II

Θεμα II

$$f(x)=\begin{cases} x^2+a, & x\le 2 \\ \dfrac{1}{x-1}, & x>2 \end{cases}$$

Συνεχω!

ⓐ $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x^2+a) = 4+a$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \dfrac{1}{x-1} = 1$

$\Leftrightarrow$ $4+a=1$

$a=-3$

$$\boxed{f(x)=\begin{cases} x^2-3, & x\le 2 \\ \dfrac{1}{x-1}, & x>2 \end{cases}}$$

ⓑ $\varepsilon: y-f(3)=f'(3)(x-3)$

$y-\dfrac{1}{2}=-\dfrac{1}{4}(x-3) \Leftrightarrow 4y-2=-x+3$

$\boxed{\varepsilon: x+4y-5=0}$

Για $x>2$

$f'(x)=\dfrac{-1}{(x-1)^2}$

(γ) Αφού f συνεχής στο 2 και $D_f = \mathbb{R}$

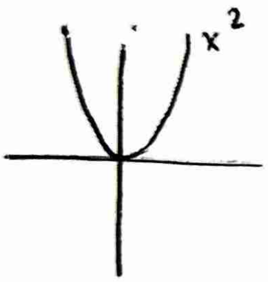
δεν έχει κατακόρυφη ασύμπτωτη.

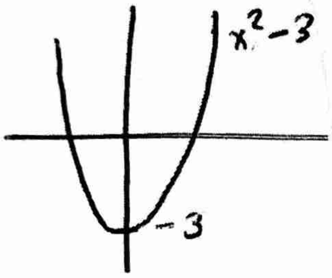
$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^2-3 = +\infty$$

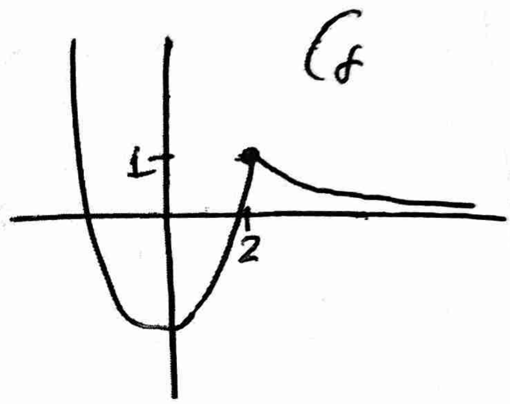
$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{1}{x-1} = 0 \qquad \boxed{\varepsilon: y=0}$$

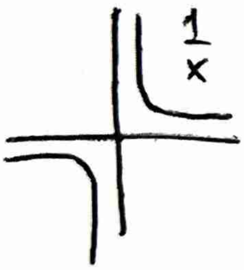
$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{x^2-3}{x} = \lim_{x\to-\infty} \frac{x^{\cancel{2}}}{\cancel{x}} = -\infty$$

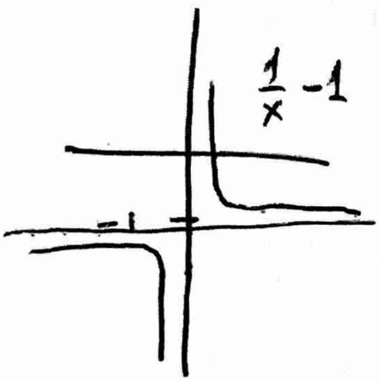
Δεν έχει ασύμπτωτη στο $-\infty$.

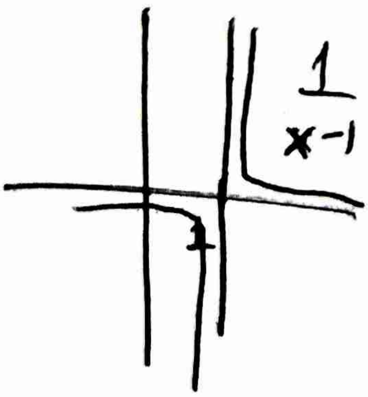












(δ) $E = \int_2^3 |f(x)|\,dx \Leftrightarrow$

$$E = \int_2^3 \frac{1}{x-1}\,dx \Leftrightarrow$$

$$E = \left(\ln|x-1|\right)_2^3 = \ln 2$$

Θεμα 12

$$f(x) = \frac{x}{x-1} \quad , x \neq 1$$

ⓐ $f(x) > 0 \iff \frac{x}{x-1} > 0$

x		0		1	
x	$-$	0	$+$		$+$
$x-1$	$-$		$-$	0	$+$
$\frac{x}{x-1}$	$+$		$-$		$+$

Όταν $x \in (-\infty, 0) \cup (1, +\infty)$ η $f(x)$ είναι πάνω από τον $x'x$

ⓑ $f'(x) = \frac{x-1-x}{(x-1)^2} = -\frac{1}{(x-1)^2} < 0$ η $f \downarrow$ στο $(-\infty, 1)$ και στο $(1, +\infty)$.

$$f''(x) = -\frac{-2(x-1)}{(x-1)^4} = \frac{2}{(x-1)^3}$$

x		1	
f''	$-$		$+$
f	$\frown$		$\smile$

ⓖ. $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{x}{x-1} = \lim_{x\to 1^+} x \cdot \frac{1}{x-1} = 1 \cdot (+\infty) = +\infty$

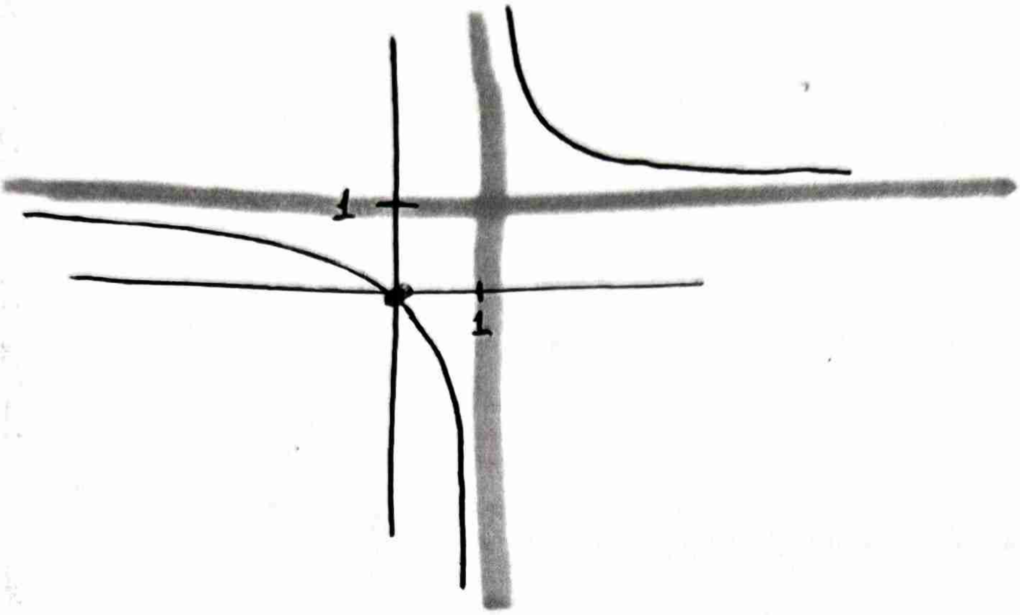
$\boxed{\varepsilon_1 : x=1}$ κατακόρυφη ασύμπτωτη.

$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{x}{x-1} = \lim_{x\to -\infty} \frac{x}{x} = 1.$

$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} \frac{x}{x-1} = \lim_{x\to +\infty} \frac{x}{x} = 1$

$\varepsilon_2 : y = 1$ οριζόντια.

$-\infty$
$+\infty$



(8) $\int_2^3 f(x)\,dx = \int_2^3 \frac{x}{x-1}\,dx = \int_2^3 \frac{(x-1)1+1}{x-1}\,dx$

$$\begin{array}{r|l} x & x-1 \\ \hline -(x-1) & 1 \\ \hline 1 & \end{array}$$

$x = (x-1)1+1$

$= \int_2^3 \frac{x-1}{x-1} + \frac{1}{x-1}\,dx = \int_2^3 1 + \frac{1}{x-1}\,dx$

$= \int_2^3 1\,dx + \int_2^3 \frac{1}{x-1}\,dx =$

$= (x)_2^3 + (\ln|x-1|)_2^3 = 1 + \ln 2$.

Θεμα 13

$$f(x)=\begin{cases} e^x+a, & x<0 \\ \ln(x+1), & x\geq 0 \end{cases}$$ Συνεχής!

(α) $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-}(e^x+a) = 1+a$

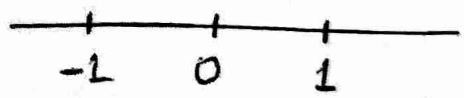
$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+}(\ln(x+1)) = 0$

$\Leftrightarrow\ a+1=0$

$a=-1$

$$\boxed{f(x)=\begin{cases} e^x-1, & x<0 \\ \ln(x+1), & x\geq 0 \end{cases}}$$

(β) Για να ισχύουν οι προϋποθέσεις του ΘΜΤ πρέπει f συνεχής $[-1,1]$ και f παρ/μη $(-1,1)$.



Η f συνεχής $[-1,0)$ και $(0,1]$ ως πράξεις συνεχών συναρτήσεων και συνεχής στο 0 άρα συνεχής στο $[-1,1]$

Η $f(x)$ ειναι παραγωγισιμη στο $(-1,0)$ και $(0,1)$ ως πραξεις παραγωγισιμων συναρτησεων

$$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-} \frac{e^x-1}{x} = \lim_{x\to 0^-} \frac{e^x}{1} = 1$$

$$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} \frac{\ln(x+1)}{x} = \lim_{x\to 0^+} \frac{1}{x+1} = 1$$

Αρα παρ/μη και στο 0 αρα ικανοποιει το ΘΜΤ στο $[-1,1]$

(γ) Εστω $\varepsilon: y - f(x_0) = f'(x_0)(x-x_0)$ η εφαπτομενη που ψαχνω. Πρεπει $f'(x_0)=1$

Πρεπει να λυσω την εξισωση $f'(x)=1$

$\underline{x<0}$

$f'(x)=1$

$e^x=1$

$\cancel{\boxed{x=0}}$

$\underline{x\geq 0}$

$f'(x)=1$

$\frac{1}{x+1}=1$

$1=x+1$

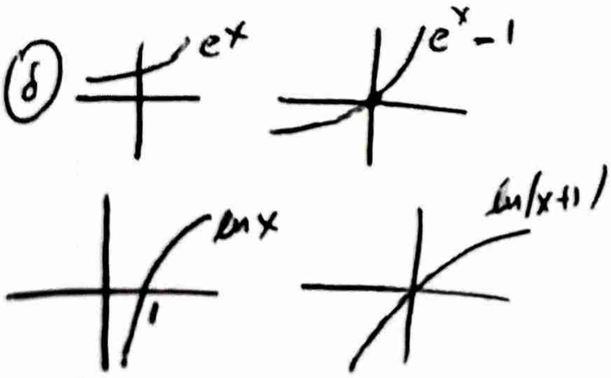
$\boxed{x=0}$

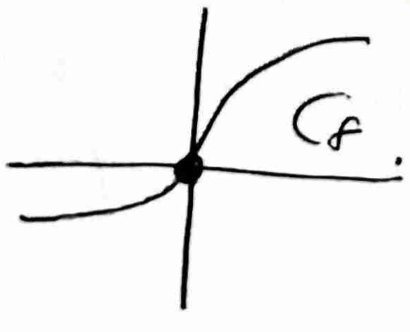
$\varepsilon: y - f(0) = f'(0)(x-0)$

$y-0 = 1(x-0)$

$\boxed{\varepsilon: y=x}$

(δ)





Θέμα 14

- $f:(0,+\infty)\longrightarrow \mathbb{R}$
- $f(1)=5$
- $f'(x)=\dfrac{x^2-4}{x^2}$, $x>0$.

Ⓐ νδο $f(x)=x+\dfrac{4}{x}$, $x>0$

$$f'(x)=\frac{x^2-4}{x^2} \Leftrightarrow f'(x)=\frac{x^2}{x^2}-\frac{4}{x^2} \Leftrightarrow f'(x)=1-\frac{4}{x^2}$$

$$f'(x)=\left(x+\frac{4}{x}\right)' \Leftrightarrow f(x)=x+\frac{4}{x}+c$$

$\underline{x=1}$

$f(1)=1+4+c \Leftrightarrow 5=5+c$

$c=0$

Άρα $\boxed{f(x)=x+\dfrac{4}{x}\quad x>0}$

Ⓑ $f'(x)=\dfrac{x^2-4}{x^2}$

x	0		2		$+\infty$
f'		$-$	0	$+$	
f		$\searrow$		$\nearrow$	

$f(x)\geq f(2)$

$\boxed{f(x)\geq 4}$

$A(2,4)$ Ο.Ε.

$$f''(x)=\frac{2x\cdot x^2-(x^2-4)2x}{x^4}=\frac{2x^2-2(x^2-4)}{x^3}=$$

$$=\frac{2x^2-2x^2+8}{x^3}=\frac{8}{x^3}>0 \quad \forall x>0$$

f κυρτή.

(γ). $D_f=(0,+\infty)$

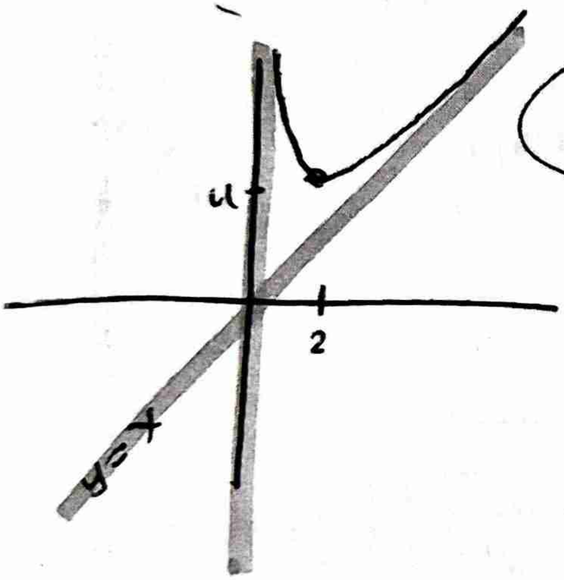
• $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(x+\frac{4}{x}\right) = +\infty$

$\boxed{\varepsilon_1 : x=0 \text{ κατακόρυφη}}$

• $\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty}\left(x+\frac{4}{x}\right) = +\infty$ Δεν έχει οριζόντια.

• $\lim_{x\to +\infty} \frac{f(x)}{x} = \lim_{x\to +\infty} \frac{x+\frac{4}{x}}{x} = \lim_{x\to +\infty} \frac{x^2+4}{x^2} = 1$

$\lim_{x\to +\infty} f(x)-x = \lim_{x\to +\infty} \cancel{x}+\frac{4}{x}-\cancel{x} = 0$



$\varepsilon_2 : y=x$ $+\infty$

(δ). ΝΔΟ $f'(x) > \frac{f(x)-5}{x-4}$ $\forall x>4$.

4 ξ x

$f'(\xi) = \frac{f(x)-f(4)}{x-4}$

$\xi < x \underset{f'\uparrow}{\Rightarrow} f'(\xi) < f'(x) \Leftrightarrow \frac{f(x)-f(4)}{x-4} < f'(x)$

$\frac{f(x)-5}{x-4} < f'(x)$.

Θεμα 15

• $f(x) = \dfrac{e^x}{e^x+1}$, $x \in \mathbb{R}$

ⓐ Εστω $f(x_1) = f(x_2) \Leftrightarrow \dfrac{e^{x_1}}{e^{x_1}+1} = \dfrac{e^{x_2}}{e^{x_2}+1}$

$\Leftrightarrow e^{x_1}(e^{x_2}+1) = e^{x_2}(e^{x_1}+1) \Leftrightarrow \cancel{e^{x_1}e^{x_2}} + e^{x_1} = \cancel{e^{x_2}e^{x_1}} + e^{x_2}$

$e^{x_1} = e^{x_2} \Leftrightarrow x_1 = x_2$ αρα f 1-1 άρα αντιστρ.

Θετω $y = \dfrac{e^x}{e^x+1} \Leftrightarrow y(e^x+1) = e^x \Leftrightarrow ye^x + y = e^x$

$\boxed{y>0}$ $\quad ye^x - e^x = -y \Leftrightarrow e^x(y-1) = -y$

$e^x = \dfrac{-y}{y-1}$ $\boxed{y \neq 1}$ $\Leftrightarrow e^x = \dfrac{y}{1-y}$

⊕

πρεπει $\dfrac{y}{1-y} > 0 \Rightarrow 1-y>0 \Rightarrow \boxed{y<1}$

Αρα $x = \ln\dfrac{y}{1-y} \Leftrightarrow$ $\boxed{f^{-1}(x) = \ln\left(\dfrac{x}{1-x}\right) \quad D_{f^{-1}} = (0,1)}$

(β) $2f(x^2-1)<1 \Rightarrow f(x^2-1)<\frac{1}{2} \Leftrightarrow f(x^2-1)<f(0)$ (6)

$$f'(x)=\frac{e^x(e^x+1)-e^xe^x}{(e^x+1)^2}=\frac{e^{2x}+e^x-e^{2x}}{(e^x+1)^2}=\frac{e^x}{(e^x+1)^2}$$

$f'(x)>0$

$f\nearrow$

$f\nearrow$

$x^2-1<0.$

$x^2<1$

$x^2<1^2$

$|x|<|1|$

$-1<x<1$

(γ) $$f''(x)=\frac{e^x(e^x+1)^2-e^x2(e^x+1)e^x}{(e^x+1)^4}$$

$$f''(x)=\frac{e^x(e^x+1)-2e^{2x}}{(e^x+1)^3}=\frac{e^{2x}+e^x-2e^{2x}}{(e^x+1)^3}=\frac{e^x-e^{2x}}{(e^x+1)^3}$$

$$f''(x)=\frac{e^x(1-e^x)}{(e^x+1)^3}$$

x		0	
f''	$+$	0	$-$
f	∪		∩

$O(0,\frac{1}{2})$

Σ.Κ.

$\rightarrow 1-e^x=0$

$1=e^x$

$\boxed{x=0}$

$\varepsilon: y-f(0)=f'(0)(x-0)$

$y-\frac{1}{2}=\frac{1}{4}x \Leftrightarrow \boxed{y=\frac{1}{4}x+\frac{1}{2}}$

$\forall x\geq 0$ η f κοίλη άρα $f(x)\leq\frac{1}{4}x+\frac{1}{2}$

$\underline{\underline{4f(x)\leq x+2}}$

5. $$E = \int_0^1 |f(x)|\,dx = \int_0^1 \left| \overset{\oplus}{\frac{e^x}{e^x+1}} \right| dx =$$

$$= \int_0^1 \frac{e^x}{e^x+1}\,dx = \left(\ln |e^x+1| \right)_0^1 =$$

$$= \ln(e+1) - \ln 2 .$$

Θεμα 16

$f(x) = x^2 + \frac{2}{x} \quad , x \neq 0$

α) $f'(x) = 2x - \frac{2}{x^2} = \frac{2x^3 - 2}{x^2} = 2\,\frac{x^3-1}{x^2}$

x^2 (+)

x		0	1	
f'	−	‖	− 0	+
f	↘	‖	↘ 3	↗

β) $f(-2) = 4 - 1 = 3$

$f(-1) = 1 - 2 = -1$

$f(-2)f(-1) < 0$

Bolzano $\exists\, \xi_1 \in (-2,-1)$ τω $f(\xi_1) = 0$.

Η f συνεχη στο $[-2,-1]$ ως πραξη συνεχων συναρτησεων.

Αφου $f\downarrow$ στο $(-\infty, 0)$ ειναι και στο $[-2,-1]$

αρα το ξ_1 μοναδικο.

γ) Το $e^x > 0$ και $\forall x > 0$ η $f(x)$ εχει ελαχιστο το $A(1,3)$ αρα μονο $f(1) = 3$ συνεπως $e^x = 1$.

$f(e^x) = 3$

$e^x = 1$

$\underline{x = 0}$

(5) $f'(x) = 2x - \frac{2}{x^2}$

$$f''(x) = 2 - \frac{-2 \cdot 2x}{x^4} = 2 + \frac{4}{x^3} = \frac{2x^3+4}{x^3}$$

$$f''(x) = 2\,\frac{x^3+2}{x^3}$$

$\rightarrow$ $x^3+2=0$

$x^3=-2$

$x=-\sqrt[3]{2}$

x		$-\sqrt[3]{2}$		0	
x^3+2	$-$		$+$		$+$
x^3	$-$		$-$		$+$
f''	$+$		$-$		$+$
f	$\cup$		$\cap$		$\cup$

f''

Θεμα 17

$$f(x)=\begin{cases} x^2 \ln x & , x>0 \\ 0 & , x=0 \end{cases}$$

(α) Η $f(x)$ είναι συνεχής στο $(0,+\infty)$ ως πράξη συνεχών συναρτήσεων.

• $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x^2 \ln x = \lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x^2}} = \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{\frac{-2x}{x^4}}$

$= \lim\limits_{x\to 0^+} -\dfrac{x^4}{+2x^2} = 0$

• $f(0)=0$ άρα $f(0)=\lim\limits_{x\to 0^+} f(x)$ συνεχής

και στο 0.

(β) $f'(x) = 2x\ln x + x^2 \dfrac{1}{x} = 2x\ln x + x = x(2\ln x + 1)$

$\longrightarrow 2\ln x + 1 = 0 \iff \ln x = -\dfrac{1}{2} \iff x = e^{-\frac{1}{2}} = \dfrac{1}{\sqrt{e}} = \dfrac{\sqrt{e}}{e}$

x	0		$\frac{\sqrt{e}}{e}$	
f'		$-$	0	$+$
f		$\searrow$		$\nearrow$

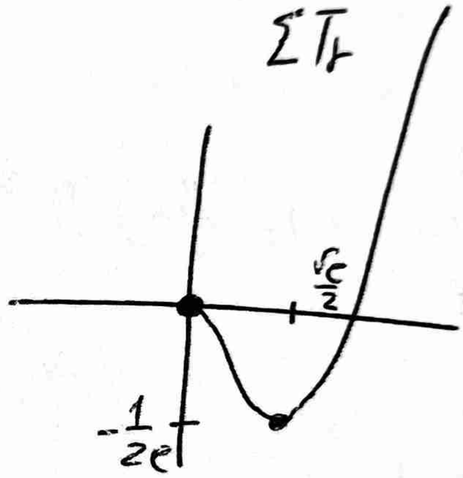
$f(x) \geq f\left(\frac{\sqrt{e}}{e}\right)$

$$f''(x) = 2\ln x + 2x\frac{1}{x} + 1 = 2\ln x + 3$$

$$\longrightarrow 2\ln x + 3 = 0 \Leftrightarrow \ln x = -\frac{3}{2} \Leftrightarrow x = e^{-\frac{3}{2}}$$

$$x = \frac{1}{e^{3/2}} = \frac{1}{\sqrt{e^3}} = \frac{1}{e\sqrt{e}} = \frac{\sqrt{e}}{e^2}$$

x	0	$\sqrt{e}/e^2$	
f''	$-$	0	$+$
f	$\frown$		$\smile$

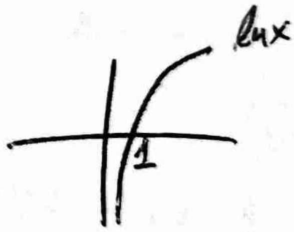


(γ) $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} x^2\ln x = +\infty$

$$f(e^{-\frac{1}{2}}) = (e^{-\frac{1}{2}})^2 \ln e^{-\frac{1}{2}} =$$

$$= -e^{-1}\frac{1}{2} = -\frac{1}{2e}$$

$$\Sigma T_f = [-\tfrac{1}{2e}, +\infty).$$



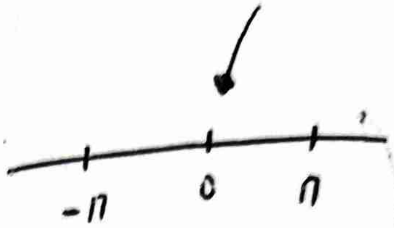
(δ) $E = \int_1^2 |f(x)|\,dx = \int_1^2 |x^2 \ln x|\,dx$

$$= \int_1^2 x^2\ln x\,dx = \int_1^2 \left(\frac{x^3}{3}\right)'\ln x\,dx =$$

$$= \left(\frac{x^3}{3}\ln x\right)_1^2 - \int_1^2 \frac{x^3}{3}\frac{1}{x}\,dx = \frac{8}{3}\ln 2 - \frac{1}{3}\int_1^2 x^2 dx = \frac{8}{3}\ln 2 - \frac{1}{9}7.$$

Θεμα 18

$$f(x) = \begin{cases} \alpha - \eta\mu x, & -\pi \le x < 0 \\ e^{-x} - 1, & x \ge 0 \end{cases}$$



(α) Αφου ισχυουν οι προυποθεσεις του ΘΜΤ η $f(x)$ συνεχης στο $[-\pi, \pi]$ αρα και στο 0

$$\left.\begin{aligned} \lim_{x \to 0^-} f(x) &= \lim_{x \to 0^-} (\alpha - \eta\mu x) = \alpha \\ \lim_{x \to 0^+} f(x) &= \lim_{x \to 0^+} (e^{-x} - 1) = 0 \end{aligned}\right\} \Leftrightarrow \underline{\underline{\alpha = 0}}$$

(β) Εστω $\varepsilon: y - f(x_0) = f'(x_0)(x - x_0)$ η εφαπτομενη που ψαχνω, η οποια ειναι καθετη στην $y = x$

αρα $f'(x_0) \cdot 1 = -1 \Leftrightarrow f'(x_0) = -1$.

Πρεπει να λυσω την εξισωση $f'(x) = -1$.

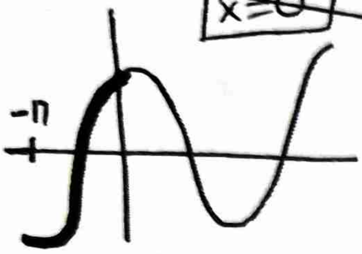
$-\pi \le x < 0$

$f'(x) = -1$

$-\sigma\upsilon\nu x = -1$

$\sigma\upsilon\nu x = 1$

$\cancel{\boxed{x=0}}$



$x \geq 0$

$f'(x) = -1$

$-e^{-x} = -1$

$e^{-x} = 1$

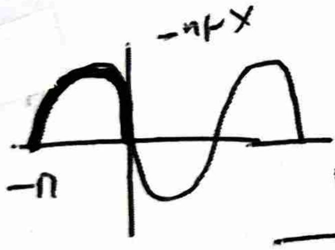
$\boxed{x=0}$

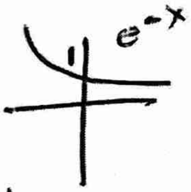
$\varepsilon: y - f(0) = f'(0)(x-0)$

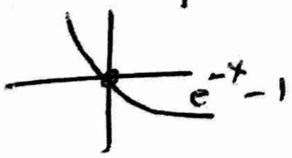
$y - 0 = -x \Leftrightarrow \boxed{y=-x}$

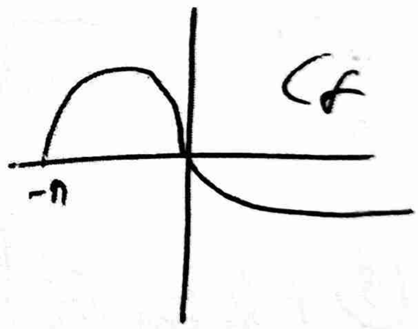
(γ). Αφού η f συνεχής στο 0 δεν έχει κατακόρυφη ασύμπτωτη.

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} (e^{-x} - 1) = -1$ $\quad \boxed{\varepsilon_1: y=-1}$









(δ) $\int_{-\pi}^{0} x f(x)\,dx = \int_{-\pi}^{0} -x\,\eta\mu x\,dx = -\int_{-\pi}^{0} x\,(-\sigma\upsilon\nu x)'\,dx$

$= -\left[(-x\sigma\upsilon\nu x)_{-\pi}^{0} - \int_{-\pi}^{0} -\sigma\upsilon\nu x\,dx \right]$

$= -\left[-(x\sigma\upsilon\nu x)_{-\pi}^{0} + \cancel{(\eta\mu x)_{-\pi}^{0}}^{\,0} \right] = +\pi\sigma\upsilon\nu(-\pi) = -\pi.$

Θεμα 19

[redacted]

$f(x) = \dfrac{x}{\sqrt{x^2+9}}$ $\qquad D_f = \mathbb{R}$.

(α) $f'(x) = \dfrac{\sqrt{x^2+9} - x \cdot \dfrac{2x}{2\sqrt{x^2+9}}}{x^2+9} =$

$= \dfrac{\sqrt{x^2+9} - \dfrac{x^2}{\sqrt{x^2+9}}}{x^2+9} = \dfrac{x^2+9-x^2}{(x^2+9)\sqrt{x^2+9}}$

$f'(x) = \dfrac{9}{(x^2+9)\sqrt{x^2+9}} > 0$ $\qquad f \nearrow$

(β) $f''(x) = \dfrac{-9\left[2x\sqrt{x^2+9} + (x^2+9)\dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2+9}}\right]}{(x^2+9)^4 (x^2+9)}$

$f''(x) = \dfrac{-9\left(2x(x^2+9) + x(x^2+9)\right)}{(x^2+9)^5\sqrt{x^2+9}}$ $(=)$

$f''(x) = \dfrac{-27x}{(x^2+9)^4\sqrt{x^2+9}}$

x		0	
f''	$+$	0	$-$
f	$\cup$		$\cap$

γ) $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2+9}} = \lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2\left(1+\frac{9}{x^2}\right)}}$

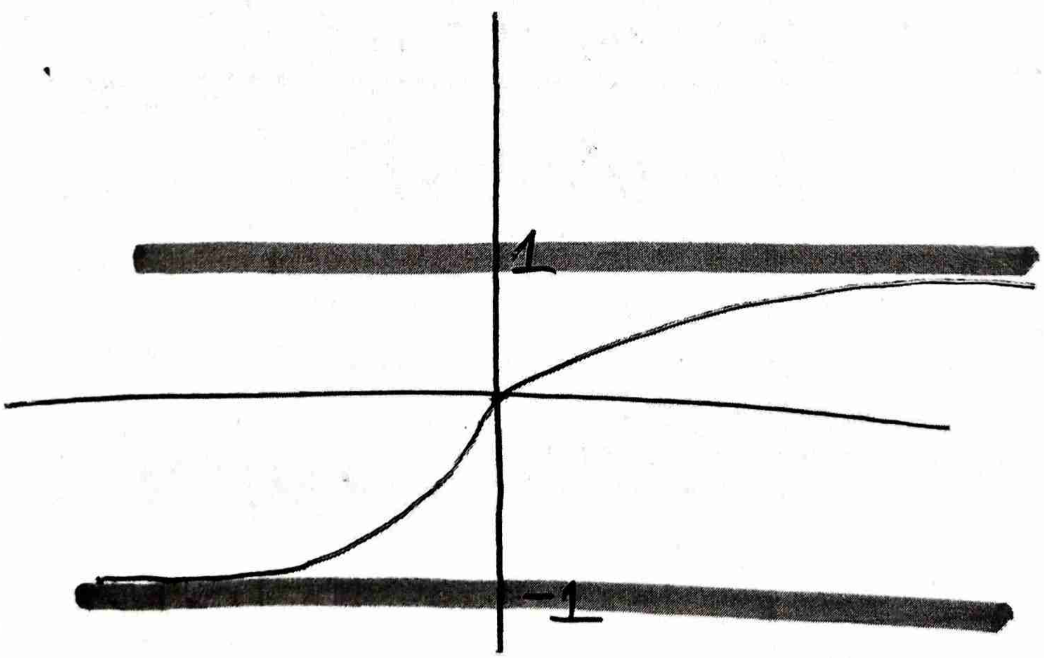
$= \lim\limits_{x\to-\infty} \dfrac{x}{\overset{\ominus}{|x|}\sqrt{1+\frac{9}{x^2}}} = \lim\limits_{x\to-\infty} \dfrac{\cancel{x}}{-\cancel{x}\sqrt{1+\frac{9}{x^2}}} = -1.$

$\varepsilon_1 : y = -1$ $-\infty$

$\lim\limits_{x\to+\infty} f(x) = 1$

$\varepsilon_2 : y = 1$ $+\infty$

δ) .



Θεμα 20

$$f(x)=\begin{cases} e^{-x}+\alpha, & x<0 \\ 1-\sigma\upsilon\nu x, & 0\le x\le 2\pi \end{cases}$$

ⓐ $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(e^{-x}+a) = 1+a$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(1-\sigma\upsilon\nu x) = 0$

Αφου f συνεχη $1+a=0 \iff \underline{\underline{a=-1}}$

$$\boxed{f(x)=\begin{cases} e^{-x}-1, & x<0 \\ 1-\sigma\upsilon\nu x, & 0\le x\le 2\pi \end{cases}}$$

ⓑ $\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-} \frac{e^{-x}-1}{x} = \lim_{x\to 0} -e^{-x} = -1$

$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} \frac{1-\sigma\upsilon\nu x - 0}{x-0} =$

$= \lim_{x\to 0^+} \frac{1-\sigma\upsilon\nu x}{x} = \lim_{x\to 0^+} \frac{\eta\mu x}{1} = 0$

Η $f(x)$ __δεν__ ειναι παρ/μη στο 0, αρα Το 0 κρισιμο σημειο

$x<0$

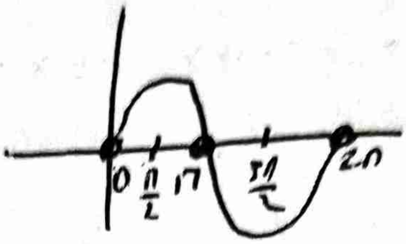
$f_1(x) = e^{-x} - 1$

$f_1'(x) = -e^{-x} < 0$

$0 < x \le 2\pi$

$f_2(x) = 1 - \text{συν}x$

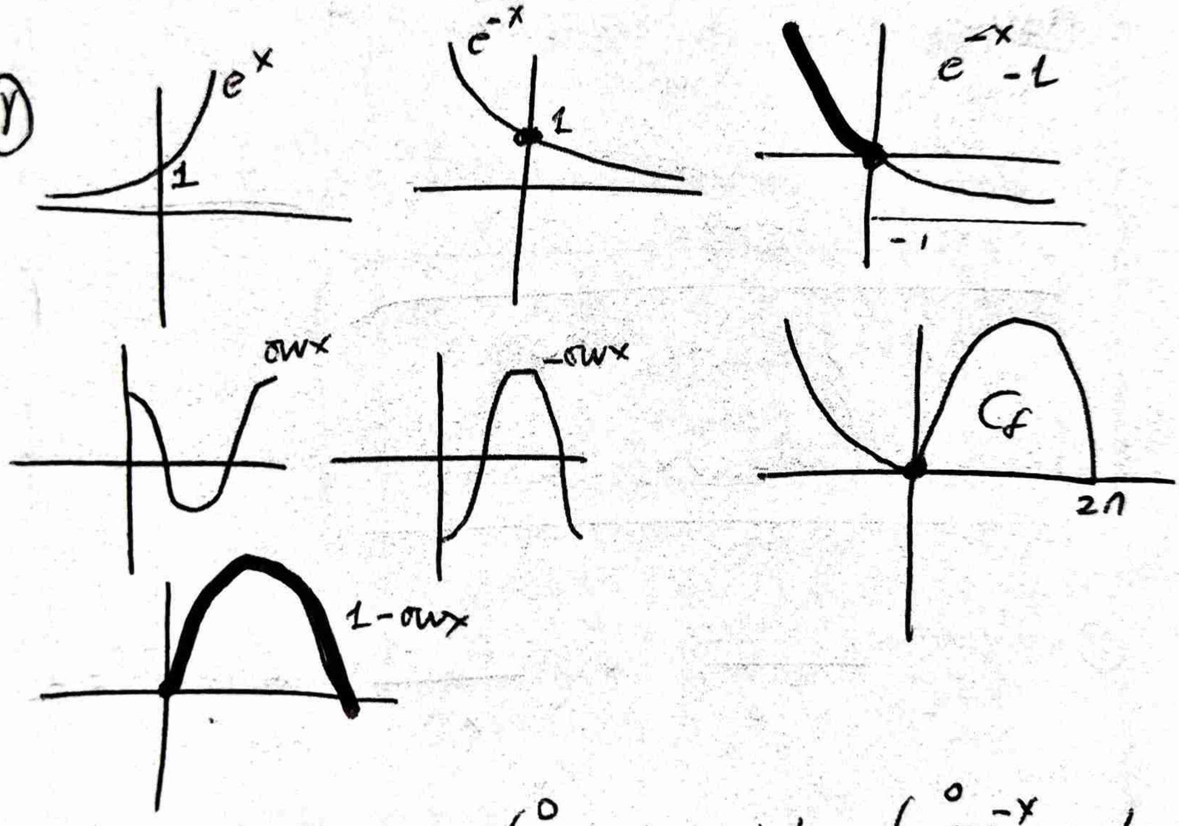
$f_2'(x) = \text{ημ}x$



Οι αποδεκτές ρίζες της $f'(x)$ είναι το π και το 2π

Το π κρίσιμο σημείο, Το 2π όχι γιατί δεν είναι εσωτερικό

(γ)



(δ) $\int_{-1}^{0} x f(x)\,dx = \int_{-1}^{0} x(e^{-x}-1)\,dx = \int_{-1}^{0} xe^{-x} - x\,dx$

$= \int_{-1}^{0} xe^{-x}dx - \int_{-1}^{0} x\,dx = \int_{-1}^{0} x(-e^{-x})'dx - \frac{1}{2}(x^2)_{-1}^{0} =$

$= (-xe^{-x})_{-1}^{0} - \int_{-1}^{0} -e^{-x}dx + \frac{1}{2} = -e - (e^{-x})_{-1}^{0} + \frac{1}{2} = -\frac{1}{2}.$

Θεμα 21

$$f(x) = \frac{e^x}{e^x - 1} \qquad D_f = \mathbb{R}^*$$

ⓐ $f(x_1) = f(x_2) \Leftrightarrow \dfrac{e^{x_1}}{e^{x_1}-1} = \dfrac{e^{x_2}}{e^{x_2}-1} \Leftrightarrow$

$$e^{x_1}(e^{x_2}-1) = e^{x_2}(e^{x_1}-1) \Leftrightarrow \cancel{e^{x_1}e^{x_2}} - e^{x_1} = \cancel{e^{x_1}e^{x_2}} - e^{x_2}$$

$$e^{x_1} = e^{x_2} \Rightarrow x_1 = x_2 \Rightarrow f \text{ 1-1 άρα αντιστρ.}$$

$$y = \frac{e^x}{e^x-1} \Leftrightarrow y(e^x - 1) = e^x \Leftrightarrow ye^x - y = e^x$$

$$ye^x - e^x = y \Leftrightarrow e^x(y-1) = y \Leftrightarrow e^x = \frac{y}{y-1}$$

$\boxed{y \neq 1}$

$$x = \ln\left(\frac{y}{y-1}\right) \text{ πρέπει } \frac{y}{y-1} > 0.$$

y		0		1	
y	$-$	0	$+$		$+$
$y-1$	$-$		$-$	0	$+$
$\frac{y}{y-1}$	$+$		$-$		$+$

$$f^{-1}(x) = \ln\left(\frac{x}{x-1}\right)$$

$D_{f^{-1}} \longrightarrow \quad y \in (-\infty, 0) \cup (1, +\infty).$

(B) $f'(x) = \dfrac{e^x(e^x-1) - e^x e^x}{(e^x-1)^2}$

$$f'(x) = \frac{\cancel{e^{2x}} - e^x - \cancel{e^{2x}}}{(e^x-1)^2} = -\frac{e^x}{(e^x-1)^2} < 0$$

$f \downarrow$ στο $(-\infty, 0)$ και στο $(0, +\infty)$.

(γ). $f''(x) = -\dfrac{e^x(e^x-1)^2 - e^x\, 2(e^x-1)e^x}{(e^x-1)^4}$

$$f''(x) = -\frac{e^x(e^x-1) - 2e^{2x}}{(e^x-1)^3}$$

$$f''(x) = -\frac{-e^x - e^{2x}}{(e^x-1)^3} = \frac{e^x(e^x+1)}{(e^x-1)^3}$$

x		0	
f''	$-$		$+$
f	$\frown$		$\smile$

(5) · $f^{-1}(x) = \ln\left(\dfrac{x}{x-1}\right)$ $\qquad D_{f^{-1}} = (-\infty, 0) \cup (1, +\infty)$

$\quad \| $

$\quad \varphi(x)$

$$\varphi'(x) = \frac{1}{\frac{x}{\cancel{x-1}}} \cdot \frac{x-1-x}{(x-1)^{\cancel{2}}} = \frac{1}{x} \cdot \frac{-1}{x-1}$$

$$\varphi'(x) = \frac{-1}{\underbrace{x(x-1)}_{+}} < 0 \qquad \varphi \searrow \text{ στο } (-\infty, 0) \text{ και } (1, +\infty)$$

$$\varphi''(x) = \frac{x-1+x}{x^2(x-1)^2} = \frac{2x-1}{x^2(x-1)^2}$$

x		0		$\frac{1}{2}$		1	
φ''	$-$		$\cancel{-}$	$\cancel{0}$	$\cancel{+}$		$+$
φ	$\frown$						$\smile$

Θεμα 22

$f(x)=\begin{cases} e^{x-1}+\alpha, & x\le 1 \\ 1+\frac{\ln x}{x}, & x>1 \end{cases}$

ⓐ Αφου η f ειναι συνεχης

$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x) \Leftrightarrow$

$\lim_{x\to 1^-}\left(e^{x-1}+\alpha\right)=\lim_{x\to 1^+}\left(1+\frac{\ln x}{x}\right)$

$1+\alpha=1 \Leftrightarrow \underline{\underline{\alpha=0}}$

$$\boxed{f(x)=\begin{cases} e^{x-1}, & x\le 1 \\ 1+\frac{\ln x}{x}, & x>1 \end{cases}}$$

ⓑ $\lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1^-}\frac{e^{x-1}-1}{x-1}=\lim_{x\to 1^-}\frac{e^{x-1}}{1}=1.$

$\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1^+}\frac{1+\frac{\ln x}{x}-1}{x-1}=\lim_{x\to 1^+}\frac{\ln x}{x^2-x}$

$=\lim_{x\to 1^+}\frac{\frac{1}{x}}{2x-1}=\frac{1}{1}=1$ $\quad f'(1)=1=\varepsilon\varphi\omega$

$\omega=45$

(γ). $\underline{x \leq 1}$ $\qquad\qquad$ $\underline{x > 1}$

$f_1'(x) = e^{x-1} > 0$ $\qquad\qquad$ $f_2'(x) = \dfrac{1 - \ln x}{x^2}$

$\rightarrow 1 - \ln x = 0$

$1 = \ln x$

$\boxed{x = e}$

x		1	e
$f_1'(x)$	+	//////	//////
$f_2'(x)$	//////	+	0 −
$f'(x)$	+	+	−
$f(x)$	↗	↗	↘

$f(x) \leq f(e)$

$\boxed{f(x) \leq 1 + \frac{1}{e}.}$

Αφού $D_f = R$ και $f(x)$ συνεχής $\underline{\text{δεν}}$ έχω

κατακόρυφη ασύμπτωτη.

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} e^{x-1} = 0$ $\qquad$ $\varepsilon_1 : y = 0$ $\quad -\infty$

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} 1 + \dfrac{\ln x}{x} = 1$ $\qquad$ $\varepsilon_2 : y = 1$ $\quad +\infty$

$\rightarrow \lim\limits_{x \to +\infty} \dfrac{\ln x}{x} = \lim\limits_{x \to +\infty} \dfrac{1}{x} = 0$

(δ) $\displaystyle\int_1^2 f(x)\,dx = \int_1^2 1 + \frac{\ln x}{x}\,dx = \int_1^2 1\,dx + \int_1^2 \frac{\ln x}{x}\,dx = (x)_1^2 + \int_0^{\ln 2} t\,dt$

$\boxed{\begin{aligned} \ln x &= t \\ \tfrac{1}{x}dx &= dt \end{aligned}}$ $\quad = 1 + \frac{1}{2}(t^2)_0^{\ln 2} = \dfrac{\ln^2 2}{2} + 1$

Θεμα 23

• $f(x) = e^{x-1} - \ln x$, $D_f = (0, +\infty)$

ⓐ $f'(x) = e^{x-1} - \frac{1}{x}$ $f'(1) = 0$

$f''(x) = e^{x-1} + \frac{1}{x^2} > 0$

x	0	1	
f''	+		+
f'	↗ −	0	↗ +
f	↘		↗

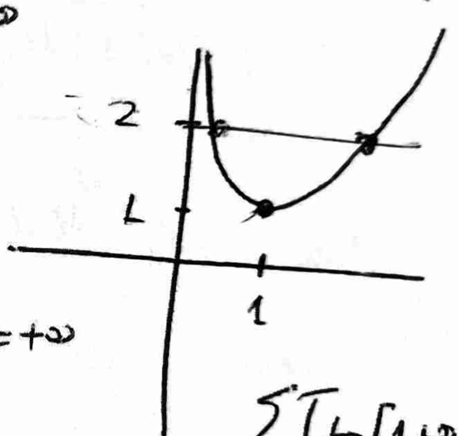
$f(x) \geq f(1)$

$f(x) \geq 1$.

ⓑ $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} (e^{x-1} - \ln x) = e^{-1} - (-\infty) = +\infty$

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} (e^{x-1} - \ln x) =$

$= \lim\limits_{x \to +\infty} e^{x-1} \left(1 - \frac{\ln x}{e^{x-1}}\right) = +\infty$



$\Sigma T_f = [1, +\infty)$.

$\rightarrow \lim\limits_{x \to +\infty} \frac{\ln x}{e^{x-1}} = \lim\limits_{x \to +\infty} \frac{\frac{1}{x}}{e^{x-1}} = \lim\limits_{x \to +\infty} \frac{1}{x e^{x-1}} = 0$

ⓖ

$x \leq 1$

• f συνεχ.
• f ↘
• $\Sigma T_f = [1, +\infty)$

Το $2 \in \Sigma T_f$ άρα $\exists x_1$ τ.υ $f(x_1) = 2$ μοναδικο/.

$x > 1$

• f συνεχ.
• f ↗
• $\Sigma T_f = [1, +\infty)$

Το $2 \in \Sigma T_f$ άρα $\exists x_2$ μοναδικο τ.υ $f(x_2) = 2$

δ) $f(x) + |x-1| = 1$.

$$\underbrace{f(x)-1}_{\oplus} + \underbrace{|x-1|}_{\oplus} = 0$$

- $f(x) \geq 1 \Rightarrow f(x) - 1 \geq 0$ Το "=" για $x=1$.
- $|x-1| \geq 0$ Το "=" για $x=1$

Μοναδική λύση $x=1$

Θεμα 24

• $f(x) = \alpha e^{x-1} + \beta x^2$

(α) Η ε: y=1 εφαπτομενη στο 1.

$\hookrightarrow y = 0 \cdot x + 1$

$$\begin{cases} f'(1) = 0 \\ f(1) = 1 \end{cases} \Leftrightarrow \begin{cases} \alpha + 2\beta = 0 \\ \alpha + \beta = 1 \end{cases} \Leftrightarrow \begin{cases} -\alpha - 2\beta = 0 \\ \alpha + \beta = 1 \end{cases} (+)$$

$-\beta = 1$

$\underline{\underline{\beta = -1}}$

$\underline{\underline{\alpha = 2}}$

$f'(x) = \alpha e^{x-1} + 2\beta x$

$$\boxed{f(x) = 2e^{x-1} - x^2}$$

(β) $f'(x) = 2e^{x-1} - 2x$. $f'(1) = 0$

$f''(x) = 2e^{x-1} - 2$

$\longrightarrow 2e^{x-1} - 2 = 0$

$2e^{x-1} = 2$

$e^{x-1} = 1$

$x - 1 = 0$

$x = 1$.

x		1	
f''	−	0	+
f'	↘ +		+ ↗
f	↗		↗

Αρα η f ↗ αρα 1-1 αρα αντιστρεψιμη.

$D_{f^{-1}} = \Sigma T_f$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}\left(2e^{x-1} - x^2\right) = -\infty$

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\left(2e^{x-1} - x^2\right) = \lim\limits_{x\to+\infty} x^2\left(2\frac{e^{x-1}}{x^2} - 1\right) = +\infty$

$\to \lim\limits_{x\to+\infty} \frac{e^{x-1}}{x^2} = \lim\limits_{x\to+\infty} \frac{e^{x-1}}{2x} = \lim\limits_{x\to+\infty} \frac{e^{x-1}}{2} = +\infty$

$\Sigma T_f = \mathbb{R}$.

⑧

x		1	
f''	$-$	0	$+$
f	$\frown$		$\smile$

$A(1, f(1))$ $\Sigma.K$.

⑨ . $f'(2e^{x-1}+1) > f'(x^2+2)$.

$e^{x-1} > 0 \Rightarrow 2e^{x-1} > 0 \Leftrightarrow 2e^{x-1}+1 > 1$

$x^2 \geq 0 \Rightarrow x^2+2 > 2$

$\forall x > 1$, $f' \nearrow$

$2e^{x-1}+1 > x^2+2$

$2e^{x-1} - x^2 > 1$.

$f(x) > 1 \Rightarrow f(x) > f(1) \Rightarrow \underline{\underline{x > 1}}$

Θεμα 25

$f(x) = x + \frac{2\ln x}{x}$, $D_f = (0, +\infty)$

ⓐ $f'(x) = 1 + 2\frac{\frac{1}{x}x - \ln x}{x^2} = 1 + 2\frac{1-\ln x}{x^2} = \frac{x^2+2-2\ln x}{x^2}$

Θετω $\boxed{\varphi(x) = x^2+2-2\ln x}$ $\varphi(1) = 0$

x	0		1	
φ'		−		+
φ		↘ +	3	+ ↗
f'		+		+
f		↗		↗

$\varphi'(x) = 2x - \frac{2}{x} = \frac{2x^2-2}{x} = 2\frac{x^2-1}{x}$

$\varphi'(x) = 0 \iff x = 1$ ή $x = -1$

$\varphi(x) \geq \varphi(1)$

$\underline{\varphi(x) \geq 3}$

ⓑ $f'(x) = 1 + 2\frac{1-\ln x}{x^2}$

$f''(x) = 2\frac{-\frac{1}{x}x^2 - (1-\ln x)2x}{x^4} = 2\frac{-x-2x+2x\ln x}{x^4}$

$f''(x) = 2\frac{-3+2\ln x}{x^3}$

x	0		$e\sqrt{e}$	
f''		−	0	+
f		∩		∪

→ $2\ln x - 3 = 0$

$\ln x = \frac{3}{2}$

$x = e^{3/2} = \sqrt{e^3} = e\sqrt{e}$

⑧ $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x + \frac{2\ln x}{x} = \lim\limits_{x\to 0^+} x + 2\ln x \, \frac{1}{x} =$

$\boxed{\varepsilon_1 : x = 0}$ $= 0 + 2(-\infty)(+\infty) = -\infty .$

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\left(x + \frac{2\ln x}{x}\right) = +\infty$

$\to \lim\limits_{x\to+\infty} \frac{2\ln x}{x} = \lim\limits_{x\to+\infty} \frac{2}{x} = 0$

$\lim\limits_{x\to+\infty} \frac{f(x)}{x} = \lim\limits_{x\to+\infty} \frac{x + \frac{2\ln x}{x}}{x} = \lim\limits_{x\to+\infty} 1 + \frac{2\ln x}{x^2} = \underline{1}$

$\to \lim\limits_{x\to+\infty} \frac{2\ln x}{x^2} = \lim\limits_{x\to+\infty} \frac{\frac{2}{x}}{2x} = \lim\limits_{x\to+\infty} \frac{2}{2x^2} = 0$

$\lim\limits_{x\to+\infty} (f(x) - x) = \lim\limits_{x\to+\infty}\left(x + \frac{2\ln x}{x} - x\right) = 0$

$\boxed{\varepsilon_2 : y = x}$

(δ) $E = \int_1^e |f(x) - x| \, dx = \int_1^e \left| \frac{2\ln x}{x} \right| dx \overset{(*)}{=}$

(+) (+)

• $f(x) = x \Rightarrow x + \frac{2\ln x}{x} = x \Leftrightarrow \frac{2\ln x}{x} = 0 \Leftrightarrow x = 1$

• $f(x) - x = \frac{2\ln x}{x}$

$\overset{(*)}{=} \int_1^e 2 \frac{\ln x}{x} dx = 2 \int_1^e \frac{\ln x}{x} dx$

$= 2\int_0^1 t \, dt =$

$= (t^2)_0^1 = 1.$

Θέτω $\ln x = t$

$\frac{1}{x} \cdot dx = dt$

Θεμα 26

$\bullet \; f(x) = \begin{cases} \frac{ημx}{x}, & x \in (0, π) \\ α, & x = 0. \end{cases}$ Συνεχής!.

ⓐ νδο $α = 1$.

$\lim_{x \to 0^+} f(x) = f(0) \Leftrightarrow \lim_{x \to 0^+} \frac{ημx}{x} = α \Leftrightarrow α = 1$

ⓑ. $ε: y - f(0) = f'(0)(x - 0)$

$\bullet \; f(0) = 1$

$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{\frac{ημx}{x} - 1}{x} = \lim_{x \to 0} \frac{ημx - x}{x^2}$

$= \lim_{x \to 0} \frac{συνx - 1}{2x} = \lim_{x \to 0} \frac{-ημx}{2} = 0$

$ε: y - 1 = 0(x - 0)$ $\boxed{ε: y = 1}$.

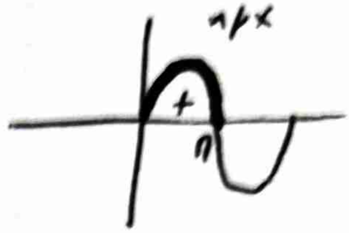
γ) $f'(x) = \dfrac{x\,συνx - ημx}{x^2}$

$\boxed{φ(x) = x\,συνx - ημx}$

$φ(0) = 0$

$φ'(x) = συνx - x\,ημx - συνx$

$φ'(x) = -\underset{\oplus}{x}\,\underbrace{ημx}_{\oplus} < 0$ στο $(0,π)$



x	0 π
φ'	−
φ	↘ −
f'	−
f	↘

$x>0 \Rightarrow φ(x)<φ(0) \Rightarrow φ(x)<0$

δ) $\lim\limits_{x\to 0}\left[ημx \cdot \ln x\right] = \lim\limits_{x\to 0} \dfrac{ημx}{x} \cdot x\ln x = 1\cdot 0 = 0$

$\to \lim\limits_{x\to 0} x\ln x = \lim\limits_{x\to 0} \dfrac{\ln x}{\frac{1}{x}} = \lim\limits_{x\to 0} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = 0.$

Θεμα 27

• $f(x) = \alpha^x - x$

• $f(x) \geq 1 \quad \forall x \in R$.

(α) νδο $\alpha = e$.

$f(x) \geq 1$

$f(x) \geq f(0)$

$\left.\begin{array}{l}\text{Ακροτατο στο } 0 \\ f \text{ παρ/μη στο } 0 \\ \text{Το } 0 \text{ Εσωτερικο του } R\end{array}\right\}$ Fermat $f'(0) = 0$

$f'(x) = a^x \ln a - 1$

$f'(0) = \ln a - 1 = 0 \Rightarrow \ln a = 1 \Leftrightarrow \alpha = e$

$$\boxed{f(x) = e^x - x}$$

(β). $e < \pi \Rightarrow f(e) < f(\pi) \Rightarrow e^e - e < e^\pi - \pi$

$\pi - e < e^\pi - e^e$

$1 < \dfrac{e^\pi - e^e}{\pi - e}$.

$f(x) = e^x - x$

$f'(x) = e^x - 1$

x		0	
f'	$-$	0	$+$
f	↘		↗

$f(x) \geq f(0) \Rightarrow f(x) \geq 1$

Γ) $\lim\limits_{x\to-\infty} \frac{f(x)}{x} = \lim\limits_{x\to-\infty} \frac{e^x - x}{x} = \lim\limits_{x\to-\infty} \frac{e^x-1}{1} = -1$

$\lim\limits_{x\to-\infty} (f(x)+x) = \lim\limits_{x\to-\infty} e^x = 0$

$y=-x$
$-\infty$

δ) $I = \int_0^{\pi} (e^x - x)\,\sigma\upsilon\nu x\, dx = \int_0^{\pi} e^x \sigma\upsilon\nu x\, dx - \int_0^{\pi} x\,\sigma\upsilon\nu x\, dx$

$I = J - R = -\frac{e^{\pi}+1}{2} + 2.$

$J = \int_0^{\pi} e^x \sigma\upsilon\nu x\, dx = \int_0^{\pi} (e^x)' \sigma\upsilon\nu x\, dx = (e^x \sigma\upsilon\nu x)_0^{\pi} + \int_0^{\pi} e^x \eta\mu x\, dx$

$J = -e^{\pi} - 1 + \int_0^{\pi} (e^x)' \eta\mu x\, dx$

$J = -e^{\pi} - 1 + \cancelto{0}{(e^x \eta\mu x)_0^{\pi}} - \int_0^{\pi} e^x \sigma\upsilon\nu x\, dx$

$J = -e^{\pi} - 1 - J \iff 2J = -e^{\pi} - 1 \iff J = -\frac{e^{\pi}+1}{2}.$

$R = \int_0^{\pi} x\,\sigma\upsilon\nu x\, dx = \int_0^{\pi} x(\eta\mu x)'\, dx = (x\,\eta\mu x)_0^{\pi} - \int_0^{\pi} \eta\mu x\, dx$

$R = +(\sigma\upsilon\nu x)_0^{\pi} = -2$

Θεμα 28

- $f: R \longrightarrow R$ παρ/μη
- $f(0) = 1$
- $f(x) \geq 2e^x - x - 1 \quad \forall x \in R.$

ⓐ $\underbrace{f(x) - 2e^x + x + 1}_{\varphi(x)} \geq 0 \iff \varphi(x) \geq 0 \iff \varphi(x) \geq \varphi(0)$

Fermat

$\varphi'(0) = 0$

γιατι ελαχιστο στο 0

παρ/μη στο 0

Εσωτερικο του R.

$\varphi'(x) = f'(x) - 2e^x + 1$

$\varphi'(0) = f'(0) - 1 = 0$

$f'(0) = 1$

$\varepsilon\varphi\hat{\omega} = 1 \iff \underline{\underline{\omega = 45}}$

ⓑ $f(x) \geq 2e^x - x - 1$

$\lim_{x \to +\infty} f(x) \geq \lim_{x \to +\infty} 2e^x - x - 1 \Longrightarrow \lim_{x \to +\infty} f(x) \geq +\infty$

$\lim_{x \to +\infty} f(x) = +\infty.$

$\rightarrow \lim_{x \to +\infty} 2e^x - x - 1 = \lim_{x \to +\infty} x\left(2\frac{e^x}{x} - 1 - \cancel{\frac{1}{x}}\right) = +\infty$

$\rightarrow \lim_{x \to +\infty} \frac{e^x}{x} = \lim_{x \to +\infty} e^x = +\infty$

(γ) ν.δ.ο $2\int_0^1 f(x)\,dx \geq 4e-7$

Αφού $f(x) \geq 2e^x - x - 1$

$$\int_0^1 f(x)\,dx \geq \int_0^1 2e^x - x - 1$$

$$\Leftrightarrow \int_0^1 f(x)\,dx \geq 2\left(e^x\right)_0^1 - \frac{1}{2}\left(x^2\right)_0^1 - (x)_0^1$$

$$\int_0^1 f(x)\,dx \geq 2(e-1) - \frac{1}{2} - 1$$

$$\Leftrightarrow 2\int_0^1 f(x)\,dx \geq 4e - 4 - 1 - 2$$

$$2\int_0^1 f(x)\,dx \geq 4e - 7.$$

(δ) $\lim\limits_{x\to 0} (f(x)-1)\ln x = \lim\limits_{x\to 0^+} \dfrac{f(x)-f(0)}{x-0} \, x\ln x$

$$= f'(0)\cdot 0 = 0$$

γιατί $\lim\limits_{x\to 0^+} x\ln x = \lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}} = \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}}$

$$= \lim_{x\to 0^+} -\frac{x^{\cancel{2}}}{\cancel{x}} = 0.$$

Θεμα 29

• $f(x) = 2e^x - x^2 - 2x$

ⓐ $f'(x) = 2e^x - 2x - 2 = 2\underbrace{(e^x - x - 1)}_{}^{\oplus} \geq 0 \quad f \nearrow$

• $e^x \geq x+1 \Rightarrow e^x - x - 1 \geq 0$

ⓑ $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} (2e^x - x^2 - 2x) = -\infty$

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} (2e^x - x^2 - 2x) =$

$= \lim\limits_{x \to +\infty} e^x\left(2 - \cancel{\frac{x^2}{e^x}}^{0} - \cancel{\frac{2x}{e^x}}^{0}\right) = +\infty$

$\rightarrow \lim\limits_{x \to +\infty} \frac{x^2}{e^x} = \lim\limits_{x \to +\infty} \frac{2x}{e^x} = \lim\limits_{x \to +\infty} \frac{2}{e^x} = 0$

$\Sigma T_f = \mathbb{R}.$

ⓖ i) $\lim\limits_{x \to 0} \frac{f(x) - 2}{\eta\mu x} = \lim\limits_{x \to 0} \frac{\frac{f(x) - f(0)}{x - 0}}{\frac{\eta\mu x}{x}} = \frac{f'(0)}{1} = 0.$

11. $\lim\limits_{x\to 0} (f(x)-2)\ln x =$

$= \lim\limits_{x\to 0} \frac{f(x)-f(0)}{x-0} \, x\ln x = f'(0)\cdot 0 = 0$

$\to \lim\limits_{x\to 0} x\ln x = \lim\limits_{x\to 0} \frac{\ln x}{\frac{1}{x}} = \lim\limits_{x\to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = 0$.

8 NSO $f(e^x) > f(x) \ \forall x \in \mathbb{R}$.

- $e^x \geq x+1$

$e^x > x$

$f \nearrow$

$f(e^x) > f(x)$.

Θεμα 30

$$f(x)=\begin{cases} 2x\ln x - x^2+2 & , x>0 \\ 2 & , x=0 \end{cases}$$

(α) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left[2x\ln x - x^2+2\right] = 2$

$\to \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} =$

$= \lim_{x\to 0^+} -\dfrac{x^2}{x} = 0$

$f(0)=2$ αρα συνεχη στο 0.

Επισης συνεχη στο $(0,+\infty)$ ως πραξεις

συνεχων συναρτησεων.

(Β) $f'(x) = 2\ln x + 2 - 2x = 2\left(\ln x - x + 1\right) \leq 0$

$\cdot\ \ln x \leq x-1 \Rightarrow \ln x - x + 1 \leq 0$

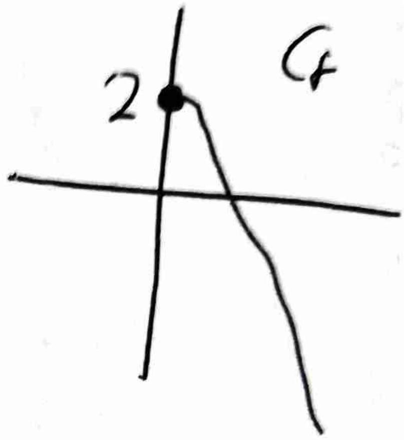
$f \downarrow$

(γ) $f(0)=2$

$$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\left(2x\ln x-x^2+2\right)=$$

$$=\lim_{x\to+\infty} x^2\Big(2\cancel{\frac{\ln x}{x}}^{\,0}-1+\cancel{\frac{2}{x^2}}^{\,0}\Big)=-\infty$$

$$\to \lim_{x\to+\infty}\frac{\ln x}{x}=\lim_{x\to+\infty}\frac{1}{x}=0$$



$\Sigma T_f=(-\infty,2]$

(δ) $f(|x|)-f(|\eta\mu x|)=0$

$f(|x|)=f(|\eta\mu x|)$

f ?1-1 αφού $f\downarrow$

$|x|=|\eta\mu x|$

όμως $|\eta\mu x|\le|x|$ και το "="

επιτυγχάνεται για $x=0$,

Επομενο Μαθημα

Δευτερα.

Στα δυο επομενα μαθηματα
Θα λυθουν απο εσας.
Ασκησεις στο Μαθημα.

Για το μαθημα (1ο μετα
το πασχα).

Θεματα πανελληνιων

2020
2021
2022
2023

κανονικη

Μπαρλα

32 40 51
41
33
42
37
49
38
50