Εκθετικά - Λογάριθμοι

Ιδιότητες δυνάμεων

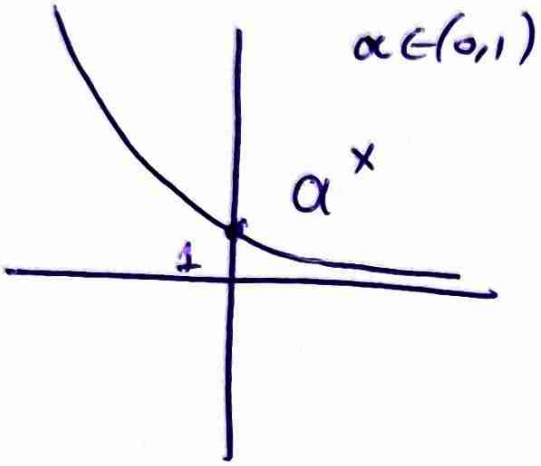
1. $\alpha^{x_1} \cdot \alpha^{x_2} = \alpha^{x_1+x_2}$
2. $\alpha^{x_1} : \alpha^{x_2} = \alpha^{x_1-x_2}$
3. $\frac{\alpha^{x_1}}{\alpha^{x_2}} = \alpha^{x_1-x_2}$
4. $(\alpha^{x_1})^{x_2} = \alpha^{x_1 x_2}$
5. $\alpha^0 = 1$
6. $\alpha^1 = \alpha$
7. $1^\alpha = 1$
8. $0^\alpha = 0$

Ρίζες

1. $\sqrt[\mu]{x^\nu} = x^{\nu/\mu}$ και $\sqrt{x} = x^{1/2}$
2. $\left(\frac{\alpha}{\beta}\right)^{-\nu} = \left(\frac{\beta}{\alpha}\right)^{\nu}$
3. $\alpha^{-\nu} = \frac{1}{\alpha^\nu}$

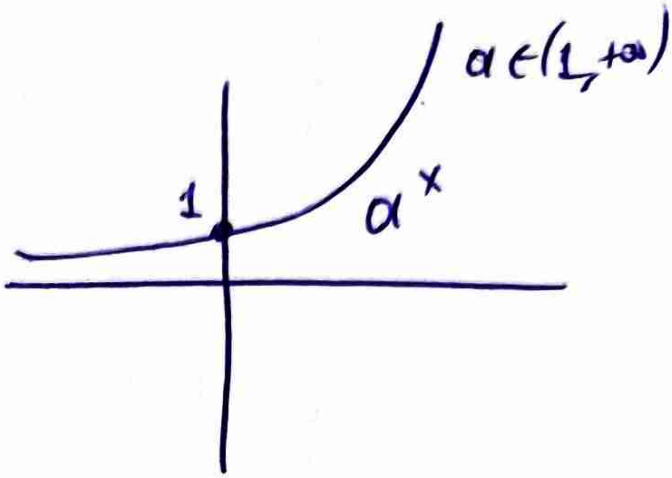
Εκθετικη συναρτηση

$$f(x) = a^x \quad , a \in (0,1) \cup (1, +\infty)$$



Παρατηρησεις

1. $D_f = R$
2. $\Sigma T_f = (0, +\infty)$
3. $a^x > 0$
4. $f \downarrow$



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ΠΡΟΣΟΧΗ

1. Η $f(x) = \alpha^x$ ορίζεται σε όλο το $\mathbb{R}$ αν $\alpha > 0$

2. Η $f(x) = \alpha^x$ είναι εκθετική αν $\alpha \in (0,1) \cup (1,+\infty)$.

3. Η $f(x)$ ↗ αν $\alpha \in (1,+\infty)$

4. Η $f(x)$ ↘ αν $\alpha \in (0,1)$.

Εκθετικές Εξισώσεις

1. $3^{x^2-1} = 9^{x-1}$

$3^{x^2-1} = (3^2)^{x-1}$

$3^{x^2-1} = 3^{2x-2}$

$x^2-1 = 2x-2$

$x^2-2x+1 = 0$

$x = 1$

2. $3^{x^3} = 81^x \cdot 27^{x+2}$

$3^{x^3} = (3^4)^x \cdot (3^3)^{x+2}$

$3^{x^3} = 3^{4x} \cdot 3^{3x+6}$

$3^{x^3} = 3^{7x+6} \Rightarrow x^3 = 7x+6$

$$x^3 - 7x - 6 = 0$$

1	0	−7	−6	(−1)
↓	−1	1	6	
1	−1	−6	0	

$$(x+1)(x^2 - x - 6) = 0$$

$x+1=0$ ve $x^2 - x - 6 = 0$

$x = -1$ $x = 3$ $x = -2$

3. $3^{x+2} + 3^{1-x} = 28$

$3^x \cdot 3^2 + 3^1 \cdot 3^{-x} = 28$

$3^x \cdot 9 + 3 \cdot \frac{1}{3^x} = 28$ $\qquad$ $3^x = t$

$9t + 3 \cdot \frac{1}{t} = 28$

$9t^2 + 3 = 28t$

$9t^2 - 28t + 3 = 0$

$\Delta = 676$

$t = \frac{28 \pm 26}{18}$ ⟨ 3, $\frac{1}{9}$

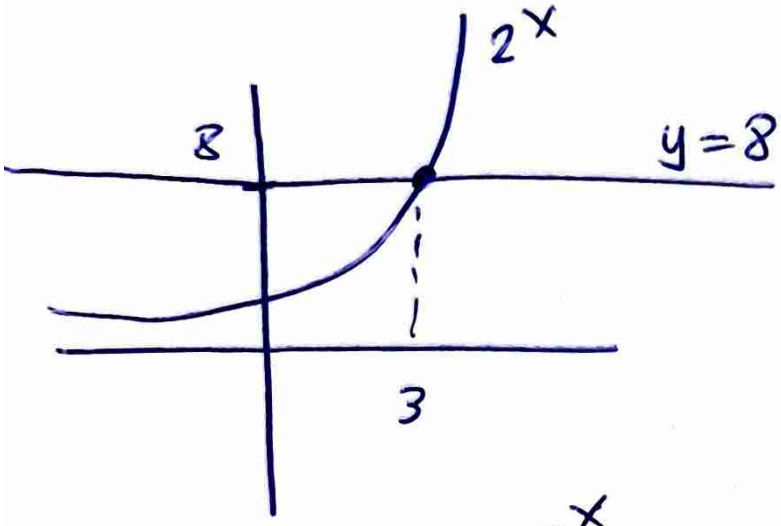
$t = 3$ ή $t = \frac{1}{9}$

$3^x = 3$ $\qquad$ $3^x = \frac{1}{9}$

$3^x = 3^1$ $\qquad$ $3^x = 3^{-2}$

$x = 1$ $\qquad$ $x = -2$

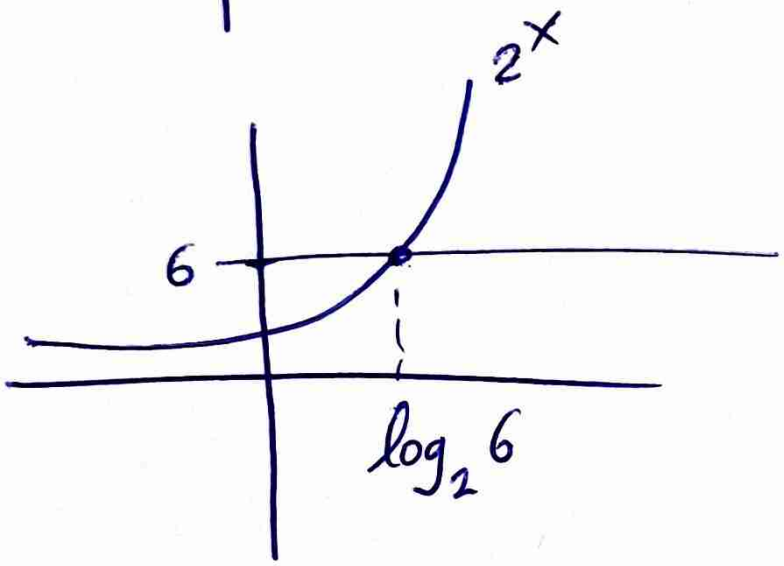
Λογαριθμοι



$2^x=8$

$2^x=2^3$

$x=3$



$2^x=6$

Γενικα

$$a^x=\Theta \iff x=\log_a \Theta$$

Ιδιότητες λογαρίθμων

1. $\log_\alpha \alpha^x = x$

2. $\alpha^{\log_\alpha x} = x$

3. $\log_\alpha 1 = 0$

4. $\log_\alpha \alpha = 1$

5. $\log_a (\theta_1 \cdot \theta_2) = \log_a \theta_1 + \log_a \theta_2$

6. $\log_a \left(\frac{\theta_1}{\theta_2}\right) = \log_a \theta_1 - \log_a \theta_2$

7. $\log_\alpha \theta^k = k \log_a \theta$.

Διαφορετικές

$\log_{10} x = \log x$

$10^x = \theta \iff x = \log \theta$

1. $\log 10^x = x$
2. $10^{\log x} = x$
3. $\log 1 = 0$
4. $\log 10 = 1$
5. $\log(\theta_1 \theta_2) = \log \theta_1 + \log \theta_2$
6. $\log\left(\frac{\theta_1}{\theta_2}\right) = \log \theta_1 - \log \theta_2$
7. $\log \theta^k = k \log \theta$

$\log_e x = \ln x$

$e^x = \theta \iff x = \ln \theta$

1. $\ln e^x = x$
2. $e^{\ln x} = x$
3. $\ln 1 = 0$
4. $\ln e = 1$
5. $\ln(\theta_1 \theta_2) = \ln \theta_1 + \ln \theta_2$
6. $\ln \frac{\theta_1}{\theta_2} = \ln \theta_1 - \ln \theta_2$
7. $\ln \theta^k = k \ln \theta$

Ενότητα 24

1. ⓐ $\log_2 8 = \log_2 2^3 = 3$

ⓑ $\log_3 81 = \log_3 3^4 = 4$

ⓒ $\log_2 \frac{1}{2} = \log_2 2^{-1} = -1$

ⓓ $\log_3 \frac{1}{9} = \log_3 3^{-2} = -2$

ⓔ $\log_7 \sqrt{7} = \log_7 7^{1/2} = \frac{1}{2}$

ⓕ $\log_5 \frac{1}{\sqrt{5}} = \log_5 \frac{1}{5^{1/2}} = \log_5 5^{-\frac{1}{2}} = -\frac{1}{2}$

3. ⓐ $\ln e^2 = 2$ ⓑ $\ln \frac{1}{e} = \ln e^{-1} = -1$

ⓒ $\ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2}$ ⓓ $\ln(e\sqrt{e}) = \ln(e^1 e^{1/2}) = \ln e^{3/2} = \frac{3}{2}$

ⓔ $\ln \frac{1}{\sqrt[3]{e^2}} = \ln \frac{1}{e^{2/3}} = \ln e^{-\frac{2}{3}} = -\frac{2}{3}$

ⓕ $\ln \frac{\sqrt[3]{e}}{e} = \ln \frac{e^{1/3}}{e^1} = \ln e^{\frac{1}{3}-1} = \frac{1}{3} - \frac{3}{3} = -\frac{2}{3}$

(7) $\ln^2 \frac{1}{e^2} = \left(\ln \frac{1}{e^2}\right)^2 = \left(\ln e^{-2}\right)^2 = (-2)^2 = 4$

(8) $\log 100 = \log 10^2 = 2$

(9) $\log \frac{1}{1000} = \log \frac{1}{10^3} = \log 10^{-3} = -3$

(10) $\log 0{,}1 = \log \frac{1}{10} = \log 10^{-1} = -1$

(11) $\log \sqrt{10} = \log 10^{1/2} = \frac{1}{2}$

(12) $\log 10\sqrt{10} = \log 10 \cdot 10^{1/2} = \log 10^{\frac{3}{2}} = \frac{3}{2}$

$$\frac{\sum \times \circ \lambda 10}{\ln^2 x = (\ln x)^2}$$

5. (a) $3^{\log_3 5} = 5$

(β) $10^{\log 3} = 3$

(γ) $e^{\ln 5} = 5$

(δ) $\log(\ln e^{100}) = \log(100) =$

$= \log 10^2 = 2$

(ε) $\ln(\log 10^e) = \ln e = 1$

(ϛ) $\log(\log \sqrt[10]{10}) =$

$= \log(\log 10^{1/10}) =$

$= \log\left(\frac{1}{10}\right) = \log 10^{-1} = -1$