

Τεστ

Όνομα: Χυμπος (Κέρρα)

Βαθμ: 17,4/20 ή 87/100

Θεμα 1ο

Το πολυώνυμο $P(x) = x^3 - (a+1)x^2 + (a-1)x + 2$ έχει παραγοντα το $x-2$

α) Νόο $a=2$

β) Να γίνει η διαίρεση $P(x) : (x+3)$

γ) Να λυθεί η ανίσωση $P(x) \leq 17x + 14$

(30)

Θεμα 2ο

Το $P(x)$ διαιρούμενο με το $x-1$ αφήνει υπόλοιπο 2

ενώ διαιρούμενο με το $x+1$ αφήνει υπόλοιπο 8

α) Ναί βρού το υπόλοιπο τμή $P(x) : (x^2-1)$

β) Αν $v(x) = -3x+5$ να λυθεί η εξίσωση $\sqrt{v(x)} = 3x+1$

(30)

Θεμα 3ο

Το $P(x) = 2x^4 + x^3 + ax^2 + bx + 2$ έχει παραγοντα το $x^2 - 2x + 1$.

α) Νόο $a=-6$ και $b=1$

β) Να λυθεί η $P(x) = 0$

γ) Να λυθεί η ανίσωση $\frac{3x-1}{x+2} \geq 2$

(40)

Θεμα 1α

Ⓐ) $P(2) = 0 \Leftrightarrow 2^3 - (a+1) \cdot 2^2 + (a-1) \cdot 2 + 2 = 0$

$\Rightarrow 8 - 4(a+1) + 2(a-1) + 2 = 0$

$8 - 4a - 4 + 2a - 2 + 2 = 0$

$-2a + 4 = 0 \Rightarrow 2a = 4 \quad \boxed{a=2}$

$P(x) = x^3 - 3x^2 + x + 2$

Ⓑ) $1 \quad -3 \quad 1 \quad 2 \quad \boxed{-3}$

$\downarrow \quad -3 \quad 18 \quad -57$

$1 \quad -6 \quad 19 \quad \boxed{-55}$

$P(x) = (x+3)(x^2 - 6x + 19) - 55$

Ⓒ) $P(x) \leq 17x + 14$

$x^3 - 3x^2 + x + 2 - 17x - 14 \leq 0$

$x^3 - 3x^2 - 16x - 12 \leq 0.$

$1 \quad -3 \quad -16 \quad -12 \quad \boxed{-1}$

$\downarrow \quad -1 \quad 4 \quad 12$

$1 \quad -4 \quad -12 \quad 0$

$(x+1)(x^2 - 4x - 12) \leq 0$

$\boxed{-1}$

$\boxed{6} \quad \boxed{-2}$

x	-2	-1	6
$x+1$	$-$	$-$	$+$
$x^2 - 4x - 12$	$+$	$-$	$+$
$P(x)$	$-$	$+$	$+$

$x \in (-\infty, -2] \cup [-1, 6]$

Θεμα 2ο

Γνωρίζω ότι $P(1)=2$ και $P(-1)=8$

$$\textcircled{a} \quad P(x) = (x^2 - 1) \pi(x) + u(x)$$

$$P(x) = (x^2 - 1) \pi(x) + \alpha x + \beta$$

$$P(1) = \alpha + \beta = 2$$

$$P(-1) = -\alpha + \beta = 8$$

⊕

$$2\beta = 10$$

$$\underline{\underline{\beta = 5}}$$

$$\underline{\underline{\alpha = -3}}$$

$$\boxed{u(x) = -3x + 5}$$

$$\textcircled{b} \quad \sqrt{-3x + 5} = 3x + 1$$

$$-3x + 5 \geq 0$$

$$3x + 1 \geq 0$$

$$-3x + 5 = (3x + 1)^2$$

$$-3x + 5 = 9x^2 + 6x + 1$$

$$9x^2 + 9x - 4 = 0$$

$$\Delta = 81 + 4 \cdot 4 \cdot 9$$

$$\Delta = 81 + 16 \cdot 9$$

$$\Delta = 225$$

$$x = \frac{-9 \pm \sqrt{225}}{18}$$

$$\frac{6}{18} = \frac{1}{3}$$

$$-\frac{24}{18} = -\frac{4}{3}$$

Делә 30

$$\begin{array}{r}
 \textcircled{a} \quad 2 \quad 1 \quad \alpha \quad \beta \quad 2 \quad \textcircled{1} \\
 \downarrow \quad 2 \quad 3 \quad 3+\alpha \quad 3+\alpha+\beta \\
 2 \quad 3 \quad 3+\alpha \quad 3+\alpha+\beta \quad | \quad \alpha+\beta+5 = 0
 \end{array}$$

$$\begin{array}{r}
 2 \quad 3 \quad 3+\alpha \quad 3+\alpha+\beta \quad \textcircled{1} \\
 \downarrow \quad 2 \quad 5 \quad 8+\alpha \\
 2 \quad 5 \quad 8+\alpha \quad | \quad 11+2\alpha+\beta = 0
 \end{array}$$

$$\begin{cases}
 \alpha + \beta + 5 = 0 \\
 11 + 2\alpha + \beta = 0
 \end{cases}
 \quad \ominus \quad -\alpha - 6 = 0$$

$\alpha = -6$
 $\beta = 1$

$$\textcircled{b} \quad 2x^4 + x^3 - 6x^2 + x + 2 = 0$$

$$\begin{array}{r}
 2 \quad 1 \quad -6 \quad 1 \quad 2 \quad \textcircled{1} \\
 \downarrow \quad 2 \quad 3 \quad -3 \quad -2 \\
 2 \quad 3 \quad -3 \quad -2 \quad 0
 \end{array}$$

$$(x-1) (2x^3 + 3x^2 - 3x - 2) = 0$$

$$\textcircled{x=1} \quad \begin{array}{r}
 2 \quad 3 \quad -3 \quad -2 \quad \textcircled{1} \\
 \downarrow \quad 2 \quad 5 \quad 2 \\
 2 \quad 5 \quad 2 \quad 0
 \end{array}$$

$(x-1)(x-1)(2x^2+5x+2) = 0$
 $\Delta = 9$
 $\frac{-5 \pm 3}{4}$
 $x = 1$
 $\left(-\frac{1}{2}\right)$
 (-2)

①

$$\frac{3x-1}{x+2} \geq 2 \Rightarrow \frac{3x-1}{x+2} - 2 \geq 0$$

$$\frac{3x-1}{x+2} - \frac{2(x+2)}{x+2} \geq 0$$

$$\frac{x-5}{x+2} \geq 0$$

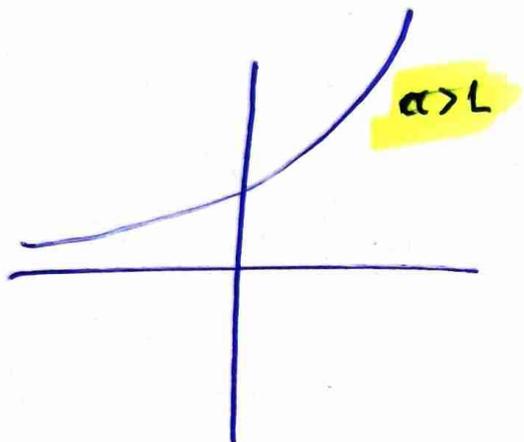
x	-2	5
x-5	-	-0+
x+2	-	+ +
f(x)	+	- +

$$x \in (-\infty, -2) \cup [5, +\infty)$$

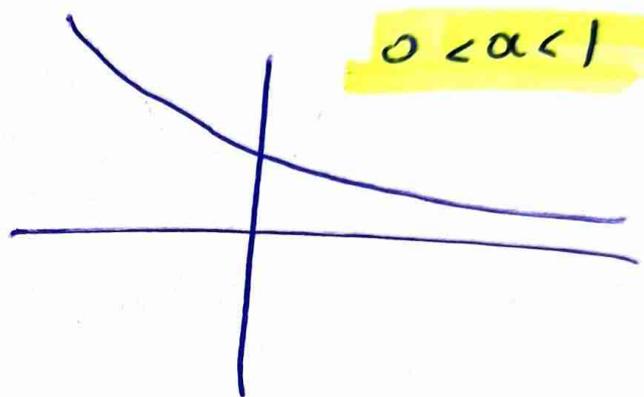
Εκθετική

Συναρτησιολογία

1. $f(x) = a^x \quad x \in \mathbb{R}, a \in (0, 1) \cup (1, +\infty)$



- $D_f = \mathbb{R}$
- $\Sigma T_f = (0, +\infty)$
- $f \nearrow$



- $D_f = \mathbb{R}$
- $\Sigma T_f = (0, +\infty)$
- $f \searrow$

$$1. \text{ (B) } 4^x = 64$$

$$4^x = 4^3$$

$$\boxed{x=3}$$

ΕΥΟΤΗΤΑ

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$$\text{(δ)} \quad 2^{2x-1} - 8 = 0$$

$$2^{2x-1} = 8$$

$$2^{2x-1} = 2^3$$

$$2x-1 = 3$$

$$2x = 4$$

$$\boxed{x=2}$$

$$\text{(εζ)} \quad \left(\frac{2}{3}\right)^x = \frac{8}{27}$$

$$\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^3$$

$$\boxed{x=3}$$

$$a^{-v} = \frac{1}{a^v}$$

$$2. \text{ (B) } 2^{-x} = \frac{1}{64}$$

$$2^{-x} = \left(\frac{1}{2}\right)^6 = \frac{1}{2^6}$$

$$2^{-x} = (2)^{-6}$$

$$-x = -6$$

$$\boxed{x=6}$$

$$\textcircled{1} \left(\frac{3}{2}\right)^x = \frac{4}{9}$$

$$\left(\frac{3}{2}\right)^x = \left(\frac{2}{3}\right)^2$$

$$\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^{-2}$$

$$x = -2$$

3.

$$\textcircled{B} 4^{2x} = 32$$

$$(2^2)^{2x} = 2^5$$

$$2^{4x} = 2^5$$

$$4x = 5$$

$$x = \frac{5}{4}$$

$$(a^v)^\mu = a^{v \cdot \mu}$$

$$\textcircled{5} 9^{2x+1} = 27$$

$$(3^2)^{2x+1} = 3^3$$

$$3^{4x+2} = 3^3$$

$$4x+2=3$$

$$4x=1$$

$$x = \frac{1}{4}$$

(52)

$$3^{x^2-1} = 9^{x-1}$$

$$3^{x^2-1} = (3^2)^{x-1}$$

$$3^{x^2-1} = 3^{2x-2}$$

$$x^2-1 = 2x-2$$

$$x^2-2x+1 = 0$$

$$(x-1)^2 = 0$$

$$x=1$$

$$3. \quad \textcircled{\beta} \quad e^{2x-3} = 1$$

$$e^{2x-3} = e^0$$

$$2x-3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$e = 2, 7$$

Leonard Euler

венерω
απ, θμω.

$$\textcircled{\delta} \quad e^x - \frac{1}{e} = 0$$

$$e^x = \frac{1}{e}$$

$$e^x = e^{-1}$$

$$x = -1$$

$$\sum x = \alpha \mu \omega$$

$$a^0 = 1$$

$$a^1 = a$$

$$a^{-v} = \frac{1}{a^v}$$

$$\textcircled{\epsilon} \quad e^{2x} - e^{x+1} = 0$$

$$e^{2x} = e^{x+1}$$

$$2x = x+1$$

$$x = 1$$

$$a^v \cdot a^p = a^{v+p}$$

$$a^v : a^p = a^{v-p}$$

$$(a^v)^p = a^{v \cdot p}$$

$$\left(\frac{a}{b}\right)^v = \left(\frac{b}{a}\right)^{-v}$$

$$5. \quad \textcircled{\beta} \quad e^x + 2 = 0$$

$$e^x = -2$$



Абсурд.

Отсюда

получается

$$\textcircled{\delta} \quad 1 - e^{|x|-2} = 0$$

$$1 = e^{|x|-2}$$

$$e^0 = e^{|x|-2}$$

$$0 = |x| - 2$$

$$|x| = 2$$

$$x = \pm 2$$

Επιτομια Μαθηματων

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① α γ ε

② α

③ α γ ε

④ α γ

⑤ α γ .