52. $f(x) = a \ln \frac{5+x}{5-x}$ $A(3, 4\ln\sqrt{2})$

ⓐ πρπι $\frac{5+x}{5-x} > 0$ και $5-x \neq 0$

$x \neq 5$

x		-5		5	
5+x	-	0	+		+
5-x	+		+	0	-
P(x)	-	0	+	0	-

$x \in (-5, 5)$

$A_f = (-5, 5)$

$f(3) = a \cdot \ln \frac{8}{2} = 4 \ln\sqrt{2}$

$a \ln 4 = \ln \sqrt{2}^4$

$\ln 4^a = \ln 4$

$a \ln 4 = \ln 4$

$a = 1$

$f(x) = \ln \frac{5+x}{5-x}$

⑧ $\ln\sqrt{x} = (\alpha + t/0)\sqrt{\ln x}$

$\ln x^{1/2} = (1 + 0)\sqrt{\ln x}$

$\frac{1}{2}\ln x = \sqrt{\ln x}$

$\sqrt{\ln x} = t$

$\ln x = t^2$

$\frac{1}{2}t^2 = t$

$t^2 = 2t$

$t^2 - 2t = 0$

$\Rightarrow t(t-2) = 0$

$t = 0$ $\quad$ $t = 2$

$\sqrt{\ln x} = 0$ $\quad$ $\sqrt{\ln x} = 2$

$\ln x = 0$ $\quad$ $\ln x = 4$

$e^{\ln x} = e^0$ $\quad$ $e^{\ln x} = e^4$

$x = 1$ ✓ $\quad$ $x = e^4$ ✓

(δ) $B, \gamma > 0 \qquad B \neq \gamma \qquad B^B \gamma^\gamma > 4e^{+|-3|} B^\gamma \gamma^B$

$$B^B \gamma^\gamma > 4e^{\ln \frac{1}{4}} B^\gamma \gamma^B$$

$$B^B \gamma^\gamma > B^\gamma \gamma^B$$

$$\frac{B^B}{\gamma^B} > \frac{B^\gamma}{\gamma^\gamma}$$

$$\left(\frac{B}{\gamma}\right)^B > \left(\frac{B}{\gamma}\right)^\gamma$$

Αν $B > \gamma$ τοτε $\frac{B}{\gamma} > 1 \Rightarrow B > \gamma$ ✓

Αν $B < \gamma$ τοτε $\frac{B}{\gamma} < 1 \Rightarrow B < \gamma$ ✓

54. $f(x) = (3-2\ln a)^x$

(α) Για να ορίζεται στο $\mathbb{R}$ πρέπει

$3-2\ln a > 0 \Rightarrow 3 > 2\ln a$

$\frac{3}{2} > \ln a$ 3/2

$a < e$

(β) Για να είναι $f \downarrow$

πρέπει $0 < 3-2\ln a < 1$

$0 < 3-2\ln a$ και $3-2\ln a < 1$

$2\ln a < 3$ $-2\ln a < -2$

$\ln a < \frac{3}{2}$ $\ln a > 1$

$a < e^{3/2}$ $a > e$

$a \in (e, e^{3/2})$

Σχολιο

$f(x) = a^x$

1. Ορίζεται στο $\mathbb{R}$ αν $a > 0$
2. εκθετική $a \in (0,1) \cup (1, +\infty)$
3. $f \uparrow$ αν $a > 1$
4. $f \downarrow$ αν $0 < a < 1$

⑧ Για $a = e\sqrt[4]{e}$

$$f(x) = \left(3 - 2\ln\left(e\sqrt[4]{e}\right)\right)^x$$

$$\ln\left(e\cdot\sqrt[4]{e}\right) = \ln e + \ln\sqrt[4]{e} =$$

$$= 1 + \ln e^{1/4} = 1 + \frac{1}{4} = \frac{5}{4}$$

$$f(x) = \left(3 - 2\cdot\frac{5}{4}\right)^x = \left(3 - \frac{5}{2}\right)^x = \left(\frac{1}{2}\right)^x$$

$$\boxed{f(x) = \left(\frac{1}{2}\right)^x}$$

i) $2f(2x) - f(x) < 1$

$$2\left(\frac{1}{2}\right)^{2x} - \left(\frac{1}{2}\right)^x < 1$$

$\left(\frac{1}{2}\right)^x = \lambda$

$2\lambda^2 - \lambda - 1 < 0$

$\Delta = 1 + 8 = 9$

$\lambda = \frac{1 \pm 3}{4}$ ⟨ 1, $\frac{1}{2}$

λ		$-\frac{1}{2}$		1
$2\lambda^2-\lambda-1$	+	0	−	0	+

$\lambda \in (-\frac{1}{2}, 1)$.

$-\frac{1}{2} < 1 < 1$

$-\frac{1}{2} < \left(\frac{1}{2}\right)^x < 1$

$\left(\frac{1}{2}\right)^x < \left(\frac{1}{2}\right)^0$

$x > 0$

ii) $f(3x) + \frac{f(x)}{9^x} = \frac{2}{27^x}$

$\left(\frac{1}{2}\right)^{3x} + \frac{\left(\frac{1}{2}\right)^x}{(3^x)^2} = \frac{2}{(3^x)^3}$

$\left(\frac{1}{2}\right)^x = a$

$3^x = b$

$a^3 + \frac{a}{b^2} = \frac{2}{b^3}$

$a^3 b^3 + ab = 2$

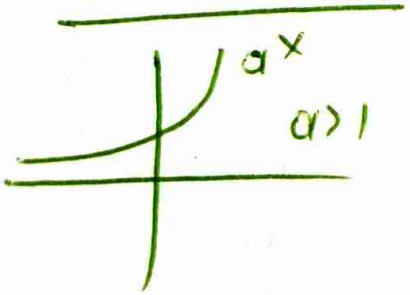
$a^3 b^3 + ab - 2 = 0$

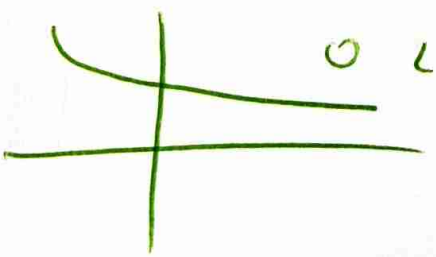
Θέτω $ab = t$

$t^3 + t - 2 = 0$

1	0	1	-2	(1)
↓	1	1	2	
1	1	2	0	

Σχόλιο





$$(t-1)(t^2+t+2)=0$$

$t=1$

$$ab=1$$

$$\left(\frac{1}{2}\right)^x \cdot 3^x = 1$$

$$\left(\frac{3}{2}\right)^x = 1$$

$$\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^0$$

$x=0$

50. $f(x) = a \cdot \dfrac{e^x+1}{e^x-1}$ $A(\ln 2, 3)$

(α) Πρεπει $e^x - 1 \neq 0 \Rightarrow e^x \neq 1 \Rightarrow \underline{x \neq 0}$

$D_f = R^*$

$f(\ln 2) = a \cdot \dfrac{e^{\ln 2}+1}{e^{\ln 2}-1} = 3$

$f(x) = \dfrac{e^x+1}{e^x-1}$
$D_f = R^*$

$\Rightarrow a \dfrac{2+1}{2-1} = 3 \Leftrightarrow 3a = 3 \Rightarrow \underline{a=1}$

(β) Το $x \in D_f$ και $-x \in D_f$.

$$f(-x) = \frac{e^{-x}+1}{e^{-x}-1} = \frac{\frac{1}{e^x}+1}{\frac{1}{e^x}-1} = \frac{1+e^x}{1-e^x} =$$

$$= -\frac{1+e^x}{e^x-1} = -f(x)$$

περιττη

(γ) Αντιστοιχη $f(x) \geq 2$

$\dfrac{e^x+1}{e^x-1} \geq 2$ Θετω $e^x = \lambda$

$\dfrac{\lambda+1}{\lambda-1} \geq 2 \Rightarrow \dfrac{\lambda+1}{\lambda-1} - 2 \geq 0 \Rightarrow \dfrac{\lambda+1-2(\lambda-1)}{\lambda-1} \geq 0$

$$\frac{3-\lambda}{\lambda-1} \geq 0$$

λ		1		3	
$3-\lambda$	+		+		−
$\lambda-1$	−		+		+
$P(\lambda)$	−	‖	+		−

$$\lambda \in (1,3]$$

$$1 < \lambda \leq 3$$

$$1 < e^x \leq 3$$

$$\ln 1 < \ln e^x \leq \ln 3$$

$$\boxed{0 < x \leq \ln 3}$$

⑧ Ndo $f(x) = 1 + \frac{2}{e^x - 1}$

$$f(x) = 1 + \frac{2}{e^x-1} = \frac{e^x-1}{e^x-1} + \frac{2}{e^x-1} = \frac{e^x+1}{e^x-1} = f(x)$$

- $x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2} \Rightarrow e^{x_1} - 1 < e^{x_2} - 1$

$$\frac{1}{e^{x_1}-1} > \frac{1}{e^{x_2}-1} \Longrightarrow$$

$\longrightarrow$ $x > 0 \Rightarrow e^x > e^0 \Rightarrow e^x > 1 \Rightarrow e^x - 1 > 0$

$$\frac{2}{e^{x_1}-1} > \frac{2}{e^{x_2}-1}$$

$$1+\frac{2}{e^{x_1}-1} > 1+\frac{2}{e^{x_2}-1}$$

$$f(x_1) > f(x_2)$$

f is $\searrow$ on $(0,+\infty)$.

Ενοτητα 26

1. (α) $\ln x = 0$

$e^{\ln x} = e^0$

$\boxed{x = 1}$

(β) $\ln x + 1 = 0$

$\ln x = -1$

$e^{\ln x} = e^{-1}$

$\boxed{x = \frac{1}{e}}$

(ε) $2\ln x + 3 = 0$

$2\ln x = -3$

$\ln x = -\frac{3}{2}$

$e^{\ln x} = e^{-\frac{3}{2}}$

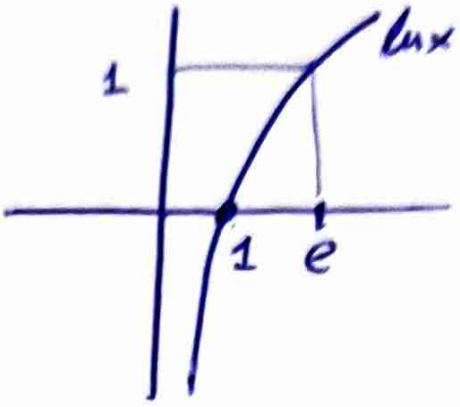
$x = \frac{1}{e^{3/2}} = \frac{1}{\sqrt{e^3}} = \frac{1}{\sqrt{e^2}\sqrt{e}} =$

$x = \frac{1}{e\sqrt{e}} =$

8. (α) $e^x = 3$

$\ln e^x = \ln 3$

$x = \ln 3$



(β) $3e^x - 1 = 0$

$3e^x = 1$

$e^x = \frac{1}{3}$

$\ln e^x = \ln \frac{1}{3}$

$x = \ln 1 - \ln 3$

$x = -\ln 3$

9. (α) $x + \ln(e^x - 1) = \ln 20$

$\ln e^x + \ln(e^x - 1) = \ln 20$

$\ln e^x(e^x - 1) = \ln 20$

$e^x(e^x - 1) = 20$ $\quad e^x = t$

$t(t-1) = 20 \Rightarrow t^2 - t - 20 = 0$

$e^x = 5 \Rightarrow x = \ln 5 \cdot t = 5$ $\quad t = -4$ $e^x \neq 4$

Εποχαο Μαθηρ

26

(2) α γ ε.

Σελ 355

(53) (55)