12. (8) $2 + \ln 15 - \ln 5 = \ln 3e^2$

$\ln e^2 + \ln \frac{15}{5} = \ln 3e^2$

$\ln e^2 + \ln 3 = \ln 3e^2$

$\ln e^2 \cdot 3 = \ln 3e^2$ ✓

11. ⑧ $\dfrac{\log \sqrt[3]{4} + \log \sqrt[3]{9} - \log \sqrt[3]{25}}{\log 6 - \log 5} = \dfrac{2}{3}$

$$\frac{\log\left(\sqrt[3]{4}\,\sqrt[3]{9}\right) - \log \sqrt[3]{25}}{\log \frac{6}{5}} = \frac{2}{3}$$

$$\frac{\log \dfrac{\sqrt[3]{4}\,\sqrt[3]{9}}{\sqrt[3]{25}}}{\log \frac{6}{5}} = \frac{2}{3}$$

$$\frac{\log \sqrt[3]{\frac{36}{25}}}{\log \frac{6}{5}} = \frac{2}{3}$$

$$\frac{\log \sqrt[3]{\left(\frac{6}{5}\right)^2}}{\log \frac{6}{5}} = \frac{2}{3} \Rightarrow \frac{\log \left(\frac{6}{5}\right)^{\frac{2}{3}}}{\log \frac{6}{5}} = \frac{2}{3}$$

$$\frac{2}{3}\,\frac{\log \frac{6}{5}}{\log \frac{6}{5}} = \frac{2}{3}$$

10. ⑧ $\log_2 64 + 2\log_3 \sqrt{27} - \log_8 4 - 2\log_8 32 = 5$

$\log_2 2^6 + 2\log_3 \sqrt{3^3} - \log_8 4 - \log_8 32^2 = 5$

$6 + 2\log_3 3^{\frac{3}{2}} - (\log_8 4 + \log_8 32^2) = 5$

$6 + \cancel{2} \cdot \frac{3}{\cancel{2}} - (\log_8 4 \cdot 32^2) = 5$

$9 - \log_8 (2^2 \cdot (2^5)^2) = 5$

$9 - \log_8 (2^{12}) = 5$

$9 - \log_8 ((2^3)^4) = 5$

$9 - \log_8 8^4 = 5$

$9 - 4 = 5$ ✓

9. в) $\ln 4 - \ln 12 + \ln 3 = 0$

$\ln \frac{4}{12} + \ln 3 = 0$

$\ln \frac{1}{3} \cdot 3 = 0$

$\ln 1 = 0$ ✓

б) $\frac{1}{2} \ln 9 - \frac{1}{3} \ln 64 + \ln 4 = \ln 3$.

$\ln 9^{1/2} - \ln 64^{1/3} + \ln 4 = \ln 3$

$\ln \sqrt{9} - \ln \sqrt[3]{64} + \ln 4 = \ln 3$

$\ln 3 - \ln 4 + \ln 4 = \ln 3$

$0 = 0$ ✓

8. (B) $3\log 2 + \log 125 =$

$= \log 2^3 + \log 125 =$

$= \log 8 + \log 125 =$

$= \log 8 \cdot 125 =$

$= \log 1000 = \log 10^3 = 3$

(8) $2\log \sqrt{2} + \log 5 =$

$= \log \sqrt{2}^2 + \log 5 =$

$= \log 2 + \log 5 =$

$= \log 2 \cdot 5 = \log 10 = 1.$

(02) $\ln 2e - \frac{1}{3}\ln 8 =$

$= \ln 2e - \ln 8^{1/3} = \ln 2e - \ln \sqrt[3]{8}$

$= \ln 2e - \ln 2$

$= \ln \frac{2e}{2} = \ln e$

$= 1.$

6. Ενότητα 24

(B) $B = \ln e^2 - e^{\ln 3} + 2\ln \frac{1}{\sqrt{e}}$

$B = 2 - 3 + 2(\ln 1 - \ln \sqrt{e})$

$B = -1 + 2(0 - \ln e^{1/2})$

$B = -1 + 2\left(-\frac{1}{2}\right) = -1 - 1 = -2$

(δ) $\log(\ln e) + 4 \ln \sqrt{e\sqrt{e}}$

$\log 1 + 4 \ln (e\sqrt{e})^{1/2}$

$= 0 + 4 \cdot \frac{1}{2} \ln(e \cdot \sqrt{e}) =$

$= 2(\ln e + \ln \sqrt{e}) =$

$= 2(1 + \ln e^{1/2}) =$

$= 2\left(1 + \frac{1}{2}\right) = 2 + 1 = 3.$

7. (B) $2\log 2 + \log 25 =$

$= \log 2^2 + \log 25 =$

$= \log 4 + \log 25 =$

$= \log 4 \cdot 25 =$

$= \log 100 = \log 10^2 = 2$

(g) $\log 3 - \log 30 =$

$= \log \frac{3}{30} = \log \frac{1}{10} = \log 1 - \log 10$

$= 0 - 1 = -1$

(62) $\log 7 - \log 70 = \log \frac{7}{70} = \log \frac{1}{10}$

$= \log 10^{-1} = -1$

(h) $\log 5 + \log 8 - \log 4 =$

$= \log 5 \cdot 8 - \log 4 = \log 40 - \log 4$

$= \log \frac{40}{4} = \log 10 = 1.$

Εβδομάδα 26

1. (β) $\ln x - 1 = 0$

$\ln x = 1$

$e^{\ln x} = e^{1}$

$\boxed{x = e}$ ✓

(δ) $2\ln x - 1 = 0$

$2\ln x = 1$

$\ln x = \frac{1}{2}$

$e^{\ln x} = e^{1/2}$

$\boxed{x = \sqrt{e}}$

(στ) $\frac{1 - 2\ln x}{2} - 1 = 0$

$1 - 2\ln x - 2 = 0$

$-1 - 2\ln x = 0$

$-1 = 2\ln x$

$\ln x = -\frac{1}{2}$

$e^{\ln x} = e^{-\frac{1}{2}}$

$x = \frac{1}{e^{1/2}} = \frac{1}{\sqrt{e}}$ $\quad \boxed{x = \frac{1}{\sqrt{e}}}$

extra

$\ln x + 1 = 0$

$\ln x = -1$

$e^{\ln x} = e^{-1}$

$\boxed{x = \frac{1}{e}}$

7. (E) $\frac{8\ln x}{\ln x - 3} = \ln x + 3$

$\ln x = t$

$\frac{8t}{t-3} = t+3$

$8t = (t+3)(t-3)$

$8t = t^2 - 9$

$t^2 - 8t - 9 = 0$

$t = 9 \qquad t = -1$

$\ln x = 9 \qquad \ln x = -1$

$e^{\ln x} = e^9 \qquad e^{\ln x} = e^{-1}$

$x = e^9 \qquad x = \frac{1}{e}$

5. (1) $\log x + \log(x-1) = \log(x+2) - \log 2$

$$\log x(x-1) = \log \frac{x+2}{2}$$

$$x(x-1) = \frac{x+2}{2}$$

$$x^2 - x = \frac{x+2}{2}$$

$$2x^2 - 2x = x + 2$$

$$2x^2 - 3x - 2 = 0$$

$$\Delta = 9 + 16 = 25$$

$$x = \frac{3 \pm 5}{4}$$

2 ✓

$-\frac{1}{2}$ ✗

6. (1) $\ln x^2 - \ln(2x-2) = \ln 2e - 1$

$$\ln \frac{x^2}{2x-2} = \ln 2e - \ln e$$

$$\ln \frac{x^2}{2x-2} = \ln \frac{2e}{e}$$

$$\frac{x^2}{2x-2} = 2 \Rightarrow x^2 = 4x - 4$$

$$x^2 - 4x + 4 = 0$$

$x = 2$ ✓

3. (f) $\log(2x-1)+1=0$

$\log(2x-1)=-1$

$10^{\log(2x-1)} = 10^{-1}$

$2x-1 = \frac{1}{10}$

$2x = \frac{1}{10}+1$

$20x = 1+10$

$20x = 11$

$x = \frac{11}{20}$

4. (f) $\log(3-x^2) - \log(x^2-1) = 0$

$\log(3-x^2) = \log(x^2-1)$

$3-x^2 = x^2-1$

$4 = 2x^2$

$x^2 = 2$

$x = \pm\sqrt{2}$

2. (B) $2\ln(3x-1)+1=0$

$$\ln(3x-1) = -\frac{1}{2}$$

$$e^{\ln(3x-1)} = e^{-\frac{1}{2}}$$

$$3x-1 = \frac{1}{e^{1/2}}$$

$$3x = \frac{1}{\sqrt{e}} + 1$$

$$\boxed{x = \frac{1}{3\sqrt{e}} + \frac{1}{3}}$$

(δ) $\ln x^2 - 1 = 0$

$$\ln x^2 = 1$$

$$e^{\ln x^2} = e^1$$

$$x^2 = e$$

$$x = \pm\sqrt{e}$$

(δ2) $\ln(1-x^2)+1=0$

$$\ln(1-x^2) = -1$$

$$e^{\ln(1-x^2)} = e^{-1}$$

$$1-x^2 = \frac{1}{e}$$

$$-x^2 = \frac{1}{e} - 1$$

$$x^2 = 1 - \frac{1}{e}$$

$$x = \pm\sqrt{1-\frac{1}{e}}$$

Εποχω Μαθημα

24

(6) α γ

(7) α γ ε ∃ϑ

(8) α γ ε

(9) α γ

(10) α β γ

(11) α β γ

(12) α β

26

(1) α γ ε

(2) α γ ε

(3) α β

(4) β ε β

(5) α β

(6) α β

(7) α β γ δ.