5.

Ενότητα 3

$f(x) = \sqrt{1-x}$

$D_f = (-\infty, 1]$

$g(x) = \ln x$

$D_g = (0, +\infty)$

(γ) $\left(\frac{1}{g}\right)(x) = \frac{1}{g(x)} = \frac{1}{\ln x}$

$x \in D_g$ και $g(x) \neq 0$

$\underline{\underline{x > 0}}$

$\ln x \neq 0$

$e^{\ln x} \neq e^0$

$\underline{\underline{x \neq 1}}$

$D_{1/g} = (0,1) \cup (1, +\infty)$.

⑧ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{1-x}}{\ln x}.$

$$\left.\begin{array}{l} D_f \cap D_g = (0,1] \\ g(x) \neq 0 \\ x \neq 1 \end{array}\right\} D_{f/g} = (0,1)$$

8. (α) $f(x) = \ln(x-1)$ $\quad D_f = (1, +\infty)$

$g(x) = \frac{1}{x}$ $\quad D_g = \mathbb{R}^*$

$(g \circ f)(x) = g(f(x)) = \ln\left(\frac{1}{x} - 1\right) = \ln\left(\frac{1-x}{x}\right)$

$x \in D_f$ και $f(x) \in D_g$

$\underline{\underline{x > 1}}$

$\ln(x-1) \neq 0$

$e^{\ln(x-1)} \neq e^0$

$x - 1 \neq 1$

$\underline{\underline{x \neq 2}}$

$D_{g \circ f} = (1, 2) \cup (2, +\infty).$

(β) $f(x) = \ln x$

$D_f = (0, +\infty)$

$g(x) = \sqrt{1-x}$

$D_g = (-\infty, 1].$

$(g \circ f)(x) = g(f(x)) = \sqrt{1 - \ln x}$

$x \in D_f$ και $f(x) \in D_g$

$\underline{\underline{x > 0}}$

$\ln x \leq 1$

$e^{\ln x} \leq e^1$

$\underline{\underline{x \leq e}}$

$D_{g \circ f} = (0, e].$

(γ). $f(x) = \dfrac{x}{x-1}$

$(g \circ f)(x) = g(f(x))$

$D_f = \mathbb{R} - \{1\}$

$$\boxed{(g \circ f)(x) = \ln \frac{x}{x-1}}$$

$g(x) = \ln x$

$D_g = (0, +\infty)$

$x \in D_f$ και $f(x) \in D_g$

$x \neq 1$ $\qquad \dfrac{x}{x-1} > 0$

x		0		1	
x	$-$	0	$+$		$+$
$x-1$	$-$		$-$		$+$
$\sim$	$+$		$-$		$+$

$x \in (-\infty, 0) \cup (1, +\infty)$

$D_{g \circ f} = (-\infty, 0) \cup (1, +\infty)$

(δ) $f(x) = \dfrac{2x-1}{x+1}$

$D_f = \mathbb{R} - \{-1\}$

$(g \circ f)(x) = g(f(x)) =$

$g(x) = \sqrt{x-1}$

$= \sqrt{\dfrac{2x-1}{x+1} - 1}$

$D_g = [1, +\infty)$

$= \sqrt{\dfrac{2x-1-x-1}{x+1}} = \sqrt{\dfrac{x-2}{x+1}}$

$x \in D_f$ και $f(x) \in D_g$

$x \neq -1$ $\qquad \dfrac{2x-1}{x+1} > 1 \Rightarrow \dfrac{2x-1}{x+1} - 1 > 0 \Rightarrow \dfrac{x-2}{x+1} > 0$

x	$-\infty$ … -1	-1 … 2	2 … $+\infty$
$x-2$	$-$	$-$	0 $+$
$x+1$	$-$	$+$	$+$
~	$+$	$-$	$+$

$$x \in (-\infty, -1) \cup (2, +\infty)$$

$$D_{g \circ f} = (-\infty, -1) \cup (2, +\infty),$$

5. $f(x) = e^{x-1} + \ln x - 1$

Ερώτημα 4

$x > 0$

(α) $x_1 < x_2 \Rightarrow x_1 - 1 < x_2 - 1$

$e^{x_1 - 1} < e^{x_2 - 1}$

(+)

$x_1 < x_2 \Rightarrow \ln x_1 - 1 < \ln x_2 - 1$

$f \nearrow$

(β) i) Αν $x > 1$

$f \nearrow$

$f(x) > f(1)$

$f(x) > 0$ ✓

ii). $\alpha < \beta$

$f \nearrow$

$f(\alpha) < f(\beta)$

$e^{\alpha-1} + \ln\alpha - \cancel{1} < e^{\beta-1} + \ln\beta - \cancel{1}$

$\ln\alpha - \ln\beta < e^{\beta-1} - e^{\alpha-1}$

$\ln\frac{\alpha}{\beta} < e^{\beta-1} - e^{\alpha-1}$

iii) $\forall x>0$ vdo $f(x)-f(x+1)<0$

$$x<x+1$$

$$f \nearrow$$

$$f(x)<f(x+1)$$

$$f(x)-f(x+1)<0$$

iv). $2x<5x$

$$f \nearrow$$

$$f(2x)<f(5x)$$

v) $\forall x>1$

$$x<x^2$$

$$f \nearrow$$

$$f(x)<f(x^2)$$

vi) Αν $a>0$ νδο $f(a)-f(a+3)<f(a+1)-f(a+2)$

$$f(a)+f(a+2)<f(a+1)+f(a+3)$$

$$\left.\begin{array}{l} \bullet\ a<a+1 \Rightarrow f(a)<f(a+1) \\ \bullet\ a+2<a+3 \Rightarrow f(a+2)<f(a+3) \end{array}\right\} (+)$$

$$f(a)+f(a+2)<f(a+1)+f(a+3)$$

12. $f(x) = x^3 + x - 10$

ⓐ $x > \dfrac{10}{x^2+1} \Rightarrow x(x^2+1) > 10$

$x^3 + x > 10 \Rightarrow x^3 + x - 10 > 0$

$f(x) > 0$

$f(x) > f(2)$

$f \nearrow$

$\underline{\underline{x > 2}}$

- $x_1 < x_2 \Rightarrow x_1^3 < x_2^3$ (+)
- $x_1 < x_2 \Rightarrow x_1 - 10 < x_2 - 10$

$f(x_1) < f(x_2)$

$f \nearrow$

$(-\infty, 0)$.

ⓑ $x^2 + \dfrac{10}{x} < -1$

$x^3 + 10 > -x$

$x^3 + x + 10 > 0$

$x^3 + x > -10$

$x^3 + x - 10 > -20$

$f(x) > f(-2)$

$f \nearrow$

$\underline{x > -2}$

7. $\ln^3 x + \ln x \geq 10$

$\ln^3 x + \ln x - 10 \geq 0$

$f(\ln x) \geq 0$

$f(\ln x) \geq f(2)$

$f \nearrow$

$\ln x \geq 2$

$e^{\ln x} \geq e^2$

$x \geq e^2$

8. $f(3x-4) + 10 < x^3 + x$

$f(3x-4) < x^3 + x - 10$

$f(3x-4) < f(x)$

$f \nearrow$

$3x - 4 < x$

$2x < 4$

$x < 2.$

ε $f(x^3+2x-2)+10-x > x^3$

$f(x^3+2x-2) > x^3+x-10$

$f(x^3+2x-2) > f(x)$

$f \nearrow$

$x^3+2x-2 > x$

$x^3+x-2 > 0$

$x^3+x > 2$

$x^3+x-10 > 2-10$

$f(x) > -8$

$f(x) > f(1)$

$f \nearrow$

$x > 1$

52 $f^3(x) + f(x) - x > x^3$

$f^3(x) + f(x) > x^3 + x$

$f^3(x) + f(x) - 10 > x^3 + x - 10$

$f(f(x)) > f(x)$

$f \uparrow$

$f(x) > x$

$x^3 + \cancel{x} - 10 > \cancel{x}$

$x^3 > 10$

$x > \sqrt[3]{10}$

13.

Ενοτητα 4

$f(x) = \text{εφ}x + e^x - 1$

$x \in [0, \frac{\pi}{2})$

Προσοχη

$\text{εφ}x \nearrow$

(α) $x_1 < x_2 \Rightarrow \text{εφ}x_1 < \text{εφ}x_2$

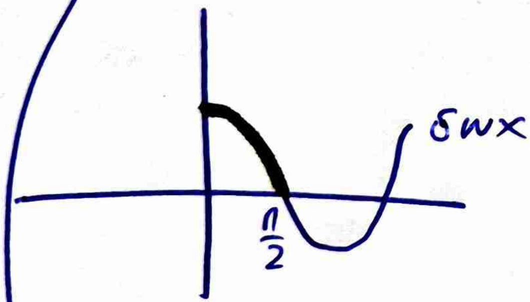
$x_1 < x_2 \Rightarrow e^{x_1} - 1 < e^{x_2} - 1$ (+)

$f(x_1) < f(x_2)$

$f \nearrow$

(β) ΝΔΟ $\forall x \in (0, \frac{\pi}{2})$ $\text{ημ}x + e^x \text{συν}x > \text{συν}x$

$$\frac{\text{ημ}x}{\text{συν}x} + \frac{e^x \text{συν}x}{\text{συν}x} > \frac{\text{συν}x}{\text{συν}x}$$



Το $\text{συν}x > 0$ στο $(0, \frac{\pi}{2})$

$\text{εφ}x + e^x > 1$

$\text{εφ}x + e^x - 1 > 0$

$f(x) > f(0)$

$f \nearrow$ $x > 0$ ✓

16. $f(x) = x + \ln(x+3)$, $x > -3$

ⓐ $x_1 < x_2 \Rightarrow \ln(x_1+3) < \ln(x_2+3)$

$x_1 < x_2$

$\oplus$

$x_1 + \ln(x_1+3) < x_2 + \ln(x_2+3)$

$f(x_1) < f(x_2)$ $\quad f \nearrow$

ⓑ i) $x^2 + \ln(x^2+3) > \ln 3$.

$f(x^2) > f(0)$

$f \nearrow$

$x^2 > 0$

$x \in \mathbb{R}^*$

11) $x^4 - x^2 < \ln \frac{x^2+3}{x^4+3}$

$$x^4 - x^2 < \ln(x^2+3) - \ln(x^4+3)$$

$$\ln(x^4+3) + x^4 < \ln(x^2+3) + x^2$$

$$f(x^4) < f(x^2)$$

$$f \nearrow$$

$$x^4 < x^2$$

$$x^4 - x^2 < 0$$

$$x^2(x^2 - 1) < 0$$

x		-1		0		1	
x^2	$+$		$+$		$+$		$+$
x^2-1	$+$		$-$		$-$		$+$
$\sim$	$+$		$-$		$-$		$+$

$x \in (-1,0) \cup (0,1)$

iii) $\ln\left(\frac{x+6}{x^2+4}\right) < x^2 - x - 2$

$\ln(x+6) - \ln(x^2+4) < x^2 - x - 2$

$$\boxed{f(x) = x + \ln(x+3)}$$

$\ln(x+3+3) + x+3 < \ln(x^2+1+3) + x^2+1$

$f(x+3) < f(x^2+1)$.

$f \nearrow$

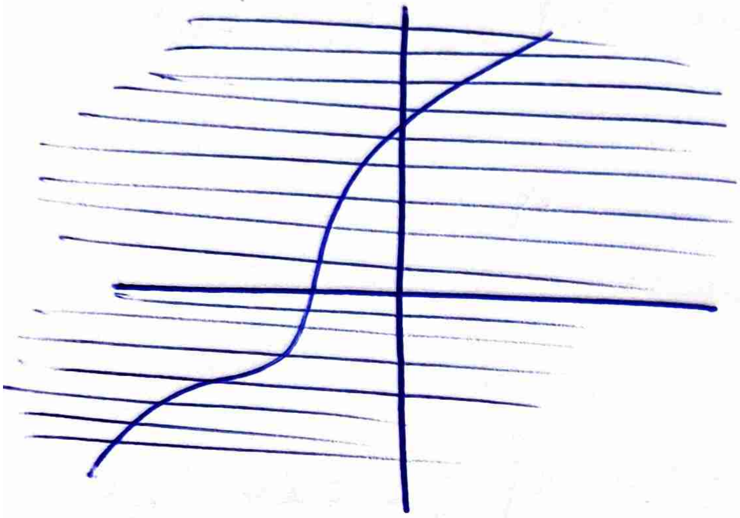
$x+3 < x^2+1$

$0 < x^2 - x - 2$

x		-1		2	
x^2-x-2	$+$		$-$		$+$

$x \in (-\infty, -1) \cup (2, +\infty)$.

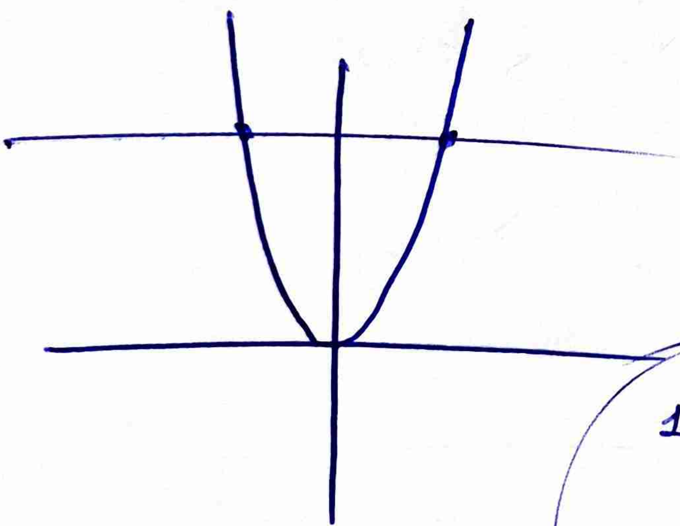
Συναρτηση 1-1



καθε οριζοντια ευθεια την τεμνει το πολυ 1 φορα.

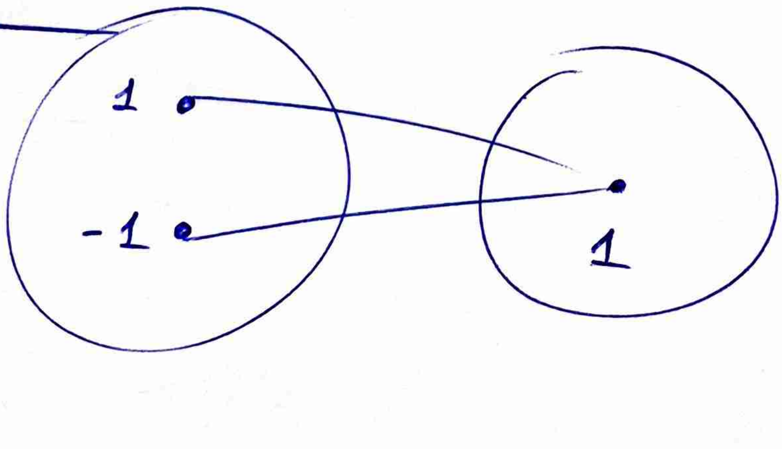
Ποτε δεν ειναι 1-1;

π.χ $f(x)=x^2$



$f(1)=1$

$f(-1)=1$



Συναρτηση 1-1

$$f(x_1) = f(x_2) \iff x_1 = x_2$$

Πως αποδεικνυω οτι μια

$f(x)$ ειναι 1-1 ;

1. $f(x) = \dfrac{e^x - 1}{e^x + 1}$. $\qquad D_f = \mathbb{R}$.

$$f(x_1) = f(x_2)$$

$$\frac{e^{x_1} - 1}{e^{x_1} + 1} = \frac{e^{x_2} - 1}{e^{x_2} + 1}$$

$$(e^{x_1} - 1)(e^{x_2} + 1) = (e^{x_1} + 1)(e^{x_2} - 1)$$

$$\cancel{e^{x_1} e^{x_2}} + e^{x_1} - e^{x_2} \cancel{- 1} = \cancel{e^{x_1} e^{x_2}} - e^{x_1} + e^{x_2} \cancel{- 1}$$

$$2e^{x_1} = 2e^{x_2}$$

$$e^{x_1} = e^{x_2}$$

$$x_1 = x_2$$

föl-1

2. $f(x) = e^{2-x} + \frac{1}{x} - \ln x \quad , x>0$

- $x_1 < x_2 \Rightarrow -x_1 > -x_2 \Rightarrow 2-x_1 > 2-x_2$

$$e^{2-x_1} > e^{2-x_2}$$

$$+$$

- $x_1 < x_2 \rightarrow \frac{1}{x_1} > \frac{1}{x_2}$

- $x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2 \Rightarrow -\ln x_1 > -\ln x_2$

$$f(x_1) > f(x_2)$$

$f \downarrow$ άρα f 1-1.

7. $f(x) = e^{x-2} + x - 3$

Ενότητα 6

(α) $x_1 < x_2 \Rightarrow x_1 - 2 < x_2 - 2 \Rightarrow$

$e^{x_1-2} < e^{x_2-2}$

(+)

$x_1 < x_2 \Rightarrow x_1 - 3 < x_2 - 3$

$f(x_1) < f(x_2)$ $\quad f \nearrow$ $\quad f: 1-1$

(β) i) $x + e^{x-2} = 3$

$e^{x-2} + x - 3 = 0$

$f(x) = 0$

$f(x) = f(2)$

$f: 1-1$

$x = 2$

II) $e^{x^2-2} + x^2 = 3$

$$e^{x^2-2} + x^2 - 3 = 0$$

$$f(x^2) = f(2)$$

$$f \text{ 1-1}$$

$$x^2 = 2$$

$$\underline{\underline{x = \pm\sqrt{2}}}$$

III) $e^{3x-2} - e^{x^2} = x^2 - 3x + 2.$

$$e^{3x-2} + 3x - 3 = e^{x^2+2-2} + x^2+2 - 3$$

$$f(3x) = f(x^2+2)$$

$$f \text{ 1-1}$$

$$3x = x^2 + 2$$

$x = 1$

$$x^2 - 3x + 2 = 0$$

$x = 2$

iii) $f\left((9-3x)e^{2-x}-1\right)=0$

$f\left((9-3x)e^{2-x}-1\right)=f(2)$

$f3)-1$

$(9-3x)e^{2-x}-1=2$

$(9-3x)e^{2-x}=3$

$(3-x)e^{2-x}=1$

$\frac{(3-x)\cancel{e^{2-x}}}{\cancel{e^{2-x}}}=\frac{1}{e^{2-x}}$

$3-x=e^{x-2}$

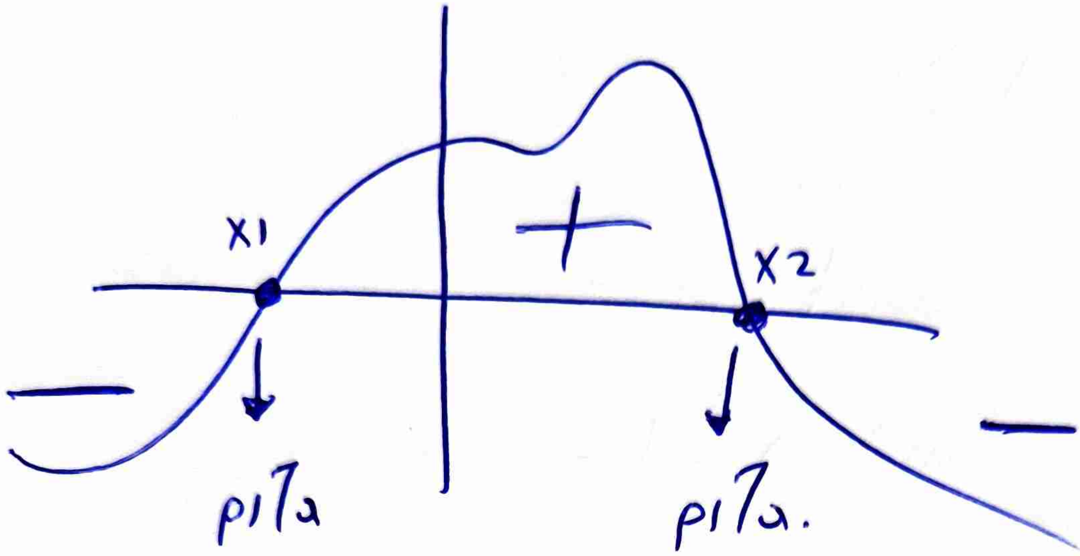
$0=e^{x-2}+x-3$

$f(2)=f(x)$

$f3)-1$

$x=2$

Πρόσημο της f(x)



x		x_1		x_2	
$f(x)$	−	0	+	0	−

$$f(x) = e^{x-1} + x - 2$$

Ριζα $f(x)$

$f(x) = 0$

$f(x) = f(1)$

f 1-1

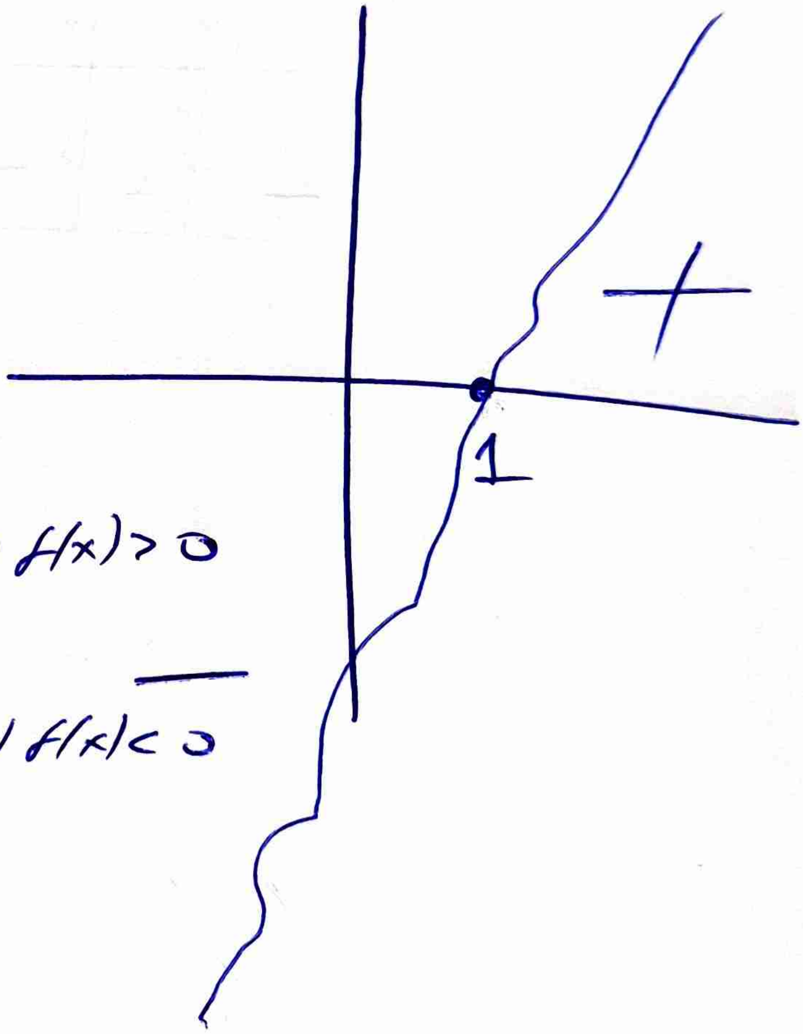
$\underline{\underline{x = 1}}$

- $x_1 < x_2 \Rightarrow x_1 - 1 < x_2 - 1$
  $e^{x_1 - 1} < e^{x_2 - 1}$ (+)
- $x_1 < x_2 \Rightarrow x_1 - 2 < x_2 - 2$

$f(x_1) < f(x_2)$

$f \nearrow$ f 1-1

x		1	
$f(x)$	$\nearrow$ $-$	0	$\nearrow$ $+$



$x > 1 \overset{f \nearrow}{\Rightarrow} f(x) > f(1) \Rightarrow f(x) > 0$

$x < 1 \overset{f \nearrow}{\Rightarrow} f(x) < f(1) \Rightarrow f(x) < 0$

Επομαο Μαθημα

Ενοτητα 4

9

10

11

14

15.

Ενοτητα 6

2 ολη

4

5

6