \alpha - 1 = 0$

$\alpha^2 - \alpha = 0$

$\alpha(\alpha-1) = 0$

$\alpha = 0$ ή $\alpha = 1$

(γ) $x'x$

$f(x) = 0$

$x^2 + x - 2 = 0$

$x = -2$ $\qquad$ $x = 1$

$\Delta(-2,0)$ $\qquad$ $E(1,0)$

$y'y$

$f(0) = -2$

$\Gamma(0,-2)$.

(8) $f(x) > 0$

$x^2 + x - 2 > 0$

x		-2	1	
x^2+x-2	$+$		$-$	$+$

$x \in (-\infty, -2) \cup (1, +\infty)$.

17. $f(x) = 3x^2 - 2x + k^3 + 1$ $\quad f(0) = 0$

$g(x) = 2x^2 + x - 2$

(α) $f(0) = 0 \Rightarrow f(0) = k^3 + 1 = 0$

$k^3 = -1$

$\underline{k = -1}$

$$\boxed{f(x) = 3x^2 - 2x}$$

(β) $f(x) = g(x)$

$3x^2 - 2x = 2x^2 + x - 2$

$x^2 - 3x + 2 = 0$

$x = 1$ $\quad x = 2$

$A(1, 1)$

$B(2, 8)$

(γ) $f(x) \le g(x)$

$x^2 - 3x + 2 \le 0$

x		1		2	
x^2-3x+2	+		−		+

$x \in [1, 2]$.

18. (α) $f(x) = |x-1| - 1$
$g(x) = \alpha x$ } $f(1) = g(1)$

$-1 = \alpha$

(β) $f(x) = g(x)$

$|x-1| - 1 = -x$

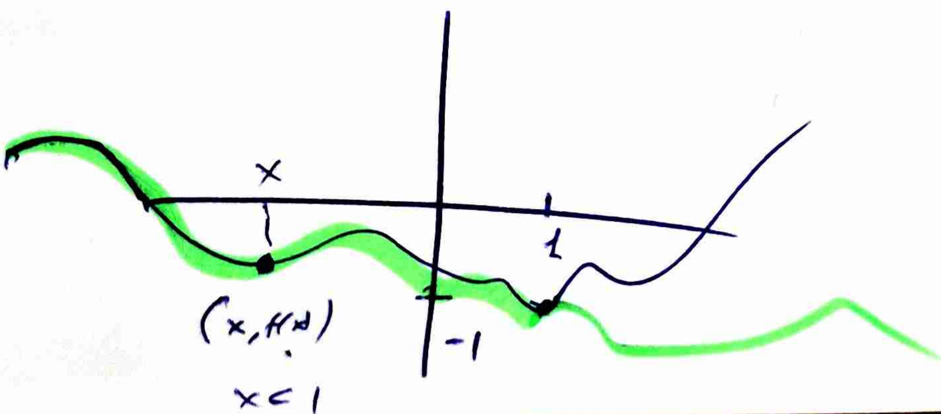
$|x-1| = 1 - x$

πρέπει $1 - x \geq 0 \Rightarrow x \leq 1$

$x - 1 = 1 - x$ ή $x - 1 = -1 + x$
Αόριστη
$2x = 2$ $x \in [-\infty, 1]$.

$x = 1$

Έχουν άπειρα σημεία όλα της μορφής $(x, f(x))$, $x \leq 1$



γ) $f(x) > g(x)$

$|x-1| - 1 > -x$

$|x-1| > 1 - x$

• $x \geq 1$

$|\overset{+}{x-1}| > 1 - x$

$x - 1 > 1 - x$

$2x > 2$

$x > 1$

$x \in (1, +\infty)$

• $x < 1$

$|\overset{-}{x-1}| > 1 - x$

$\cancel{1} - \cancel{x} > \cancel{1} - \cancel{x}$

$0 > 0$

Αδύνατη

19. $f(x) = x^2 - (\lambda-1)x - \lambda + 1$

(α) $f(x) = 0 \rightarrow x^2 - (\lambda-1)x - \lambda + 1 = 0$

Δυο λυσεις.

$\Delta > 0$

$(\lambda-1)^2 - 4(-\lambda+1) > 0$

$\lambda^2 - 2\lambda + 1 + 4\lambda - 4 > 0$

$\lambda^2 + 2\lambda - 3 > 0$

λ	-3		1	
$\lambda^2+2\lambda-3$	+		−	+

$\lambda \in (-\infty, -3) \cup (1, +\infty)$.

Σχολιο

Οταν θελω νδο η $f(x)$ τεμνει τον $x'x$ σε δυο σημεια αρκει νδο η εξισωση $f(x) = 0$ εχει 2 λυσεις.

3) Απαιτώ $\Delta = 0$

$\Rightarrow \lambda^2 + 2\lambda - 3 = 0$

$\lambda = -3$ $\lambda = 1$

γ Απαιτώ $\Delta < 0$

$\lambda^2 + 2\lambda - 3 < 0$

$\lambda \in (-3, 1)$

20. $f(x) = x^2 - \lambda x + \lambda^2 + \lambda + 1$

$\Delta = \lambda^2 - 4(\lambda^2 + \lambda + 1)$

$\Delta = \lambda^2 - 4\lambda^2 - 4\lambda - 4$

$\Delta = -3\lambda^2 - 4\lambda - 4$

$\Delta^* = (-4)^2 - 4(-3)(-4)$

$\Delta^* = 16 - 48$

$\Delta^* < 0$

Διατηρει σταθερο προσημο ομοσημο του α

$\Delta < 0$

21. (α) $f(x) = x^2 - (3\lambda - 1)x + 1 - 3\lambda$.

πάνω από

τον $x'x$

$\Delta < 0$ και $1 > 0$

$(3\lambda-1)^2 - 4(1-3\lambda) < 0$

$9\lambda^2 - 6\lambda + 1 - 4 + 12\lambda < 0$

$9\lambda^2 + 6\lambda - 3 < 0$

$3\lambda^2 + 2\lambda - 1 < 0$

$\Delta = 4 + 12 = 16$ $\qquad \lambda = \dfrac{-2 \pm 4}{6}$ -1, $\dfrac{1}{3}$

λ		-1		$1/3$	
$3\lambda^2+2\lambda-1$	$+$		$-$		$+$

$\lambda \in \left(-1, \dfrac{1}{3}\right)$.

B) $f(x) = (\lambda-1)x^2 + (\lambda+1)x - 2\lambda$

$\Delta < 0$ $\qquad\qquad$ $\lambda - 1 < 0$

$B^2 - 4ac < 0$ $\qquad\qquad$ $\lambda < 1$

$(\lambda-1)^2 - 4(\lambda-1)(-2\lambda) < 0$

$\lambda^2 - 2\lambda + 1 + 8\lambda(\lambda-1) < 0$

$\lambda^2 - 2\lambda + 1 + 8\lambda^2 - 8\lambda < 0$

$9\lambda^2 - 10\lambda + 1 < 0$

$\Delta = 100 - 36 = 64$

$\lambda = \frac{10 \pm 8}{18}$ — 1, $\frac{1}{9}$

λ		$1/9$		1	
$9\lambda^2 - 10\lambda + 1$	$+$		$-$		$+$

$\lambda \in \left(\frac{1}{9}, 1\right)$

22. $f(x)=\lambda x^2-4\lambda x-2 \quad , \lambda\neq 3$

$g(x)=3x^2-7x+\lambda$

C_f κάτω C_g

$f(x)<g(x)$

$\lambda x^2-4\lambda x-2<3x^2-7x+\lambda$

$\lambda x^2-4\lambda x-2-3x^2+7x-\lambda<0$

$$\boxed{(\lambda-3)x^2+(7-4\lambda)x-2-\lambda<0}$$

$\Delta<0$ και $\lambda-3<0$

$\underline{\lambda<3}$

$(7-4\lambda)^2-4(\lambda-3)(-2-\lambda)<0$

$49-56\lambda+16\lambda^2-4(-2\lambda-\lambda^2+6+3\lambda)<0$

$49-56\lambda+16\lambda^2-4\lambda+4\lambda^2-24<0$

$20\lambda^2-60\lambda+25<0$

$4\lambda^2-12\lambda+5<0$

$\Delta=144-80=64$

$\lambda=\dfrac{12\pm8}{8}$ → $\dfrac{20}{8}$ = $\dfrac{5}{2}$, $\dfrac{1}{2}$

λ		$1/2$		$5/2$	
$4\lambda^2-12\lambda+5$	$+$		$-$		$+$

$\underline{\lambda\in\left(\frac{1}{2},\frac{5}{2}\right)}$

24. $f(x) = \dfrac{x - x^2}{2x^2 - 5x + 3}$

(α) Πρέπει $2x^2 - 5x + 3 \neq 0$

$D_f = \mathbb{R} - \{\frac{3}{2}, 1\}$.

(β) $f(x) = \dfrac{x(1-x)}{2(x-\frac{3}{2})(x-1)} = \dfrac{-x\cancel{(x-1)}}{(2x-3)\cancel{(x-1)}}$

$f(x) = \dfrac{x}{3-2x}$

(γ). $f(x) = 1$

$\dfrac{x}{3-2x} = 1$

$x = 3 - 2x$

$3x = 3$

$\cancel{x = 1}$

Αδύνατη.

23. $f(x) = 2x^2 + \alpha x - 1$ $\quad A(-1, 2)$

ⓐ $f(-1) = 2(-1)^2 + \alpha(-1) - 1 = 2$

$2 - \alpha - 1 = 2$

$\underline{\underline{-1 = \alpha}}$

$f(x) = 2x^2 - x - 1$

β) $\underline{x'x}$

$f(x) = 0$

$2x^2 - x - 1 = 0$

$x = 1 \qquad x = \frac{1}{2}$

$B(1, 0) \qquad \Gamma(-\frac{1}{2}, 0)$

$\underline{y'y}$

$f(0) = -1$

$\Delta(0, -1)$.

γ) i) $f(x) > 0$

x		$-1/2$		1	
$f(x)$	$+$		$-$		$+$

$x \in (-\infty, -\frac{1}{2}) \cup (1, +\infty)$.

ii). $f(x) < g(x)$

$2x^2 - \cancel{x} - 1 < x^2 - \cancel{x}$

$x^2 - 1 < 0$

x		-1		1	
$x^2 - 1$	$+$		$-$		$+$

$x \in (-1, 1)$.

25. $f(x) = \begin{cases} ax^3+1, & x<0 \\ Bx^2-4, & x\geq 0 \end{cases}$ $\quad A(-2,-7)$ $\quad B(\sqrt{2},-2)$

(a) $f(-2)=-7$

$a(-2)^3+1=-7$

$-8a=-8$

$a=1$

$f(\sqrt{2})=-2$

$B(\sqrt{2})^2-4=-2$

$2B=2$

$B=1$

$$\boxed{f(x) = \begin{cases} x^3+1, & x<0 \\ x^2-4, & x\geq 0 \end{cases}}$$

(b) $\underline{x'x}$

$f(x)=0$

$\underline{x<0}$

$f(x)=0$

$x^3+1=0$

$x^3=-1$

$x=-1$

$P(-1,0)$

$\underline{x\geq 0}$

$f(x)=0$

$x^2-4=0$

$x^2=4$

$x=2$ $\quad \cancel{x=-2}$

$K(2,0)$.

$\underline{y'y}$

$f(0)=0^2-4=-4$

$K(0,-4)$.

(8) $f(x^2)=0$

$(x^2)^2-4=0$

$(x^2)^2=4$

$(x^2)^2=2^2$

$|x^2|=|2|$

$x^2=2$

$x=\sqrt{2}$ $x=-\sqrt{2}$,

Επομενο Μαθημα

Παραδιδω $f(x) = ax + B$

και τελειωνουμε την

υλη.

Την Μ.Τεταρτη λυνουμε

ολα τα ασκησεα τη

τελευταια ενοτητα.