Παραγοντοποίηση

1 $x^2-4x = x(x-4)$

2 $2x^3-4x^2 = 2x^2(x-2)$

3 $5x-10 = 5(x-2)$

4 $x^3-x^4 = x^3(1-x)$

5 $7x^2-14x = 7x(x-2)$

6 $x^2-1 = x^2-1^2 = (x-1)(x+1)$

7 $x^2-4 = x^2-2^2 = (x-2)(x+2)$.

8 $4x^2-9 = (2x)^2-3^2 = (2x-3)(2x+3)$

9 $25-9x^2 = 5^2-(3x)^2 = (5-3x)(5+3x)$

10 $100-x^2 = 10^2-x^2 = (10-x)(10+x)$

extra.

$1 - x^3 = 1^3 - x^3 = (1-x)(1^2 + 1 \cdot x + x^2)$

$= (1-x)(1 + x + x^2)$

$8x^3 - 27 = (2x)^3 - 3^3 =$

$= (2x-3)((2x)^2 + 2x \cdot 3 + 3^2)$

$= (2x-3)(4x^3 + 6x + 9)$.

11. $x^2 - 2x + 1 = (x-1)^2$

$\boxed{a=1 \quad B=-2 \quad \gamma=1}$

$\Delta = B^2 - 4a\gamma$

$\Delta = (-2)^2 - 4 \cdot 1 \cdot 1$

$\Delta = 4 - 4$

$\Delta = 0$

$x_1 = -\frac{B}{2a}$

$x_1 = -\frac{(-2)}{2 \cdot 1} = \frac{2}{2} = 1$

12. $4x^2 - 4x + 1 = 4\left(x - \frac{1}{2}\right)^2$

$\boxed{a=4 \quad B=-4 \quad \gamma=1}$

$\Delta = B^2 - 4a\gamma$

$\Delta = (-4)^2 - 4 \cdot 4 \cdot 1$

$\Delta = 16 - 16 = 0.$

$x = \frac{-B}{2a}$

$x = \frac{-(-4)}{2 \cdot 4} = \frac{1}{2}.$

13. $x^2 - 6x + 9 = (x-3)^2.$

$\boxed{a=1 \quad B=-6 \quad \gamma=9}$

$\Delta = B^2 - 4a\gamma$

$\Delta = (-6)^2 - 4 \cdot 1 \cdot 9$

$\Delta = 36 - 36 = 0$

$x = \frac{-B}{2a} = \frac{-(-6)}{2 \cdot 1} = 3.$

14. $x^2 - 4x + 4 = (x-2)^2$

$a = 1 \quad B = -4 \quad \gamma = 4$

$\Delta = B^2 - 4a\gamma$

$\Delta = (-4)^2 - 4 \cdot 1 \cdot 4$

$\Delta = 16 - 16$

$\Delta = 0$

$x = \frac{-B}{2a}$

$x = \frac{-(-4)}{2} = 2$

15. $x^2 + 2x + 1 = (x+1)^2$.

$a = 1 \quad B = 2 \quad \gamma = 1$

$\Delta = B^2 - 4a\gamma$

$\Delta = 2^2 - 4 \cdot 1 \cdot 1$

$\Delta = 4 - 4$

$\Delta = 0.$

$x = \frac{-B}{2a}$

$x = \frac{-2}{2} = -1$

16. $x^2 - 4x + 3 = (x-3)(x-1)$.

$a = 1 \quad B = -4 \quad \gamma = 3$

$\Delta = B^2 - 4a\gamma$

$\Delta = (-4)^2 - 4 \cdot 1 \cdot 3$

$\Delta = 16 - 12$

$\Delta = 4$

$x_{1,2} = \frac{-B \pm \sqrt{\Delta}}{2a}$

$x = \frac{-(-4) \pm \sqrt{4}}{2 \cdot 1}$

$x = \frac{4 \pm 2}{2}$ → $\frac{4+2}{2} = 3$; $\frac{4-2}{2} = 1$

17. $x^2 - 6x + 5 = (x-5)(x-1)$.

$a = 1 \quad B = -6 \quad \gamma = 5$

$\Delta = B^2 - 4a\gamma$

$\Delta = (-6)^2 - 4 \cdot 1 \cdot 5$

$\Delta = 36 - 20$

$\Delta = 16$.

$x = \frac{-B \pm \sqrt{\Delta}}{2a}$.

$x = \frac{-(-6) \pm 4}{2}$

$x = \frac{6 \pm 4}{2}$ → 5 ; 1

18. $2x^2 - x - 1 = 2(x-1)\left(x+\frac{1}{2}\right)$

$a=2 \quad B=-1 \quad \gamma=-1$

$\Delta = B^2 - 4a\gamma$

$\Delta = (-1)^2 - 4 \cdot 2 \cdot (-1)$

$\Delta = 1 + 8 = 9$

$$x = \frac{-B \pm \sqrt{\Delta}}{2a}$$

$$x = \frac{1 \pm 3}{4} \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$$

19. $x^2 + 3x + 2 = (x+1)(x+2)$.

$a=1 \quad B=3 \quad \gamma=2$

$\Delta = B^2 - 4a\gamma$

$\Delta = 3^2 - 4 \cdot 1 \cdot 2$

$\Delta = 9 - 8$

$\Delta = 1$

$$x = \frac{-B \pm \sqrt{\Delta}}{2a}$$

$$x = \frac{-3 \pm 1}{2} \begin{cases} -1 \\ -2 \end{cases}$$

$$A = \frac{4x^2-7x+3}{x^3+2x^2+4x} : \frac{x^2-3x+2}{8-x^3}$$

$\Delta = B^2 - 4ac$

$\Delta = (-7)^2 - 4 \cdot 4 \cdot 3$

$\Delta = 49 - 48$

$\Delta = 1$

$x = \frac{-B \pm \sqrt{\Delta}}{2a}$

$x = \frac{7 \pm 1}{8}$ → (1) ; $\frac{3}{4}$.

$$A = \frac{4(x-1)(x-\frac{3}{4})}{x(x^2+2x+4)} : \frac{(x-2)(x-1)}{2^3-x^3}$$

$$A = \frac{(x-1)\cdot(4x-3)}{x(x^2+2x+4)} : \frac{(x-2)(x-1)}{(2-x)(2^2+2x+x^2)}$$

$$A = \frac{(x-1)(4x-3)}{x(x^2+2x+4)} \cdot \frac{(2-x)(4+2x+x^2)}{(x-2)(x-1)}$$

$$A = -\frac{4x-3}{x}$$

$$\left(x-\frac{1}{x}\right):\left(\frac{x^3-x}{x^2+x}\right)^{-1} =$$

$$= \left(\frac{x}{1}-\frac{1}{x}\right) : \frac{x^2+x}{x^3-x}$$

$$= \left(\frac{x^2}{x}-\frac{1}{x}\right) \cdot \frac{x^3-x}{x^2+x}$$

$$= \frac{x^2-1}{x} \cdot \frac{x^3-x}{x^2+x}$$

$$= \frac{(x-1)(x+1)}{x} \cdot \frac{x(x^2-1)}{x(x+1)}$$

$$= \frac{x-1}{x} \cdot \frac{(x-1)(x+1)}{1}$$

Παραγοντοποίηση Τριωνύμου

1. $\alpha x^2+\beta x+\gamma = \alpha(x-x_1)(x-x_2)$

αν $\Delta > 0$ και $x_1 = \frac{-\beta+\sqrt{\Delta}}{2\alpha}$ $x_2 = \frac{-\beta-\sqrt{\Delta}}{2\alpha}$.

2. $\alpha x^2+\beta x+\gamma = \alpha(x-x_1)^2$

αν $\Delta = 0$ και $x_1 = \frac{-\beta}{2\alpha}$.

3. Αν $\Delta < 0$ το $\alpha x^2+\beta x+\gamma$ <u>δεν</u>

παραγοντοποιείται.

$$\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2).$$

Ασκηση

$$\left(x - \frac{1}{x}\right)^2 \cdot \frac{x^3 + x^2}{x^3 + 3x^2 + 3x + 1} =$$

$$= \left(\frac{x^2}{x} - \frac{1}{x}\right)^2 \cdot \frac{x^2(x+1)}{(x+1)^3} =$$

$$= \left(\frac{x^2 - 1}{x}\right)^2 \cdot \frac{x^2 \cancel{(x+1)}}{\cancel{(x+1)^3}}$$

$$= \left[\frac{(x-1)(x+1)}{x}\right]^2 \cdot \frac{x^2}{(x+1)^2} =$$

$$= \frac{(x-1)^2 \cancel{(x+1)^2}}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{\cancel{(x+1)^2}}$$

$$= (x-1)^2$$

Ασκηση

$$A = \frac{63^2 - 21^2}{24^2 + 2 \cdot 18 \cdot 24 + 18^2}$$

$$A = \frac{(63-21)(63+21)}{(24+18)^2}$$

$$A = \frac{42 \cdot 84}{42^2} = \frac{84}{42} = 2.$$

Προσοχή

$$\alpha^3+\beta^3+\gamma^3=3\alpha\beta\gamma$$

αν $\alpha+\beta+\gamma=0$

Χρήσιμο

Λύσε την εξίσωση

$$(x-100)^3+(100+2x)^3-27x^3=0.$$

$$(x-100)^3+(100+2x)^3+(-3x)^3=0$$

$$x-100+100+2x-3x=0$$

$$3\cdot(x-100)(100+2x)(-3x)=0$$

$x-100=0$ ή $100+2x=0$ ή $-3x=0$

$x=100$ $x=-50$ $x=0$

Εκτυπα 7

1. ⓐ Αν $\alpha<1$ και $\beta>2$

→ $\alpha<1 \Rightarrow \alpha-1<0$

→ $\beta>2 \Rightarrow \beta-2>0$

$$(\alpha-1)(\beta-2) < 0$$
$$(-)\quad(+)$$

2. ⓑ $\alpha<\beta<\gamma$

• $\alpha<\beta \Rightarrow \alpha-\beta<0$

• $\beta<\gamma \Rightarrow \beta-\gamma<0$

$$\frac{\overset{(-)}{\alpha-\beta}}{\underset{(-)}{\beta-\gamma}} > 0.$$

3. (α) $a^2-4a+4 \geq 0$

$(a-2)^2 \geq 0$ που ισχύει!

(β) $a^2+25 \geq 10a$

$a^2-10a+25 \geq 0$

$(a-5)^2 \geq 0$ ✓

(δ) $(\alpha+\beta)^2 \geq 4\alpha\beta$

$a^2+2\alpha\beta+\beta^2 \geq 4\alpha\beta$

$a^2-2\alpha\beta+\beta^2 \geq 0$

$(a-\beta)^2 \geq 0$ ✓

(ε) $2(a^2+\beta^2) \geq (\alpha-\beta)^2$

$2a^2+2\beta^2 \geq a^2-2\alpha\beta+\beta^2$

$a^2+2\alpha\beta+\beta^2 \geq 0$ $(a+\beta)^2 \geq 0$ ✓

28. ⓐ $x^2-4x+5>0$

$x^2-4x+4+1>0$

$(x-2)^2+1>0$ ✓

ⓑ $9x^2-6x+2>0$

$9x^2-6x+1+1>0$

$(3x-1)^2+1>0$ ✓

ⓕ $2x^2+2x+1>0$

$x^2+x^2+2x+1>0$

$x^2+(x+1)^2>0$ ✓

ⓖ $a^2+ab+b^2 \geq 0$.

$2a^2+2ab+2b^2 \geq 0$

$a^2+\underbrace{a^2+2ab+b^2}+b^2 \geq 0$

$a^2+(a+b)^2+b^2 \geq 0$ ✓

5. (a) $x^2+y^2-2x+1 \geq 0$

$x^2-2x+1+y^2 \geq 0$

$(x-1)^2+y^2 \geq 0$ ✓

26. (a) $x^3y^2+x^3 \geq 2x^3y$, $x \geq 0$

$x^3y^2+x^3-2x^3y \geq 0$

$x^3(y^2-2y+1) \geq 0$

$x^3(y-1)^2 \geq 0$ ✓

x (+) $(y-1)^2$ (+)

Αποδείξτε ανισότητα με δεδομένη συνθήκη

1. Αν $\alpha < 3 < \beta$ τότε

$$\alpha\beta + 9 < 3(\alpha + \beta).$$

$$\alpha\beta + 9 < 3\alpha + 3\beta$$

$$\alpha\beta + 9 - 3\alpha - 3\beta < 0$$

$$\alpha(\beta - 3) - 3(\beta - 3) < 0$$

$$\boxed{(\beta - 3)(\alpha - 3) < 0}$$

$(+)$ $(-)$

- $3 < \beta \Rightarrow 0 < \beta - 3$
- $\alpha < 3 \Rightarrow \alpha - 3 < 0$

2. Av $\alpha < -2 < \beta$

vsa $\alpha^2 - 2\beta > \alpha(\beta - 2)$

$\alpha^2 - 2\beta > \alpha\beta - 2\alpha$

$\alpha^2 - 2\beta - \alpha\beta + 2\alpha > 0$

$\alpha(\alpha - \beta) + 2(-\beta + \alpha) > 0$

$(\alpha - \beta)(\alpha + 2) > 0$

$(-)$ $(-)$

- $\alpha < \beta \Rightarrow \alpha - \beta < 0$
- $\alpha < -2 \Rightarrow \alpha + 2 < 0$

3. Av $\alpha < \beta < -4$

vsa $4\beta - \alpha(4-\beta) < \alpha^2$

$4\beta - 4\alpha + \alpha\beta < \alpha^2$

$4\beta - 4\alpha + \alpha\beta - \alpha^2 < 0$

$4(\beta-\alpha) + \alpha(\beta-\alpha) < 0$

$(\beta-\alpha)(4+\alpha) < 0$

(+) (−)

- $\alpha < \beta \Rightarrow 0 < \beta - \alpha$
- $\alpha < -4 \Rightarrow \alpha + 4 < 0$

4. Αν $x \geq -1$ νδο $x^3 - x^2 - x + 1 \geq 0$.

$$x^2(x-1) - (x-1) \geq 0$$

$$(x-1)(x^2-1) \geq 0$$

$$(x-1)(x-1)(x+1) \geq 0$$

$$(x-1)^2 (x+1) \geq 0$$

(+) (+)

✓

$\longrightarrow x \geq -1 \Rightarrow x+1 \geq 0$

5. Av $a \geq b$

vsa $a^3 + a^2 b - ab^2 - b^3 \geq 0$

$a^2(a+b) - b^2(a+b) \geq 0$

$(a+b)(a^2 - b^2) \geq 0$

$(a+b)(a+b)(a-b) \geq 0$

$(a+b)^2 (a-b) \geq 0$

(+) (+) ✓

• $a \geq b \Rightarrow a - b \geq 0$