7. (a) $f(x) = 1 - \sqrt{1+e^x}$, $x \in R$

$x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2} \Rightarrow 1+e^{x_1} < 1+e^{x_2}$

$$\sqrt{1+e^{x_1}} < \sqrt{1+e^{x_2}}$$

$$-\sqrt{1+e^{x_1}} > -\sqrt{1+e^{x_2}}$$

$f \downarrow$

$\Rightarrow f$ 1-1

$\Rightarrow f$ αντιστρ.

$$1-\sqrt{1+e^{x_1}} > 1-\sqrt{1+e^{x_2}}$$

$$f(x_1) > f(x_2)$$

$y = 1 - \sqrt{1+e^x}$

$\sqrt{1+e^x} = 1-y$

$1 - y \geq 0$

$1 \geq y$

$1 + e^x = (1-y)^2$

$e^x = (1-y)^2 - 1$

$x = \ln\left[(1-y)^2 - 1\right]$

$(1-y)^2 - 1 > 0$

$(1-y)^2 > 1$

$(1-y)^2 > 1^2$

$|1-y| > |1|$

$|1-y| > 1$

$1-y>1$ ή $1-y<-1$

$0>y$ $\qquad$ $y>2$

$y\in(-\infty,0)\cup(2,+\infty)$

$$\boxed{\begin{gathered} f^{-1}(x)=\ln\left((1-x)^2-1\right) \\ D_{f^{-1}}=(-\infty,0) \end{gathered}}$$

7. B $f(x) = 1 - \ln(1+e^x)$, $x \in \mathbb{R}$.

$x_1 < x_2 \Rightarrow 1+e^{x_1} < 1+e^{x_2}$

$\ln(1+e^{x_1}) < \ln(1+e^{x_2})$

$1 - \ln(1+e^{x_1}) > 1 - \ln(1+e^{x_2})$

$f(x_1) > f(x_2)$

$f \downarrow$

$y = 1 - \ln(1+e^x)$

$\ln(1+e^x) = 1 - y$

$1 + e^x = e^{1-y}$

$e^x = e^{1-y} - 1$ $\qquad\qquad$ $e^{1-y} - 1 > 0$

$x = \ln(e^{1-y} - 1)$ $\qquad\qquad$ $e^{1-y} > 1$

$1 - y > 0$

$f^{-1}(x) = \ln(e^{1-x} - 1)$ $\qquad\qquad$ $1 > y$

$D_{f^{-1}} = (-\infty, 1)$,

7. γ $f(x) = \dfrac{2x-1}{x-2}$, $x \neq 2$.

$f(x_1)=f(x_2)$

$$\frac{2x_1-1}{x_1-2} = \frac{2x_2-1}{x_2-2}$$

$$(2x_1-1)(x_2-2) = (2x_2-1)(x_1-2)$$

$$\cancel{2x_1x_2} - 4x_1 - x_2 + \cancel{2} = \cancel{2x_2x_1} - 4x_2 - x_1 + \cancel{2}$$

$$-3x_1 = -3x_2$$

$f: 1-1$

$$x_1 = x_2$$

$y \neq 2$

$$y = \frac{2x-1}{x-2}$$

$$y(x-2) = 2x-1$$

$$yx - 2y = 2x-1$$

$$yx - 2x = 2y-1$$

$$x(y-2) = 2y-1$$

$$x = \frac{2y-1}{y-2}$$

$$f^{-1}(x) = \frac{2x-1}{x-2}$$

$D_{f^{-1}} = \mathbb{R} - \{2\}$.

Tc2.1

$x \neq 2$

$$\frac{2y-1}{y-2} \neq 2$$

$2y-1 \neq 2(y-2)$

$\cancel{2y}-1 \neq \cancel{2y}-4$

$-1 \neq -4$ now alway

7. στ $\quad f(x) = x - \ln(1+e^x) \quad , x \in \mathbb{R}.$

$$f(x) = \ln e^x - \ln(1+e^x)$$

$$\boxed{f(x) = \ln \frac{e^x}{1+e^x}}$$

$$f(x_1) = f(x_2)$$

$$\ln \frac{e^{x_1}}{1+e^{x_1}} = \ln \frac{e^{x_2}}{1+e^{x_2}}$$

$$\frac{e^{x_1}}{1+e^{x_1}} = \frac{e^{x_2}}{1+e^{x_2}}$$

$\Leftrightarrow$ ο.ω $\quad x_1 = x_2.$

$$y = \ln \frac{e^x}{1+e^x}$$

$$e^y = \frac{e^x}{1+e^x}$$

$$e^y(1+e^x) = e^x$$

$$e^y + e^y e^x = e^x$$

$$e^y = e^x - e^y e^x$$

$$e^y = e^x(1-e^y)$$

$$\frac{e^y}{1-e^y} = e^x$$

$y \neq 0$

$$x = \ln \frac{e^y}{1-e^y}$$

$$f^{-1}(x) = \ln \frac{e^x}{1-e^x}$$

7. (η) $f(x)=\dfrac{\ln x}{\ln x-1}$, $x>0$ και $\ln x-1\neq 0$

$\ln x\neq 1$

$x\neq e$.

$D_f=(0,e)\cup(e,+\infty)$.

$f(x_1)=f(x_2)$

$\dfrac{\ln x_1}{\ln x_1-1}=\dfrac{\ln x_2}{\ln x_2-1}$

$\circ\circ\circ$

$x_1=x_2$

$y=\dfrac{\ln x}{\ln x-1}$

$y(\ln x-1)=\ln x$

$y\ln x-y=\ln x$

$y\ln x-\ln x=y$

$\ln x(y-1)=y$

$\ln x=\dfrac{y}{y-1}$ $\quad y\neq 1$

$\to x=e^{\frac{y}{y-1}}$

$f^{-1}(x)=e^{\frac{x}{x-1}}$

Todo ʃ

$0 < x < e$ ó $x > e$

$0 < e^{\frac{y}{y-1}} < e$; $e^{\frac{y}{y-1}} > e$

$\underbrace{0 < e}$ ✓ ... $y > 1$

$e^{\frac{y}{y-1}} < e^{1}$

$\frac{y}{y-1} < 1$

$\frac{y}{y-1} - 1 < 0$

$\frac{\cancel{y} - \cancel{y} + 1}{y-1} < 0$

$\frac{1^{(+)}}{y-1} < 0$

$y - 1 < 0$

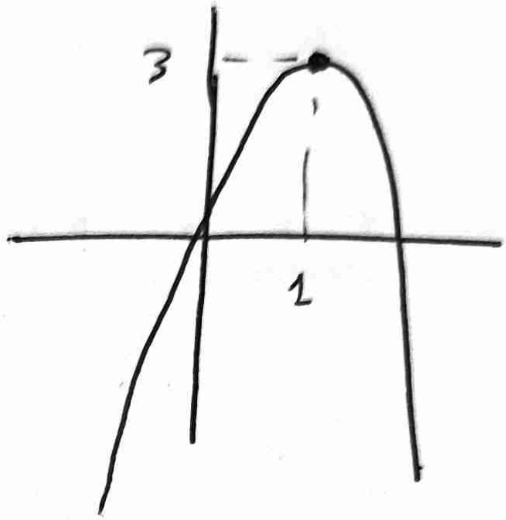
$y < 1$

$y \neq 1$

$D_{f^{-1}} = \mathbb{R} - \{1\}$.

Διαγώνισμα 1ο

Αα: ⓐ Σ ⓑ Σ ⓒ Σ ⓓ Σ ⓔ Σ



B1: $\Sigma T_f = (-\infty, 3]$

B2: $\frac{1}{2} > \frac{1}{3}$

$f \nearrow$

$f\left(\frac{1}{2}\right) > f\left(\frac{1}{3}\right)$

B3: $f(2x^2+1) - f(x^2+2) > 0$

$f(2x^2+1) > f(x^2+2)$.

$\left.\begin{array}{l} \cdot\ 2x^2+1 > 1 \\ \cdot\ x^2+2 > 1 \end{array}\right\} f \searrow$

$2x^2+1 < x^2+2$

$x^2 < 1$

$x^2 - 1 < 0$

x		-1		1	
x^2-1	$+$		$-$		$+$

$x \in (-1, 1)$.

B4. $f(e^x)=3$. Αφου το $A(1,3)$ Ο.Μ μονο το $f(1)=3$

$$e^x = 1$$

$$x=0$$

Γ1: $g(1)=1 \Leftrightarrow \frac{\alpha-1}{1}=1 \Leftrightarrow \alpha=2$

$$\boxed{g(x)=\frac{1}{x}}$$

$$g(x)>1 \Rightarrow \frac{1}{x}>1 \Rightarrow \frac{1}{x}-1>0 \Rightarrow \frac{1-x}{x}>0$$

x		0		1	
$1-x$	+		+		−
x	−		+		+
$\frac{1-x}{x}$	−		+		−

$x \in (0,1)$.

Γ2: Έστω $f(x) < g(x) \Rightarrow \ln x + 1 < \frac{1}{x}$

$$\underbrace{\ln x + 1 - \frac{1}{x}}_{\varphi(x)\nearrow} < 0 \rightarrow \varphi(x) < 0$$

$$\varphi(x) < \varphi(1)$$

$$\varphi \nearrow$$

$$0 < x < 1$$

Όταν $x \in (0,1)$ η C_f κάτω C_g

Όταν $x \in (1,+\infty)$ η C_f πάνω C_g.

Γ3: $h(x) = (g \circ f)(x) = g(f(x)) = \frac{1}{\ln x + 1}$

$x \in D_f$ και $f(x) \in D_g$

$x > 0$

$$\ln x + 1 \neq 0$$

$$\ln x \neq -1$$

$$x \neq e^{-1}$$

$$x \neq \frac{1}{e}$$

$D_h = (0, \frac{1}{e}) \cup (\frac{1}{e}, +\infty)$.

Γ4: $h(x) = \dfrac{1}{\ln x + 1}$

$h(x_1) = h(x_2) \Rightarrow \dfrac{1}{\ln x_1 + 1} = \dfrac{1}{\ln x_2 + 1} \Leftrightarrow x_1 = x_2$

$y = \dfrac{1}{\ln x + 1} \Rightarrow y(\ln x + 1) = 1$

$y \ln x + y = 1$

$y \ln x = 1 - y$

$\ln x = \dfrac{1-y}{y}$ $y \neq 0$

$x = e^{\frac{1-y}{y}}$

$h^{-1}(x) = e^{\frac{1-x}{x}}$

$D_{h^{-1}} = \mathbb{R}^*$

Τελος

$0 < x < \frac{1}{e}$ κ $x > \frac{1}{e}$

όμοια

$x > 0$

$0 < e^{\frac{1-x}{x}} < e^{-1}$

✓

$\dfrac{1-x}{x} < -1$

$x \neq 0$

$\dfrac{1-x}{x} + 1 < 0 \Rightarrow \dfrac{1}{x} < 0$

$x < 0$

Δ1: $f(x) = x^5 + x$

$f\uparrow \Rightarrow f$ 1-1

Δ2: $f^{-1}(x) = x$

$x = f(x)$

$x = x^5 + x$

$x^5 = 0$

$x = 0$

$A(0,0)$.

Δ3: $f(x^5 + 2x - 2) - x^5 > x$

$f(x^5 + 2x - 2) > x^5 + x$

$f(x^5 + 2x - 2) > f(x)$

$x^5 + 2x - 2 > x$

$x^5 + x > 2$

$f(x) > f(1)$

$x > 1$,

Δu:

$$f^{-1}\left(f(e^{x+1}) - f^{-1}(34)\right) = 0$$

$$f(e^{x+1}) - f^{-1}(34) = f(0)$$

$$f(e^{x+1}) - 2 = 0$$

$f(2) = 34$

$f^{-1}(34) = 2$

$$f(e^{x+1}) = 2$$

$$f(e^{x+1}) = f(1)$$

$f\,\overset{\circ}{3}) - 1$

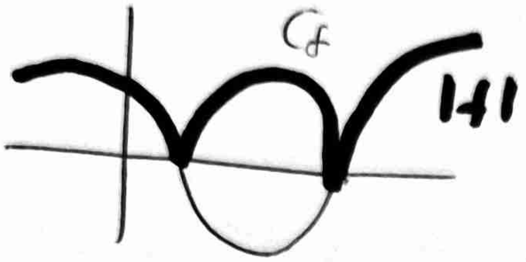
$$e^{x+1} = 1$$

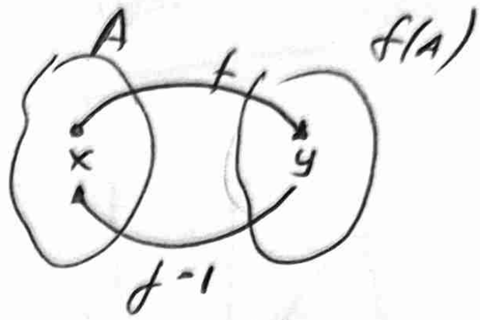
$$x+1 = 0$$

$$x = -1$$

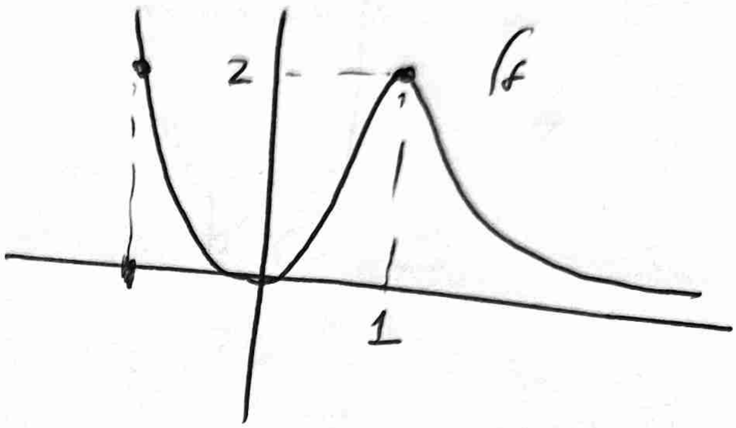
Διαγωνισμα 2ο

Αα: (α) Λ (β) Σ (γ) Λ. (δ) Λ (ε) Λ.





B1: $[0,+\infty)$



B2: $e<\pi$

$-e>-\pi$

$f(-e)<f(-\pi)$

B3: i) $f(x^2+2)<f(2x^2+1)$

$\left.\begin{array}{l} \bullet\ x^2+2>1 \\ \bullet\ 2x^2+1>1 \end{array}\right\} f\downarrow$

$x^2+2>2x^2+1$

$1>x^2$

$x\in(-1,1)$

ii) $f(e^{-x^2})<f\left(\frac{1}{e}\right) \overset{f\nearrow}{\Rightarrow} e^{-x^2}<\frac{1}{e} \Rightarrow e^{-x^2}<e^{-1}$ ——)

- $x^2\geq 0 \Rightarrow -x^2\leq 0 \Rightarrow e^{-x^2}\leq e^0 \Rightarrow e^{-x^2}\leq 1$

$\underline{0<e^{-x^2}\leq 1}$

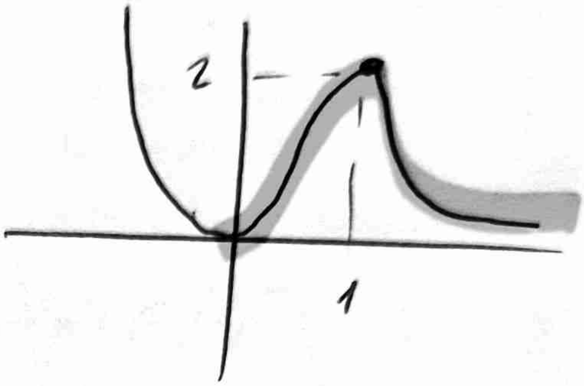
- $\frac{1}{e}\in(0,1)$

$-x^2 < -1 \Rightarrow x^2 > 1 \qquad x \in (-\infty,-1) \cup (1,+\infty)$.

B4: i) $2f(x) - 3 = 0$

$f(x) = \frac{3}{2}$ 3 ρίζ

ii). $f(1 - ημx) = 2$



$-1 \leq ημx \leq 1$

$1 \geq -ημx \geq -1$

$2 \geq 1 - ημx \geq 0$

$\forall x > 0$

Το $A(1,2)$ Ο.Μ

Μόνο ω $f(1) = 2$.

$1 - ημx = \underline{1}$

$ημx = 0$

$ημ\, x = ημ\, 0$

$x = 2κπ + 0$ ή $x = 2κπ + π - 0$

Γ_1: $f(x) = \ln x$, $x > 0$

$g(x) = \sqrt{1-x}$, $x \leq 1$

$h(x) = (f-g)(x) = f(x) - g(x) = \ln x - \sqrt{1-x}$

$D_h = D_f \cap D_g = (0, 1]$

$\varphi(x) = (g \circ f)(x) = g(f(x)) = \sqrt{1 - \ln x}$

$x \in D_f$ και $f(x) \in D_g$

$x > 0$ $\qquad$ $\ln x \leq 1$

$x \leq e$

$D_\varphi = (0, e]$.

Γ_2: Έστω $f(x) < g(x) \Rightarrow \ln x < \sqrt{1-x}$

$\ln x - \sqrt{1-x} < 0$

$h(x) < 0$

$h(x) < h(1)$

$h \nearrow$ ευκολα.

$0 < x < 1$

Όταν $x \in (0,1)$ η $f(x)$ κάτω $g(x)$

Όταν $x \in (1,+\infty)$ η $f(x)$ πάνω $g(x)$:

$A(1,0)$ κοινό σημείο.

Γ3: $\varphi(x) = \sqrt{1-\ln x}$

$x \in (0,e]$

$\varphi \downarrow \Rightarrow \varphi$ 1-1

$y = \sqrt{1-\ln x}$ $\qquad y \geq 0$

$y^2 = 1-\ln x$

$\ln x = 1-y^2$

$x = e^{1-y^2}$

$$\boxed{\varphi^{-1}(x) = e^{1-x^2}}$$

$D_{\varphi^{-1}} = [0,+\infty)$.

Τελ/

$0 < x \leq e$

$\underbrace{0 < e^{1-y^2}}_{\checkmark} \leq e^{1}$

$\cancel{1} - y^2 \leq \cancel{1}$

$-y^2 \leq 0$

$y^2 \geq 0$

Γ4: $\ln \varepsilon\varphi x = \sqrt{1-\eta\mu x} - \sqrt{1-\sigma\upsilon\nu x}$

$$\ln \eta\mu x - \ln \sigma\upsilon\nu x = \sqrt{1-\eta\mu x} - \sqrt{1-\sigma\upsilon\nu x}$$

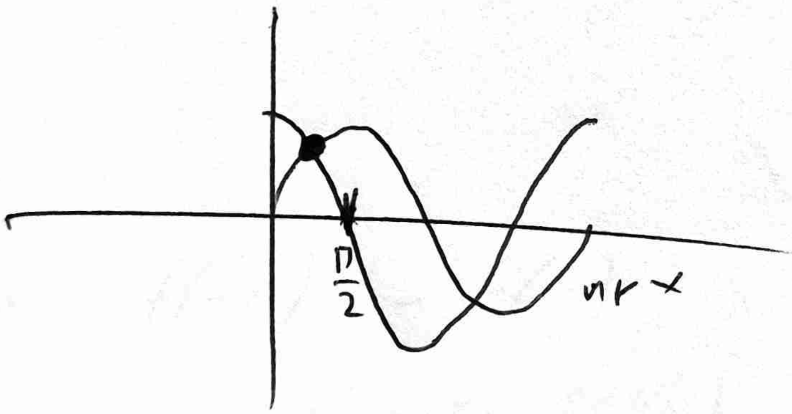
$$\ln \eta\mu x - \sqrt{1-\eta\mu x} = \ln \sigma\upsilon\nu x - \sqrt{1-\sigma\upsilon\nu x}$$

$$h(\eta\mu x) = h(\sigma\upsilon\nu x)$$

$h \uparrow 1-1$

$\eta\mu x = \sigma\upsilon\nu x \qquad x \in \left(0, \frac{\pi}{2}\right).$

$x = \frac{\pi}{4}$.



Δ1: $f(x) = e^{x} - e^{-x} + x \quad f \nearrow$

Δ2: Έστω $f^{-1}(x_1) < f^{-1}(x_2)$

$$f \nearrow$$

$$f(f^{-1}(x_1)) < f(f^{-1}(x_2))$$

$$x_1 < x_2 \quad f^{-1} \nearrow$$

$\boxed{\text{Αρκεί ν.δ.ο } f^{-1}(-x) = -f^{-1}(x).}$

$$f(-x) = e^{-x} - e^{x} - x = -(e^{x} - e^{-x} + x)$$

$$= -f(x)$$

περιττή.

Άρα $f(-x) = -f(x)$

όπου x το $f^{-1}(x)$

$$f(-f^{-1}(x)) = -f(f^{-1}(x))$$

$$f(-f^{-1}(x)) = -x$$

$$f^{-1}(f(-f^{-1}(x))) = f^{-1}(-x) \Rightarrow -f^{-1}(x) = f^{-1}(-x)$$

Δ_3: $$e^{2f^{-1}(x)} - 1 = \left(f(x) - f^{-1}(x)\right) \cdot e^{f^{-1}(x)}$$

$$e^{f^{-1}(x)} - \frac{1}{e^{f^{-1}(x)}} = f(x) - f^{-1}(x)$$

$$\underbrace{e^{f^{-1}(x)} - e^{-f^{-1}(x)} + f^{-1}(x)} = f(x)$$

$$f\left(f^{-1}(x)\right) = f(x)$$

$$x = f(x)$$

$$\cancel{x} = e^{x} - e^{-x} + \cancel{x}$$

$$e^{-x} = e^{x}$$

$$-x = x$$

$$2x = 0$$

$$x = 0.$$

Du: $f^{-1}(x-2) + f^{-1}(x-4) > f(3-x)$.

$$\underbrace{f^{-1}(x-2) + f^{-1}(x-4) - f(3-x)}_{\varphi(x)} > 0$$

- $x_1 < x_2 \Rightarrow x_1 - 2 < x_2 - 2 \Rightarrow f^{-1}(x_1-2) < f^{-1}(x_2-2)$
- $x_1 < x_2 \Rightarrow f^{-1}(x_1-4) < f^{-1}(x_2-4)$ $+$
- $x_1 < x_2 \Rightarrow 3 - x_1 > 3 - x_2 \Rightarrow f(3-x_1) > f(3-x_2)$

$$-f(3-x_1) < -f(3-x_2)$$

$\varphi \nearrow$

$$\varphi(x) > 0$$

$\boxed{f(0)=0}$

$$\varphi(x) > \varphi(3)$$

$\varphi \nearrow$ $\underline{x > 3}$

$$\varphi(3) = f^{-1}(1) + f^{-1}(-1) - f(0) =$$

$$= \cancel{f^{-1}(1)} - \cancel{f^{-1}(1)} - 0 = 0$$

Ενότητα 8

19. (B) $\displaystyle\lim_{x\to -1} \frac{\overset{\oplus}{|x^3-3x-1|}+x}{\overset{\ominus}{|x^3+5x+4|}-2} =$

$$= \lim_{x\to -1} \frac{x^3-3x-1+x}{-x^3-5x-4-2} =$$

$$= \lim_{x\to -1} \frac{x^3-2x-1}{-x^3-5x-6} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x^2-x-1)}{\cancel{(x+1)}(-x^2+x-6)}$$

1	0	-2	-1	-1
↓	-1	1	1	
1	-1	-1	0	

$$\frac{1}{-8}$$

-1	0	-5	-6	-1
↓	1	-1	6	
-1	1	-6	0	

20. ג)
$$\lim_{x\to 3} \frac{x^2-9-\sqrt{x^2-6x+9}}{|\overset{+}{x-1}|-2}$$

$$= \lim_{x\to 3} \frac{x^2-9-\sqrt{(x-3)^2}}{x-1-2}$$

$$= \lim_{x\to 3} \frac{(x-3)(x+3)-|\overset{0}{x-3}|}{x-3}$$

x	3
x−3	− 0 +

בודק ב
של
אקסטרמום!

$$\rightarrow \lim_{x\to 3^-} \frac{(x-3)(x+3)-|\overset{-}{x-3}|}{x-3} =$$

$$= \lim_{x\to 3^-} \frac{\cancel{(x-3)}(x+3)+\cancel{(x-3)}}{\cancel{x-3}} = 7$$

$$\rightarrow \lim_{x\to 3^+} \frac{(x-3)(x+3)-|\overset{+}{x-3}|}{x-3} = \lim_{x\to 3^+} \frac{\cancel{(x-3)}(x+3)-\cancel{(x-3)}}{\cancel{x-3}}$$

$$= 5$$

$$\frac{1}{x}$$

$$\frac{1}{0{,}1} = \frac{1}{\frac{1}{10}} = 10$$

$$\frac{1}{0} = \infty$$

$$\frac{1}{0{,}001} = \frac{1}{\frac{1}{1000}} = 1000$$

$$\frac{1}{0{,}0000001} = 10000000$$

$$\frac{1}{10} = 0{,}1$$

$$\frac{1}{\infty} = 0$$

$$\frac{1}{1000} = 0{,}001$$

$$\frac{1}{10000000} = 0{,}0000001$$

Ενοτητα 9

5. ε) $\lim\limits_{x\to 1} \dfrac{1}{|x-1|} = +\infty$

στ) $\lim\limits_{x\to 5} \dfrac{1}{x-5} = \infty$;

x		5	
$x-5$	$-$		$+$

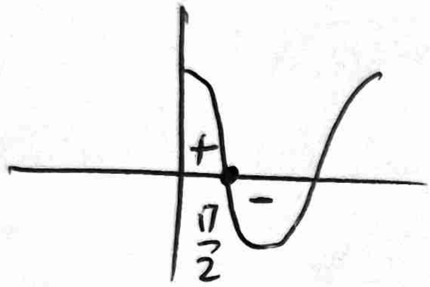
$$\left.\begin{aligned} &\lim_{x\to 5^-} \frac{1}{x-5} = -\infty \\ &\lim_{x\to 5^+} \frac{1}{x-5} = +\infty \end{aligned}\right\} \text{ Το όριο δεν υπάρχει.}$$

6. β) $\lim\limits_{x\to 2} \dfrac{2x-5}{x^2-4x+4} = \lim\limits_{x\to 2} (2x-5)\cdot \dfrac{1}{(x-2)^2}$

$= (-1)\cdot(+\infty) = -\infty.$

7. (στ) $\lim\limits_{x \to \frac{\pi}{2}} \varepsilon\varphi x = \lim\limits_{x \to \frac{\pi}{2}} \frac{\eta\mu x}{\sigma\upsilon\nu x} =$

$\lim\limits_{x \to \frac{\pi}{2}} \eta\mu x \cdot \frac{1}{\sigma\upsilon\nu x} = 1 \cdot \infty;$



$\lim\limits_{x \to \frac{\pi}{2}^-} \eta\mu x \frac{1}{\sigma\upsilon\nu x} = 1 \cdot (+\infty) = +\infty$

$\lim\limits_{x \to \frac{\pi}{2}^+} \eta\mu x \frac{1}{\sigma\upsilon\nu x} = 1(-\infty) = -\infty$

Το όριο δεν υπάρχει!

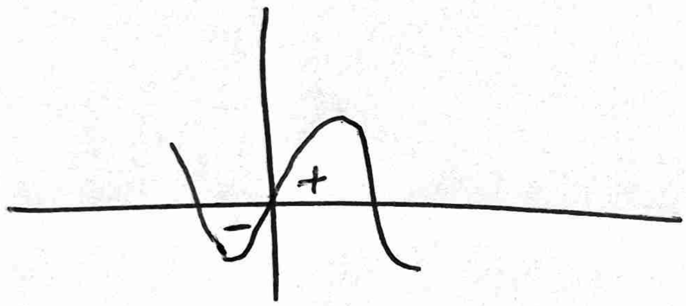
6. (Ε) $\lim\limits_{x\to 0} \dfrac{2x-1}{1-\sigma\upsilon\nu x} = \lim\limits_{x\to 0} (2x-1)\cdot \dfrac{1}{1-\sigma\upsilon\nu x} =$

$= -1 \cdot (+\infty) = -\infty.$

• $\sigma\upsilon\nu x \leq 1 \Rightarrow \underline{\underline{1-\sigma\upsilon\nu x \geq 0}}$

7. (Ε). $\lim\limits_{x\to 0} \dfrac{2x-1}{\eta\mu x} = \lim\limits_{x\to 0} (2x-1) \dfrac{1}{\eta\mu x}$

$= -1 \cdot \infty$



$\rightarrow \lim\limits_{x\to 0^-} (2x-1) \dfrac{1}{\eta\mu x} = -1(-\infty) = +\infty$

$\rightarrow \lim\limits_{x\to 0^+} (2x-1) \dfrac{1}{\eta\mu x} = -1(+\infty) = -\infty$

Το όριο δεν υπάρχει.

10. (B) $\lim\limits_{x \to 3} \dfrac{2x-5}{x^2-9} = \lim\limits_{x \to 3} \dfrac{2x-5}{x+3} \cdot \dfrac{1}{x-3}$

x		3	
$x-3$	$-$		$+$

$= \dfrac{1}{6} \cdot \infty$;

$\rightarrow \lim\limits_{x \to 3^-} \dfrac{2x-5}{x+3} \cdot \dfrac{1}{x-3} = \dfrac{1}{6}(-\infty) = -\infty$

$\rightarrow \lim\limits_{x \to 3^+} \dfrac{2x-5}{x+3} \dfrac{1}{x-3} = \dfrac{1}{6}(+\infty) = +\infty$

Το όριο δεν υπάρχει!.

Ενότητα 10

5. (δ) $\lim\limits_{x \to +\infty} (-5x^4 + x - 1) = \lim\limits_{x \to +\infty} -5x^4$

$= -5 \cdot +\infty = -\infty$

(στ) $\lim\limits_{x \to -\infty} (-2x^5 + 3x - 1) = \lim\limits_{x \to -\infty} -2x^5 =$

$= -2 \cdot (-\infty) = +\infty$

6. (i) $\lim\limits_{x \to -\infty} \dfrac{2x^3 - 3x + 1}{5x^3 + x + 1} = \lim\limits_{x \to -\infty} \dfrac{2\cancel{x^3}}{5\cancel{x^3}} = \dfrac{2}{5}$

(ζ) $\lim\limits_{x \to -\infty} \dfrac{x^3}{4 + x^2} = \lim\limits_{x \to -\infty} \dfrac{x^3}{x^2} = \lim\limits_{x \to -\infty} x = -\infty$

(η) $\lim\limits_{x \to -\infty} \dfrac{2x^2 + 1}{x^2 - x + 1} = \lim\limits_{x \to -\infty} \dfrac{2\cancel{x^2}}{\cancel{x^2}} = 2$.

7. (δ) $\lim_{x \to -\infty} \left(\frac{x^3}{1+x^2} - x \right) =$

$$= \lim_{x \to -\infty} \left(\frac{x^3}{1+x^2} - x \right) =$$

$$= \lim_{x \to -\infty} \frac{x^3}{1+x^2} - \frac{x(1+x^2)}{1+x^2}$$

$$= \lim_{x \to -\infty} \frac{\cancel{x^3} - x - \cancel{x^3}}{1+x^2}$$

$$= \lim_{x \to -\infty} \frac{-\cancel{x}}{x^{\cancel{2}}} = \lim_{x \to -\infty} -\frac{1}{x} = 0$$

Επομενο Μαθημα

Δευτερα 6/10/25.

8

ΟΧΙ

DLH

2

3

8 α β

9 α β γ δ στ.

10 α β γ.

11 α β

12 α.

13 α β γ δ ε στ ζ η

19 α γ

20 α β.