6. $f(x) = x^2 - 2x + 3$

Εναλλακτικά 17

(01) $y - f(x_0) = f'(x_0)(x - x_0)$

$f'(x_0) = \varepsilon\varphi 135 = -1$

$f'(x) = 2x - 2$

$f'(x) = -1$

$2x - 2 = -1$

$2x = 1$

$x = \frac{1}{2}$

$y - f(\frac{1}{2}) = f'(\frac{1}{2})(x - \frac{1}{2})$

- $f(\frac{1}{2}) = \frac{1}{4} - 2 \cdot \frac{1}{2} + 3 = \frac{1}{4} - 1 + 3 = \frac{1}{4} + 2 = \frac{9}{4}$
- $f'(\frac{1}{2}) = 2\frac{1}{2} - 2 = -1$

$y = -x + \frac{1}{2} + \frac{9}{4}$

$y - \frac{9}{4} = -1(x - \frac{1}{2})$ → $y = -x + \frac{11}{4}$

(7) $y - f(x_0) = f'(x_0)(x - x_0)$ $\perp$ $y = x$

$f'(x) \cdot 1 = -1$

$2x - 2 = -1$

$2x = 1$

$x = \frac{1}{2}$

$y = -x + \frac{11}{4}$

(8) $y - f(x_0) = f'(x_0)(x - x_0)$ $\parallel x'x$

$f'(x_0) = 0$

$2x - 2 = 0$

$x = 1$

$y - f(1) = f'(1)(x - 1)$

$y - 2 = 0$

$y = 2$

8. $f(x) = \frac{2x-1}{x}$

$y - f(x_0) = f'(x_0)(x - x_0) \longrightarrow (0, 0)$

$0 - f(x_0) = f'(x_0)(0 - x_0)$

$-f(x) = f'(x)(-x)$

$-\frac{2x-1}{x} = \frac{2x-(2x-1)}{x^2}(-x)$

$-\frac{2x-1}{x} = -\frac{1}{x}$

$2x-1 = 1$

$2x = 2$

$x = 1$

$y - f(1) = f'(1)(x-1)$

$y - 1 = 1(x-1)$

$y = x$

11. $f(x) = x^2 - x + 2$

(B) $y - f(x_0) = f'(x_0)(x - x_0)$ $\perp$ ε_{AB}

$$f'(x_0) \cdot \lambda_{AB} = -1$$

$$(2x - 1) \cdot \frac{y_B - y_A}{x_B - x_A} = -1$$

$$(2x - 1) \cdot \frac{f(2) - f(0)}{2 - 0} = -1$$

$$(2x - 1) \cdot \frac{4 - 2}{2} = -1$$

$$(2x - 1) = -1$$

$$2x = 0$$

$$x = 0$$

12. $f(x) = ax^3 + bx - 1$ $\quad M(1,-1)$

$y - f(1) = f'(1)(x-1)$ $\quad \perp \delta: 3x - y + 1 = 0$ $\quad f(1) = -1$

$\hookrightarrow y = 3x + 1$

$f'(1) \cdot 3 = -1$

$(3a+b) \cdot 3 = -1$

$f'(x) = 3ax^2 + b$

$9a + 3b = -1$

$\hookrightarrow -9b + 3b = -1$

$-6b = -1$

$b = \frac{1}{6}$

$f(1) = a + b - 1 = -1$

$a + b = 0$

$a = -b$

$x = -\frac{1}{6}$

20. $f(x) = x^2 - x + \frac{5}{4}$ $\quad f'(x) = 2x - 1$

$g(x) = e^{-2x}$

Νδο η εφαπτομένη της f στο $-\frac{1}{2}$

είναι και εφαπτομένη της $g(x)$

$$y - f\left(-\tfrac{1}{2}\right) = f'\left(-\tfrac{1}{2}\right)\left(x - \left(-\tfrac{1}{2}\right)\right)$$

$$y - 2 = -2\left(x + \tfrac{1}{2}\right)$$

$$y - 2 = -2x - 1$$

$$f\left(-\tfrac{1}{2}\right) = \tfrac{1}{4} + \tfrac{1}{2} + \tfrac{5}{4} = 2$$

$$y = -2x + 1$$

$$f'\left(-\tfrac{1}{2}\right) = 2\left(-\tfrac{1}{2}\right) - 1 = -2$$

$$\begin{cases} g'(x) = -2 \\ g(x) = -2x + 1 \end{cases}$$

$g'(x) = -2 \Rightarrow -2e^{-2x} = -2$

$e^{-2x} = 1$

$e^{-2x} = -2x + 1$ ⟵ $x = 0$

$e^{0} = 0 + 1$

$1 = 1$ ✓

19. $f(x) = x^2 - x + 2$

Νδο η $y = x + 1$ εφαπτεται της C_f

$$\begin{cases} f'(x) = 1 \\ f(x) = x + 1 \end{cases}$$

$f'(x) = 1 \Rightarrow 2x - 1 = 1 \Rightarrow 2x = 2$

$\boxed{x = 1}$

$x^2 - x + 2 = x + 1$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$\boxed{x = 1}$

Η $y = x + 1$ είναι η εφαπτομένη

της $f(x)$ στο 1.

21. $f(x) = x^2 + x + 2$

$\varepsilon : y = 2x + 1$.

$$\begin{cases} f'(x) = 2 \\ f(x) = 2x + 1 \end{cases} \Rightarrow \begin{cases} 2x + 1 = 2 \\ x^2 + x + 2 = 2x + 1 \end{cases}$$

$x^2 + x + 2 = (2x+1)x + 1$

$x^2 + x + 2 = 2x^2 + x + 1$

$1 = x^2$

$x = 1$ $\quad$ $x = -1$

$2 = 3$ $\quad$ $2 = -1$

22. $y = ax + 1$ $\quad f(x) = Bx^2 - ax + 2$.

$f'(x) = 2Bx - a$

$\hookrightarrow -1$

$$\begin{cases} f'(-1) = a \\ f(-1) = a(-1) + 1 \end{cases} \Rightarrow \quad -2B - a = a$$

$$-2B = 2a$$

$\alpha = -\beta$

$\beta + a + 2 = -a + 1$

$\beta + 2a = -1$

$\beta + 2(-\beta) = -1$

$-\beta = -1$

$\alpha = -1$

$\beta = 1$

23. $f(x) = x^2 + 2\alpha x + \beta$

Ο x'x εφάπτεται στο $x_0 = 1$

$$\begin{cases} f'(1) = 0 \\ f(1) = 0 \end{cases}$$

$f'(x) = 2x + 2\alpha$

$$\begin{cases} 2 + 2\alpha = 0 \\ 1 + 2\alpha + \beta = 0 \end{cases}$$

$\alpha = -1$

$1 - 2 + \beta = 0$

$\beta = 1$

24. $f(x) = x^2 - 4x$ $\qquad f'(x) = 2x - 4$

$$g(x) = \frac{a}{x}$$

$$y - f(-1) = f'(-1)(x+1)$$

$$y - 5 = -6(x+1)$$

$$y - 5 = -6x - 6$$

$$y = -6x - 1 \quad \text{(уравнение оси } 6\text{)}$$

$$\begin{cases} g'(x) = -6 \\ g(x) = -6x - 1 \end{cases} \Rightarrow -\frac{a}{x^2} = -6 \Rightarrow a = x^2$$

$$a = \frac{1}{20}$$

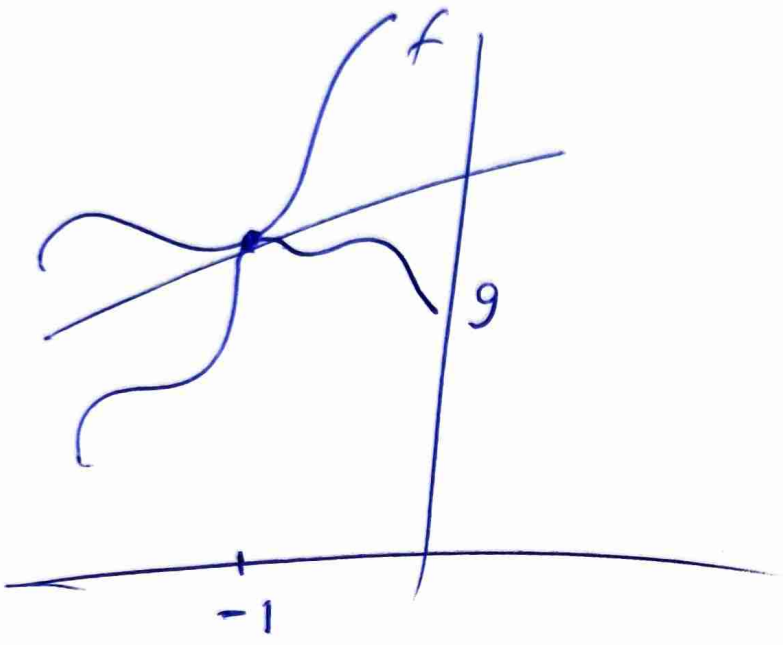
$$\frac{a}{x} = -6x - 1 \longrightarrow \frac{6x^2}{x} = -6x - 1$$

$$12x = -1$$

$$x = -\frac{1}{12}$$

25. $f(x) = kx^2 - 2\lambda x + k$

$g(x) = x^3 - x + \lambda$



$$\begin{cases} f'(-1) = g'(-1) \\ f(-1) = g(-1) \end{cases}$$

$f'(x) = 2kx - 2\lambda$

$g'(x) = 3x^2 - 1$

$$\begin{cases} -2k - 2\lambda = 2 \Rightarrow -k - \lambda = 1 \\ k + 2\lambda + k = \lambda \Rightarrow 2k + \lambda = 0 \end{cases} \quad (+)$$

$k = -1$

$\lambda = -2$

26. $f(x) = \frac{2x}{x^2+1}$ C_f, C_g

έχουν εφαπτομένη

στο -1

$g(x) = ax^2 + \beta x + 1$

$$\begin{cases} f'(-1) = g'(-1) \Rightarrow 0 = -2a + \beta \\ f(-1) = g(-1) \Rightarrow -1 = a - \beta + 1 \end{cases} \Big| \ominus$$

$a - \beta = -2$

$$f'(x) = \frac{2(x^2+1) - 2x \cdot 2x}{(x^2+1)^2}$$

$-a = -2$

$a = 2$

$\beta = 4$

$$f'(x) = \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

$g'(x) = 2ax + \beta$

$$f'(x) = \frac{2-2x^2}{(x^2+1)^2}$$

27. $f(x) = x^2 - 3x + 3$

$g(x) = \frac{1}{x}$

Έστω $f(x) < g(x)$ $\quad x > 0$

$x^2 - 3x + 3 < \frac{1}{x}$

$x^3 - 3x^2 + 3x - 1 < 0$

$(x-1)^3 < 0$

$x < 1$

Όταν $x < 1$ η C_f κάτω C_g

Όταν $x > 1$ η C_f πάνω C_g

Τέμνονται $x = 1$

(B) $\begin{cases} f'(1) = g'(1) \Rightarrow -1 = -1 \\ f(1) = g(1) \Rightarrow 1 = 1 \end{cases}$ ✓ ✓

$f'(x) = 2x - 3$

$g'(x) = -\frac{1}{x^2}$

30. $f(x) = x^2$ $\quad f'(x) = 2x$

$g(x) = \frac{1}{x}$ $\quad g'(x) = -\frac{1}{x^2}$

$$\begin{cases} f'(a) = g'(B) \\ f(a) - a f'(a) = g(B) - B g'(B) \end{cases}$$

$$\begin{cases} 2a = -\frac{1}{B^2} \\ a^2 - a \cdot 2a = \frac{1}{B} - 0 \cdot \left(-\frac{1}{B^2}\right) \end{cases}$$

$$\begin{cases} \boxed{a = -\frac{1}{2B^2}} \\ -a^2 = \frac{1}{B} + \frac{1}{B} \end{cases}$$

$$-a^2 = \frac{2}{B}$$

$$-\left(\frac{1}{2B^2}\right)^2 = \frac{2}{B} \quad \Rightarrow -\frac{1}{4B^4} = \frac{2}{B}$$

$$\Rightarrow -B = 8B^4 \quad \Rightarrow -1 = 8B^3$$

$$2B = -1$$

$$\boxed{B = -\frac{1}{2}}$$

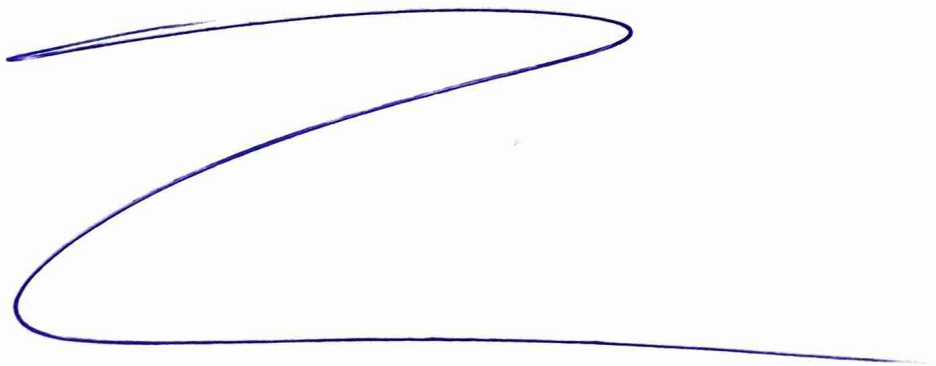
$$y - g\left(-\frac{1}{2}\right) = g'\left(-\frac{1}{2}\right)\left(x + \frac{1}{2}\right)$$

$$y - (-2) = -\frac{1}{\frac{1}{4}}\left(x + \frac{1}{2}\right)$$

$$y + 2 = -4\left(x + \frac{1}{2}\right)$$

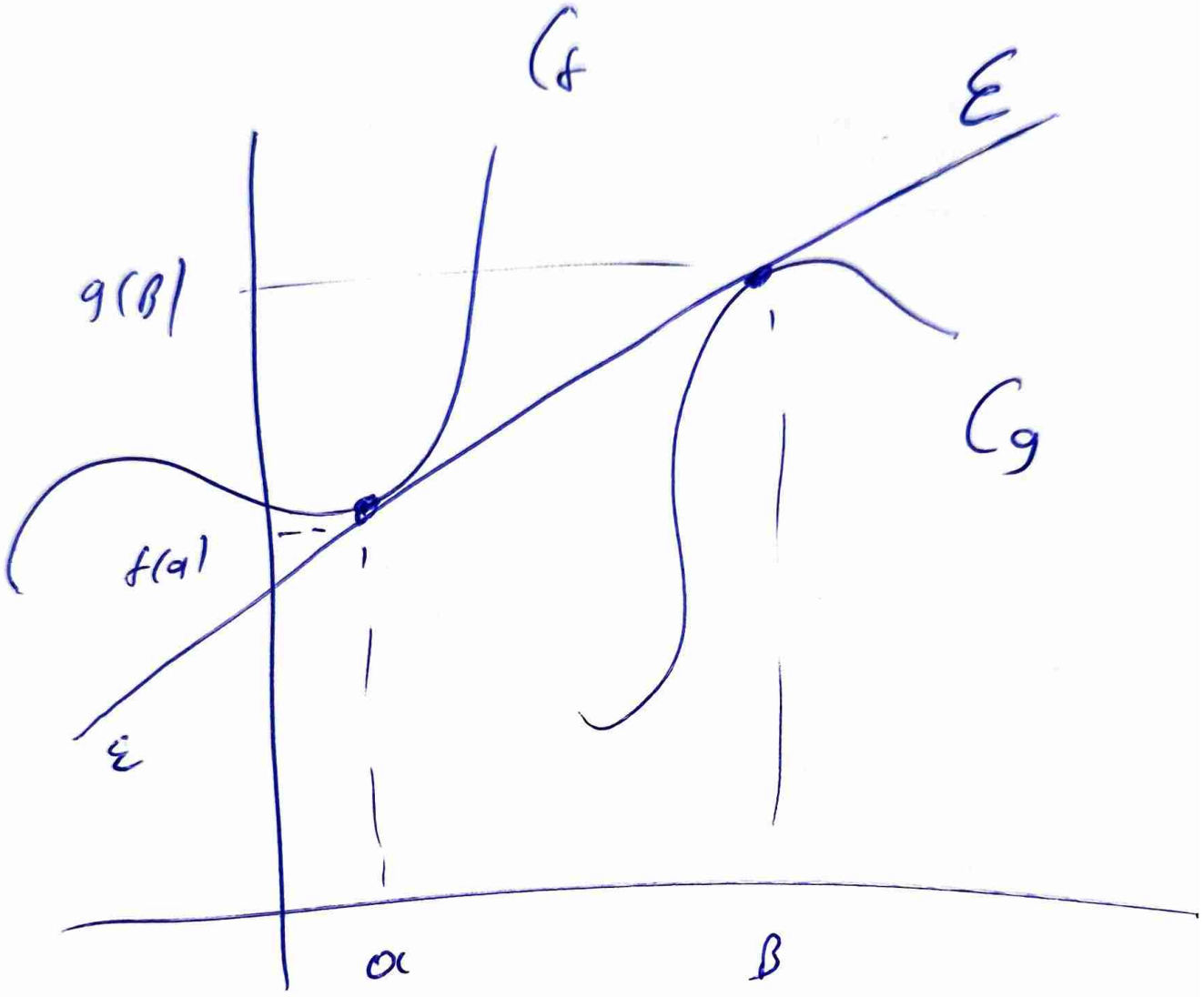
$$y + 2 = -4x - 2$$

$$y = -4x - 4$$



κοινη εφαπτομενη

σε μη κοινο σημειο



$y - f(\alpha) = f'(\alpha)(x-\alpha) \Rightarrow y - f(\alpha) = f'(\alpha)x - \alpha f'(\alpha)$

$\boxed{y = f'(\alpha)x + f(\alpha) - \alpha f'(\alpha)}$

$y - g(\beta) = g'(\beta)(x-\beta)$

$y - g(\beta) = g'(\beta)x - \beta g'(\beta) \quad \boxed{y = g'(\beta)x + g(\beta) - \beta g'(\beta)}$

$$\begin{cases} f'(\alpha) = g'(\beta) \\ f(\alpha) - \alpha f'(\alpha) = g(\beta) - \beta g'(\beta). \end{cases}$$

34. Δίνεται $f(x) = \frac{x^4}{4} + \frac{x^2}{2} - 1$

Να $\exists! x_0 \in (0,1)$ τ.ω η εφαπτομένη στο x_0 $\perp$ $x+y-3=0$

$y - f(x_0) = f'(x_0)(x-x_0)$ $\perp$ $y = -x+3$

$f'(x_0)(-1) = -1$

$f'(x_0) = 1$

$f'(x) = 1$

$\frac{4x^3}{4} + \frac{2x}{2} = 1$

$x^3 + x - 1 = 0.$

Дрка убо и єблюсь

$$x^3 + x - 1 = 0$$

єхи роковіки $\exists ! x \in (0,1)$.

$$\varphi(x) = x^3 + x - 1$$

$$\left.\begin{array}{l} \varphi(0) = -1 \\ \varphi(1) = 1 \end{array}\right\} \varphi(0)\,\varphi(1) < 0$$

Волгин $\exists ! x_0 \in (0,1)$

т.ч $\varphi(x_0) = 0$

$\varphi'(x) = 3x^2 + 1 > 0$

$\varphi \nearrow$

32. $f(x) = x \ln x + \frac{x^3}{3} - 1$

Ψάχνω την εφαπτομένη της C_f

η οποία είναι κάθετη

στη $x + 2y + 1 = 0$.

$y - f(x_0) = f'(x_0)(x - x_0)$ $\perp$ $x + 2y + 1 = 0$

$2y = -x - 1$

$y = -\frac{1}{2}x - \frac{1}{2}$

$f'(x_0) \cdot \left(-\frac{1}{2}\right) = -1$

$f'(x_0) = 2$

$f'(x) = 2$

$\ln x + x \cdot \frac{1}{x} + 3\frac{x^2}{3} = 2$

$\ln x + 1 + x^2 = 2$

$\ln x + x^2 - 1 = 0$.

$x = 1$

$h(x) = \ln x + x^2 - 1 \quad , x > 0$

$h'(x) = \frac{1}{x} + 2x > 0$

$h \nearrow$

$\ln x + x^2 - 1 = 0$

$h(x) = 0$

$h(x) = h(1)$

$h(3) - 1$

$x = 1$

$y - f(1) = f'(1)(x-1)$

$y - \left(\frac{1}{3} - 1\right) = 2(x-1)$

$y - \frac{1}{3} + 1 = 2x - 2$

$y = 2x - 3 + \frac{1}{3}$

35. $f(x) = (x-2)\ln x$

Νδο υπάρχει μοναδική εφαπτομένη που διέρχεται από αρχή των αξόνων

$y - f(x_0) = f'(x_0)(x - x_0) \longrightarrow (0,0)$

$0 - f(x) = f'(x)(0 - x)$

$-(x-2)\ln x = \left(\ln x + \frac{x-2}{x}\right)(-x)$

$(x-2)\ln x = x\ln x + x - 2$

$x\ln x - 2\ln x = x\ln x + x - 2$

$0 = 2\ln x + x - 2$

$$2\ln x + x - 2 = 0$$

$\varphi(x) = 2\ln x + x - 2$

$\varphi'(x) = \frac{2}{x} + 1 > 0$

$\varphi \nearrow$



- $\lim_{x \to 0^+} \varphi(x) = -\infty$
- $\lim_{x \to +\infty} \varphi(x) = +\infty$

$\left.\begin{array}{l}\\\\\\\end{array}\right\} \Sigma T_\varphi = \mathbb{R}$

φ συνεχής

$\varphi \nearrow$

$\Sigma T_\varphi = \mathbb{R}$ $\quad \exists!\ \xi$ τ.ω

το $0 \in \Sigma T_\varphi$ αφού $\quad \varphi(\xi) = 0$

Επομαι Μαθημα

Τριτη 20/10/26

Ενοτητα 17

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