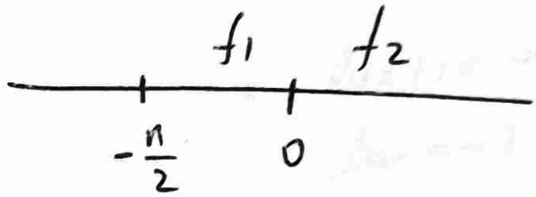
Ενοτητα 24

71. $f(x) = \begin{cases} \dfrac{\varepsilon\varphi x}{x} + \alpha, & x \in \left(-\frac{\pi}{2}, 0\right) \\ 1, & x = 0 \\ x^x, & x > 0 \end{cases}$

συνεχης

ⓐ



• $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \left(\dfrac{\varepsilon\varphi x}{x} + \alpha \right) = 1 + \alpha$

$\longrightarrow \lim\limits_{x \to 0^-} \dfrac{\varepsilon\varphi x}{x} = \lim\limits_{x \to 0^-} \dfrac{\frac{1}{\sigma\upsilon\nu^2 x}}{1} = 1$

• $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} x^x = \lim\limits_{x \to 0^+} e^{\ln x^x}$

$= \lim\limits_{x \to 0^+} e^{x \ln x} = e^0 = 1.$

$\longrightarrow \lim\limits_{x \to 0^+} x \ln x = \lim\limits_{x \to 0^+} \dfrac{\ln x}{\frac{1}{x}} = \lim\limits_{x \to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = 0.$

Πρέπει $1+\alpha=1 \Rightarrow \underline{\underline{\alpha=0}}$

⑬ $-\frac{\pi}{2}<x<0$

$$f_1(x)=\frac{\varepsilon\varphi x}{x}$$

$$f_1'(x)=\frac{\frac{1}{\sigma\upsilon\nu^2 x}x-\varepsilon\varphi x}{x^2}$$

$$f_1'(x)=\frac{x-\varepsilon\varphi x\,\sigma\upsilon\nu^2 x}{x^2\sigma\upsilon\nu^2 x}$$

$$f_1'(x)=\frac{x-\frac{\eta\mu x}{\sigma\upsilon\nu x}\sigma\upsilon\nu^2 x}{x^2\sigma\upsilon\nu^2 x}$$

$$f_1'(x)=\frac{x-\eta\mu x\,\sigma\upsilon\nu x}{x^2\sigma\upsilon\nu^2 x}$$

$x>0$

$$f_2(x)=x^x$$

$$f_2(x)=e^{x\ln x}$$

$$f_2'(x)=e^{x\ln x}(\ln x+1)$$

$\rightarrow \ln x+1=0$

$\ln x=-1$

$x=\frac{1}{e}$

$$\boxed{g(x) = x - \eta\mu x\,\sigma\upsilon\nu x} \qquad g(0)=0$$

$$g'(x) = 1 - \sigma\upsilon\nu^2 x + \eta\mu^2 x$$

$$g'(x) = 2\eta\mu^2 x \geq 0$$

x	$-\frac{\pi}{2}$ 0
g'	$+$
g	$\nearrow$ $-$
f_1'	$-$

$x<0$

$g \nearrow$

$g(x) < g(0)$

$g(x) < 0$

Τελικό πινακάκι μονοτονίας

x	$-\pi/2$ … 0	0 … $1/e$	$1/e$ … $+\infty$
f_1'	$-$	//////	//
f_2'	//////	$-$	0 $+$
f'	$-$	$-$	$+$
f	$\searrow$	$\searrow$	$\nearrow$

(γ) $f(x) = e^{1-x^2}$ $(0,+\infty)$

Προφανή ρίζα το $\underline{x=1}$

• $x^2 \geq 0 \Rightarrow -x^2 \leq 0$

$1-x^2 \leq 1$

$e^{1-x^2} \leq e$

$f(x) = e^{1-x^2}$ $\Rightarrow x^x = e^{1-x^2}$

$\ln x^x = \ln e^{1-x^2}$

$x \ln x = 1-x^2$

$\ln x = \frac{1}{x} - x$

$\underbrace{\ln x - \frac{1}{x} + x}_{h(x)} = 0$

$h'(x) = \frac{1}{x} + \frac{1}{x^2} + 1 > 0$

$h(x) = 0$

$h(x) = h(1)$

h 1-1

$h \nearrow$

$\underline{x=1}$

(δ) $f\left(-\frac{\pi x}{2}\right) = f\left(-\frac{\pi}{2e^x}\right)$ $\quad (0,1)$

- $0 < x < 1 \Rightarrow 0 > -\frac{\pi x}{2} > -\frac{\pi}{2}$
- $0 < x < 1 \Rightarrow e^0 < e^x < e^1 \Rightarrow 1 < e^x < e$

$$\Rightarrow 1 > \frac{1}{e^x} > \frac{1}{e}$$

$$-\frac{\pi}{2} < -\frac{\pi}{2e^x} < -\frac{\pi}{2e}$$

Όταν $x \in \left(-\frac{\pi}{2}, 0\right)$ η $f \downarrow$.

$$-\frac{\cancel{\pi} x}{\cancel{2}} = -\frac{\cancel{\pi}}{\cancel{2} e^x}$$

$$x = \frac{1}{e^x} \iff x - \frac{1}{e^x} = 0$$

$$x - e^{-x} = 0$$

$\varphi(x) = x - e^{-x}$ $\qquad \varphi \nearrow$

$\varphi(0) = -1 < 0$

$\varphi(1) = 1 - \frac{1}{e} > 0$

$\varphi(0)\varphi(1) < 0$

Bolzano $\exists\, \xi \in (0,1)$

T.w $\varphi(\xi) = 0$

65. $f(x) = e^x + x - 1$

$F(x) = f(2x) - f(x)\ ,\ x \geq 0$

(α) $f'(x) = e^x + 1$

$f''(x) = e^x > 0 \qquad f' \nearrow$

$$F'(x) = 2f'(2x) - f'(x) = \underset{(+)}{f'(2x)} + \underbrace{f'(2x) - f'(x)}_{\oplus}$$

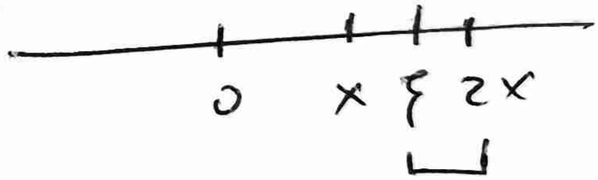
• $x < 2x \Rightarrow f'(x) < f'(2x)$

$f'(2x) - f'(x) > 0$

$F'(x) > 0 \qquad F \nearrow$

(β) ΝΔο $F(x) < x\, f'(2x) \quad \forall x > 0$.

$$f'(\xi) = \frac{f(2x) - f(x)}{2x - x} = \frac{F(x)}{x}$$



$\xi < 2x$

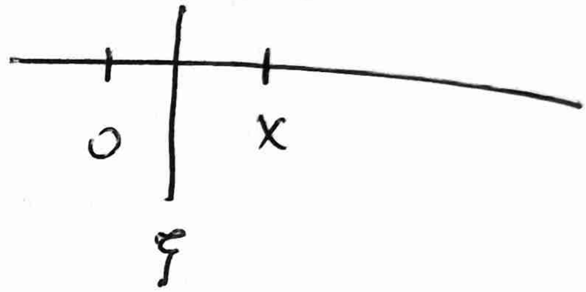
$f'(\xi) < f'(2x)$

$\frac{F(x)}{x} < f'(2x) \qquad F(x) < x\, f'(2x).$

(8) $g(x) = \begin{cases} \dfrac{F(x)}{x}, & x > 0 \\ 2, & x = 0 \end{cases}$

$$g'(x) = \frac{F'(x)\,x - F(x)}{x^2}$$

Αρκει νδο $F'(x)\,x - F(x) > 0$



$$F'(\xi) = \frac{F(x) - F(0)}{x - 0}$$

$$F'(\xi) = \frac{F(x) - 0}{x} = \frac{F(x)}{x}$$

$\xi < x$

$F'(\xi) < F'(x)$

$\dfrac{F(x)}{x} < F'(x) \Rightarrow 0 < F'(x)x - F(x)$

$F'(x) = 2f'(2x) - f'(x)$

$F''(x) = 4f''(2x) - f''(x) =$

$$= \underbrace{3f''(2x)}_{+} + \underbrace{f''(2x) - f''(x)}_{+}$$

$x < 2x \Rightarrow f''(x) < f''(2x)$

γcqu $f'''(x) = e^x$

$\Rightarrow f''(x) \nearrow$

$F' \nearrow$

(8) $f(2x^2) + f(x) < f(x^2) + f(2x)$

$$f(2x^2) - f(x^2) < f(2x) - f(x)$$

$$F(x^2) < F(x)$$

$$F \nearrow$$

$$x^2 < x$$

$$0 < x < 1$$