Ενότητα 23

1. $f(x) = 2x^2 - x - 1$

(α) $D_f = \mathbb{R}$

(β) $f(0) = 2 \cdot 0^2 - 0 - 1$

$f(0) = -1$

$f(f(1)) = f(0)$

$= -1$

$f(1) = 2 \cdot 1^2 - 1 - 1$

$f(1) = 0$

$f(3\alpha) = 2 \cdot (3\alpha)^2 - 3\alpha - 1$

$f(3a) = 2 \cdot 9a^2 - 3a - 1$

$f(3a) = 18a^2 - 3a - 1$

$f(x-2) = 2(x-2)^2 - (x-2) - 1$

$f(x-2) = 2 \cdot (x^2 - 4x + 4) - x + 2 - 1 = 2x^2 - 9x + 9$

3. $f(x) = x^2 - ax + B$

$f(0) = -1$

$f(-1) = -4$

} Nδo $a = -4$, $B = -1$

$\longrightarrow f(0) = 0^2 - a \cdot 0 + B$

$-1 = B$

$\longrightarrow f(-1) = (-1)^2 - a(-1) + B$

$-4 = 1 + a - B$

$a = -4$

9. ⓐ $f(x) = \frac{1}{|x|-1}$

Prin $|x|-1 \neq 0$

$\rightarrow |x|-1=0$

$|x|=1$

$x=1$ ν $x=-1$

$D_f = \mathbb{R} - \{1, -1\}$.

ⓑ $f(x) = \frac{2}{|x-1|-3}$

prin $|x-1|-3 \neq 0$

$D_f = \mathbb{R} - \{4, -2\}$

$\rightarrow |x-1|-3=0$

$|x-1|=3$

$x-1=3$ ν $x-1=-3$

$x=4$ $x=-2$

8. (B) $f(x) = \frac{x-1}{x-2}$

pripi $x-2 \neq 0$

$\rightarrow x-2=0$

$\boxed{x=2}$

$A_f = R-\{2\}$.

(Y) $f(x) = \frac{x}{x^2-1}$

pripi $x^2-1 \neq 0$

$\rightarrow x^2-1=0$

$x^2=1$

$x=1$ vi $x=-1$

$A_f = R-\{1,-1\}$.

(E) $f(x) = \frac{x}{x^3-1}$

pripi $x^3-1 \neq 0$

$\rightarrow x^3-1=0$

$x^3=1$

$x=1$

$A_f = R-\{1\}$.

11. ⓐ $f(x)=\sqrt{x-1}+\frac{1}{x-2}$

πρεπει $x-1\geq 0$ και $x-2\neq 0$

$x\neq 2$

$x\geq 1$

$$D_f=[1,2)\cup(2,+\infty)$$

ⓑ $H(x)=\frac{\sqrt{4-x^2}}{x}$

πρεπει $4-x^2\geq 0$ και $x\neq 0$

x		-2		2
$4-x^2$	$-$	0	$+$	0	$-$

$x\in[-2,2]$

$$D_f=[-2,0)\cup(0,2].$$

(ε) $f(x) = \dfrac{x}{\sqrt{x-1}-3}$

πρεπει $x-1 \geq 0$ και $\sqrt{x-1}-3 \neq 0$

$x \geq 1$ $\sqrt{x-1} \neq 3$

$x-1 \neq 9$

$D_f = [1,10) \cup (10,+\infty)$ $x \neq 10$

10. ⓐ $f(x) = \sqrt{x-2}$

πρεπι $x-2 \geq 0 \Rightarrow x \geq 2$

$D_f = [2, +\infty)$

ⓑ $f(x) = \sqrt{x^2-x-6}$

πρεπι $x^2-x-6 \geq 0$

x		-2		3
x^2-x-6	$+$	$-$		$+$

$D_f = (-\infty, -2] \cup [3, +\infty)$

ⓒ $f(x) = \sqrt{x-x^2}$

πρεπι $x-x^2 \geq 0$

x		0		1
$x-x^2$	$-$	$+$		$-$

$D_f = [0, 1]$

ⓔ $f(x) = \sqrt{|x|^3-1}$

πεσπι $|x|^3-1 \geq 0$

$|x|^3 \geq 1^3$

$|x| \geq 1$

$x \geq 1$ ή $x \leq -1$

$D_f = (-\infty, -1] \cup [1, +\infty)$

extra

Έστω $f(x) = x^2$

Να δείξετε ότι είναι άρτια

1. $f(-x) = +4$

$\rightarrow f(-x) = (-x)^2 = x^2$

$x^2 = 4 \Rightarrow x = 2$ ή $x = -2$

2. $f(x-1) - f(x+1) = 2(x-2)$

$f(x-1) = (x-1)^2 = x^2 - 2x + 1$

$f(x+1) = (x+1)^2 = x^2 + 2x + 1$

$x^2 - 2x + 1 - (x^2 + 2x + 1) = 2x - 4$

$x^2 - 2x + 1 - x^2 - 2x - 1 = 2x - 4$

$-4x = 2x - 4 \Rightarrow -6x = -4$

$x = \frac{4}{6}$

extra

$$f(x) = \begin{cases} x^2-2x, & x \le 1 \\ 2x+4, & x>1 \end{cases}$$

$f(-1) = (-1)^2 - 2(-1) = 1+2 = 3$

$f(1) = 1^2 - 2\cdot 1 = 1-2 = -1$

$f(2) = 2\cdot 2 + 4 = 8$.

$f(x^2+2) = 2(x^2+2)+4 = 2x^2+8$.

→ $x^2 \ge 0 \Rightarrow \underline{x^2+2 \ge 2}$

Ας υποθέσουμε ότι θέλω να λύσω την εξίσωση $f(x) = 3$

$\underline{x \le 1}$

$f(x) = 3$

$x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$

$x = 3$ ✗ $x = -1$ ✓

$\underline{x > 1}$

$f(x) = 3$

$2x + 4 = 3$

$2x = -1$

$x = -\frac{1}{2}$ ✗

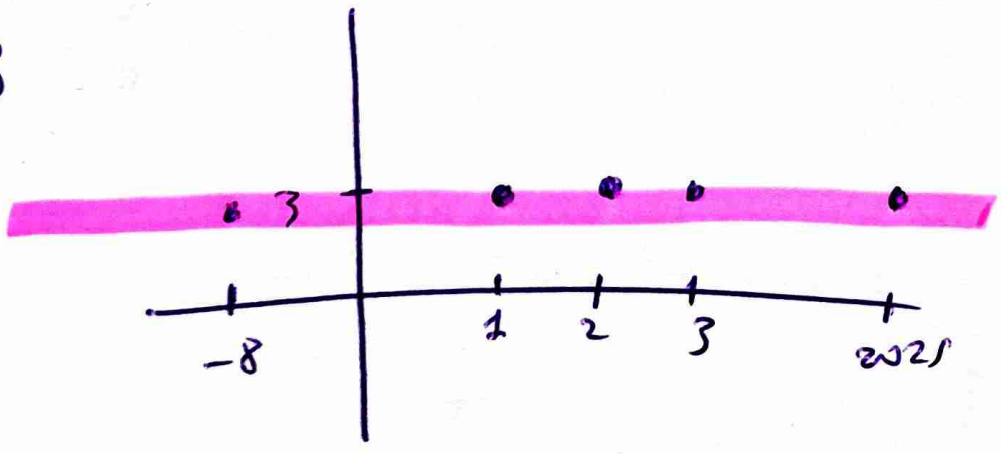
23.

$$f(x) = \begin{cases} x, & x \le 0 \\ \frac{2}{x}, & 0 < x < 1 \\ 3, & x \ge 1 \end{cases}$$

- $f(-5) = -5$
- $f(0) = 0$
- $f\left(\frac{1}{2}\right) = \frac{2}{\frac{1}{2}} = 4$
- $f(1) = 3$
- $f(2) = 3$.

$$\sum_{x=2}^{10}$$

$f(x) = 3$



(α) $f(-x^2) = -x^2$

(β) $f(x^2+1) = 3$

(γ) $f\left(\frac{1}{x}\right) = \frac{2}{\frac{1}{x}} = 2x$

όταν $x>1$.

Όταν $x>1$ τότε $0<\frac{1}{x}<1$

(δ) $f(1-2x) = 3$

• $x \le 0 \Rightarrow -2x \ge 0 \Rightarrow 1-2x \ge 1$

(ε) $f\left(x+\frac{1}{x}\right) = 3$.

• $x>0$

• $x+\frac{1}{x} \ge 2 \Rightarrow x^2+1 \ge 2x \Rightarrow x^2-2x+1 \ge 0$

$x>0$

$(x-1)^2 \ge 0$ ✓

28. $f(x) = \begin{cases} \frac{3}{x} - 1, & 0 < x \leq 1 \\ x+1, & x > 1 \end{cases}$

(α) $f\left(\frac{1}{x^2+1}\right) = f(x^2+3)$ εξίσωση

- $x^2 \geq 0 \Rightarrow x^2+1 \geq 1$

$$\frac{1}{x^2+1} \leq 1$$

- $x^2 \geq 0 \Rightarrow x^2+3 \geq 3$

$$\frac{3}{\frac{1}{x^2+1}} - 1 = x^2+3+1$$

$$3(x^2+1) - 1 = x^2+4$$

$$3x^2+3-1 = x^2+4$$

$$2x^2+2-4=0$$

$$2x^2-2=0$$

$$x^2-1=0$$

$x=1$ $x=-1$

(B) $f\left(\frac{1}{x}\right) = 5$ $\quad x \geq 1$ $\quad [1, +\infty)$

- $x \geq 1 \Rightarrow \frac{1}{x} \leq 1$

$$\frac{3}{\frac{1}{x}} - 1 = 5$$

$$3x - 1 = 5$$

$$3x = 6$$

$$x = 2$$

(8) $f(3-2x) > 3$ $\quad (0, 1)$

$$0 < x < 1$$

$$0 > -2x > -2$$

$$3 > 3-2x > 1$$

$$3-2x+1 > 3$$

$$-2x+4 > 3$$

$$-2x > -1$$

$x \in \left(0, \frac{1}{2}\right)$

$x < \frac{1}{2}$

27.

$$f(x) = \begin{cases} \frac{ax}{x-3}, & x < 2 \\ \frac{ax}{2} - 1, & x \geq 2 \end{cases}$$

(a)

$f(3) = 4f(-1)$

- $f(3) = \frac{a \cdot 3}{2} - 1 = \frac{3}{2}a - 1$
- $f(-1) = \frac{a(-1)}{-1-3} = \frac{-a}{-4} = \frac{a}{4}$

$\frac{3}{2}a - 1 = 4 \left| -\frac{a}{4} \right.$

$\frac{3}{2}a - 1 = -a$

$3a - 2 = -2a$

$a = 2$

$a = 2.$

$$f(x) = \begin{cases} \frac{2x}{x-3}, & x < 2 \\ x - 1, & x \geq 2 \end{cases}$$

(β) $f(|x|+2) = 3$

$|x|+2-1 = 3$

$|x| + 1 = 3$

$|x| = 2$

$x = 2$ ή $x = -2$

(γ) $f(1-x^2) = 1$

$$\frac{2(1-x^2)}{1-x^2-3} = 1 \Rightarrow \frac{2-2x^2}{-x^2-2} = 1$$

$2 - 2x^2 = -x^2 - 2$

$-2x^2 + x^2 = -2 - 2$

$-x^2 = -4$

$x^2 = 4$

$x = 2$ $x = -2$

24. (α) $f(x) = \sqrt{x^2 - \lambda x + 1}$

Έχει πεδίο ορισμού το $\mathbb{R}$.

Βρίσκω λ.

Πρέπει $x^2 - \lambda x + 1 \geq 0$

$\Delta \leq 0$ και $1 > 0$ ✓

$\lambda^2 - 4 \leq 0$

λ		-2		2
$\lambda^2 - 4$	$+$	$-$	$+$	

$\lambda \in [-2, 2]$

(B) $H(x) = \dfrac{x}{3x^2 - 2\lambda x + 3}$

при $3x^2 - 2\lambda x + 3 \neq 0$

$\Delta < 0$

$B^2 - 4ac < 0$

$(-2\lambda)^2 - 4 \cdot 3 \cdot 3 < 0$

$4\lambda^2 - 36 < 0$

$\lambda^2 - 9 < 0$

$\lambda \in (-3, 3)$.

Επομενο Μαθημα

23.

4
5
6
7