Σχολικό (Σελ 52)

1. $A = \left[(x^2y^3)^{-2} \cdot (xy^3)^4 \right] : \left(\frac{x^3}{y^{-1}}\right)^{-3}$

i) Ν.δ.ο. $A = x^9y^9$

$$A = \left[\left(x^{-4} y^{-6} \cdot x^{4} y^{12} \right) : \frac{x^{-9}}{y^3} \right.$$

$$A = \left[\left(x^0 y^6 \right) : \frac{x^{-9}}{y^3} \right]$$

$$A = \left[y^6 \cdot \frac{y^3}{x^{-9}} \right]$$

$$A = \frac{y^6}{1} \cdot \frac{y^3}{x^{-9}} = \frac{y^9}{x^{-9}} = y^9 x^9$$

ii) Για $x = 2010$ και $y = \frac{1}{2010}$

$$A = x^9y^9 = (xy)^9 = \left(\cancel{2010} \frac{1}{\cancel{2010}}\right)^9 = 1^9 = 1$$

2. $A = \left[(xy^{-1})^2 : (x^3y^7)^{-1}\right]^2$

$\boxed{x = 0{,}4 \qquad y = -2{,}5}$

$A = \left[(x^2y^{-2}) : (x^{-3} \cdot y^{-7})\right]^2$

$A = (x^5 y^5)^2$

$A = x^{10} y^{10} = (xy)^{10}$

$A = \left[0{,}4 \cdot (-2{,}5)\right]^{10}$

$A = (-1)^{10} = 1.$

3. (i) $1001^2 - 999^2 =$

$$= (1001-999)(1001+999) = 2 \cdot 2000 = 4000$$

(ii) $99 \cdot 101 = (100-1)(100+1) = 100^2 - 1^2 = 10000 - 1$

$$= 9999$$

(iii) $$\frac{7{,}23^2 - 4{,}23^2}{11{,}46} = \frac{(7{,}23-4{,}23)(7{,}23+4{,}23)}{11{,}46}$$

$$= \frac{3 \cdot \cancel{11{,}46}}{\cancel{11{,}46}} = 3$$

4. i) Νδο $(\alpha+\beta)^2-(\alpha-\beta)^2=4\alpha\beta$

$$\alpha^2+2\alpha\beta+\beta^2-(\alpha^2-2\alpha\beta+\beta^2)=4\alpha\beta$$

$$\cancel{\alpha^2}+2\alpha\beta\cancel{+\beta^2}\cancel{-\alpha^2}+2\alpha\beta\cancel{-\beta^2}=4\alpha\beta$$

$$4\alpha\beta=4\alpha\beta.$$

ii) $\left(\frac{999}{1000}+\frac{1000}{999}\right)^2-\left(\frac{999}{1000}-\frac{1000}{999}\right)^2=$

$$=4\cdot\cancel{\frac{999}{1000}}\cdot\cancel{\frac{1000}{999}}$$

$$=4$$

5. (α) Ndo $\alpha^2-(\alpha-1)(\alpha+1)=1$

$$\alpha^2-(\alpha^2-1)=1$$

$$\cancel{\alpha^2}-\cancel{\alpha^2}+1=1$$

$$1=1$$ ✓

(β) $1,3265^2-0,3265\cdot 2,3265=$

$$=1,3265^2-(1,3265-1)\cdot(1+1,3265)=$$

$$=\underline{1}.$$

6. Νδο η διαφορα των τετραγωνων δυο διαδοχικων φυσικων αριθμων. ισουται με το αθροισμα τωλ.

1ος : x

2ος : $x+1$.

$$(x+1)^2 - x^2 = x+1+x$$

$$\cancel{x^2}+2\cancel{x}+1-\cancel{x^2} = 2\cancel{x}+1$$

$$1 = 1$$ ✓

7. $\nu \in \mathbb{N}$

νδο $2^{\nu} + 2^{\nu+1} + 2^{\nu+2}$ πολ/σιο
του 7.

$$2^{\nu} + 2^{\nu+1} + 2^{\nu+2} =$$

$$= 2^{\nu} + 2^{\nu} \cdot 2^{1} + 2^{\nu} \cdot 2^{2} =$$

$$= 2^{\nu} + 2 \cdot 2^{\nu} + 4 \cdot 2^{\nu}$$

$= 7 \cdot 2^{\nu}$ πολ/σιο του 7.

$$a^{\nu} \cdot a^{\mu} = a^{\nu+\mu}$$

Β' ομαδα

1. i) $$\frac{a^3-2a^2+a}{a^2-a}=\frac{\cancel{a}(a^2-2a+1)}{\cancel{a}(a-1)}=\frac{(a-1)^{\cancel{2}}}{\cancel{a-1}}$$

$$=a-1$$

ii) $$\frac{(a^2-a)+2a-2}{a^2-1}=\frac{a(a-1)+2(a-1)}{(a-1)(a+1)}$$

$$=\frac{\cancel{(a-1)}(a+2)}{\cancel{(a-1)}(a+1)}=\frac{a+2}{a+1}$$

2. i) $$\left(a-\frac{1}{a}\right)^2\frac{a^3+a^2}{(a+1)^3}=\left(\frac{a}{1}-\frac{1}{a}\right)^2\frac{a^2\cancel{(a+1)}}{(a+1)^{\cancel{3}}}$$

$$=\left(\frac{a^2}{a}-\frac{1}{a}\right)^2\frac{a^2}{(a+1)^2}=\left(\frac{a^2-1}{a}\right)^2\frac{a^2}{(a+1)^2}$$

$$=\left(\frac{(a-1)(a+1)}{a}\right)^2\frac{a^2}{(a+1)^2}=\frac{(a-1)^2\cancel{(a+1)^2}}{\cancel{a^2}}\cdot\frac{\cancel{a^2}}{\cancel{(a+1)^2}}$$

$$=(a-1)^2$$

2. ii) $\dfrac{a^2+a+1}{a+1} \cdot \dfrac{a^2-1}{a^3-1} =$

$$= \frac{\cancel{a^2+a+1}}{\cancel{a+1}} \cdot \frac{\cancel{(a-1)(a+1)}}{\cancel{(a-1)(a^2+a+1)}} = \underline{1}$$

3. i) $(x+y)^2 \cdot (x^{-1}+y^{-1})^{-2} =$

$$= (x+y)^2 \cdot \left(\frac{1}{x} + \frac{1}{y}\right)^{-2}$$

$$= (x+y)^2 \cdot \left(\frac{y}{xy} + \frac{x}{xy}\right)^{-2}$$

$$= (x+y)^2 \cdot \left(\frac{x+y}{xy}\right)^{-2} =$$

$$= (x+y)^2 \cdot \left(\frac{xy}{x+y}\right)^2 = \cancel{(x+y)^2} \cdot \frac{(xy)^2}{\cancel{(x+y)^2}}$$

$$= x^2y^2.$$

11).

$$\frac{x+y}{x-y} \cdot \frac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}} =$$

$$= \frac{x+y}{x-y} \cdot \frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}} =$$

$$= \frac{x+y}{x-y} \cdot \frac{\frac{y}{xy}-\frac{x}{xy}}{\frac{y^2}{x^2y^2}-\frac{x^2}{x^2y^2}} =$$

$$= \frac{x+y}{x-y} \cdot \frac{\frac{y-x}{xy}}{\frac{y^2-x^2}{xy}} =$$

$$= \frac{x+y}{x-y} \cdot \frac{(y-x)\,xy}{xy\,(y^2-x^2)} =$$

$$= \frac{\cancel{x+y}}{x-y} \cdot \frac{\cancel{(y-x)}\,\cancel{xy}}{\cancel{xy}\,\cancel{(y-x)}\cancel{(y+x)}} = \frac{1}{x-y}.$$

4. Ndo $\left(\frac{x^3+y^3}{x^2-y^2}\right):\left(\frac{x^2}{x-y}-y\right)=1$

$$\frac{\cancel{(x+y)}(x^2-xy+y^2)}{(x-y)\cancel{(x+y)}}:\left(\frac{x^2}{x-y}-\frac{y(x-y)}{x-y}\right)=1$$

$$\frac{x^2-xy+y^2}{x-y}:\left(\frac{x^2-yx+y^2}{x-y}\right)=1$$

$$\cancel{\frac{x^2-xy+y^2}{x-y}}\cdot\cancel{\frac{x-y}{x^2-yx+y^2}}=1$$

$$1=1$$

5. $\frac{\alpha}{\beta} = \frac{\beta}{\gamma} = \frac{\gamma}{\alpha} = \lambda$

$\alpha = \lambda\beta$

$\beta = \lambda\gamma \longrightarrow \beta = \lambda \cdot \lambda\alpha$

$\boxed{\gamma = \lambda\alpha}$

$\beta = \lambda^2\alpha$

$\alpha = \lambda \cdot \lambda^2\alpha$

$\alpha = \lambda^3\alpha$

$\alpha - \lambda^3\alpha = 0$

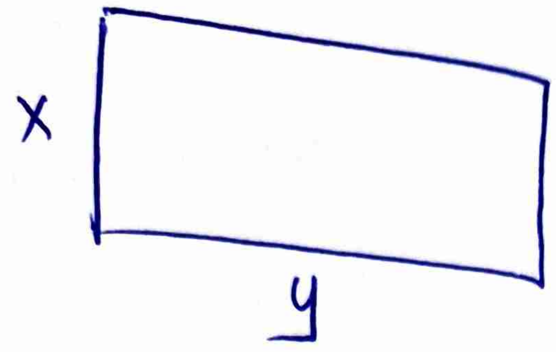
$\alpha(1 - \lambda^3) = 0$

$\cancel{\alpha \neq 0}$ ή $1 - \lambda^3 = 0$

$\lambda^3 = 1$ $\quad$ $\lambda = 1$

$\alpha = \beta = \gamma$

6.



Περίμετρος

$L=4a$

$E=a^2$

$\Pi = 2x+2y = 4a$

$\boxed{x+y=2a} \Rightarrow a = \frac{x+y}{2}$

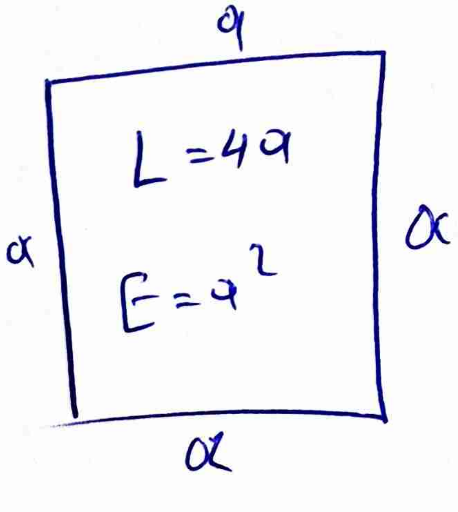
$E = xy = a^2$

$xy = \left(\frac{x+y}{2}\right)^2$

$xy = \frac{(x+y)^2}{4}$

$4xy = (x+y)^2$

$4xy = x^2+2xy+y^2$



$0 = x^2-2xy+y^2$

$0 = (x-y)^2$

$0 = x-y$

$x=y$