Ενότητα 3

2. (α) $f(x) = \frac{x^3+2x}{x^2+2}$ $D_f = \mathbb{R}$.

$g(x) = x$, $D_g = \mathbb{R}$.

$$f(x) = \frac{x^3+2x}{x^2+2} = \frac{x\cancel{(x^2+2)}}{\cancel{x^2+2}} = x = g(x).$$

Είναι ίσες!.

(γ) $f(x) = x - \ln(1+e^x)$ $D_f = \mathbb{R}$

$g(x) = \ln \frac{e^x}{1+e^x}$ $D_g = \mathbb{R}$.

$$g(x) = \ln \frac{e^x}{1+e^x} = \ln e^x - \ln(1+e^x) =$$
$$= x - \ln(1+e^x) = f(x).$$

Είναι ίσες!

3. (α) $f(x) = \sqrt{x-1}\,\sqrt{x-2}$

πρέπει $x-1 \geq 0$ και $x-2 \geq 0$

$\underline{\underline{x \geq 1}}$ $\underline{\underline{x \geq 2}}$

$$D_f = [2, +\infty)$$

$$g(x) = \sqrt{x^2-3x+2}$$

πρέπει $x^2-3x+2 \geq 0$

x		1		2	
x^2-3x+2	+	0	−	0	+

$$D_g = (-\infty, 1] \cup [2, +\infty).$$

$$f(x) = \sqrt{x-1}\,\sqrt{x-2} = \sqrt{(x-1)(x-2)}$$

$$= \sqrt{x^2-2x-x+2} = \sqrt{x^2-3x+2} = g(x)$$

Εχουν ιδιο τυπο

Δεν εχουν ιδιο πεδιο ορισμου.

Αρα ση γενικα δεν ειναι ισα.

αλλα στο $[2,+\infty)$ ταυτιζονται

ειναι ισα!.

Ανισώσεις 2ου Βαθμού

$$\alpha x^2 + \beta x + \gamma \geq 0 \ , \ \alpha > 0$$

→ Αν $\Delta > 0$ έχω 2 ρίζες x_1, x_2

x		x_1		x_2	
$\alpha x^2+\beta x+\gamma$	+	0	−	0	+

$x \in (-\infty, x_1] \cup [x_2, +\infty)$

→ Αν $\Delta = 0$ έχω 1 ρίζα διπλή x_1

x		x_1	
$\alpha x^2+\beta x+\gamma$	+	0	+

$x \in \mathbb{R}$.

→ Αν $\Delta < 0$ δεν έχω ρίζες.

x	
$\alpha x^2+\beta x+\gamma$	+

$x \in \mathbb{R}$.

3. (B) $f(x) = \ln x^2$

πρέπει $x^2 > 0$

$D_f = \mathbb{R}^*$

$g(x) = 2\ln x$

$D_g = (0, +\infty)$.

$g(x) = 2\ln x = \ln x^2 = f(x)$.

Έχουν ίδιο τύπο.

Εν γένει δεν είναι ίσες.

Είναι ίσες στο $(0, +\infty)$.

3. (β) $f(x) = \dfrac{x-2\sqrt{x}+1}{\sqrt{x}-1}$

πρέπει $x \geq 0$ και $\sqrt{x}-1 \neq 0$

$\sqrt{x} \neq 1$

$x \neq 1$.

$D_f = [0,1) \cup (1,+\infty)$

$g(x) = \sqrt{x}-1$

πρέπει $x \geq 0$

$D_g = [0,+\infty)$

$$f(x) = \frac{x-2\sqrt{x}+1}{\sqrt{x}-1} = \frac{\sqrt{x}^2-2\sqrt{x}+1}{\sqrt{x}-1} = \frac{(\sqrt{x}-1)^{\cancel{2}}}{\cancel{\sqrt{x}-1}}$$

$$= \sqrt{x}-1 = g(x).$$

Είναι ισα μονο στο $[0,1) \cup (1,+\infty)$.

5. $f(x) = \sqrt{1-x}$

πρέπει $1 - x \geq 0 \Rightarrow \underline{\underline{x \leq 1}}$

$D_f = (-\infty, 1]$.

$g(x) = \ln x$

$D_g = (0, +\infty)$

(α) $(f+g)(x) = f(x) + g(x) = \sqrt{1-x} + \ln x$

$D_{f+g} = D_f \cap D_g = (0, 1]$.

(β) $(f \cdot g)(x) = f(x) \cdot g(x) = \sqrt{1-x}\,\ln x$

$D_{f \cdot g} = D_f \cap D_g = (0, 1]$

(γ) $\left(\frac{1}{g}\right)(x) = \frac{1}{g(x)} = \frac{1}{\ln x}$ $\quad D_{1/g} = (0, 1) \cup (1, +\infty)$

$x \in D_g$ και $g(x) \neq 0$

$\underline{\underline{x > 0}}$ $\quad \ln x \neq 0 \Rightarrow e^{\ln x} \neq e^0$

$\underline{x \neq 1}$

(δ) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{1-x}}{\ln x}$

$x \in D_f \cap D_g$ και $g(x) \neq 0$

$x \in (0,1]$ $\quad$ $x \neq 1$

$D_{f/g} = (0,1)$.

6. $f(x) = \dfrac{1}{x-1}$ $\qquad D_f = \mathbb{R} - \{1\}$.

$g(x) = \dfrac{x}{x+1}$ $\qquad D_g = \mathbb{R} - \{-1\}$.

ⓐ $(f-g)(x) = f(x) - g(x) = \dfrac{1}{x-1} - \dfrac{x}{x+1} =$

$$= \frac{x+1}{(x-1)(x+1)} - \frac{x(x-1)}{(x-1)(x+1)}$$

$$= \frac{x+1-x^2+x}{(x-1)(x+1)} = \frac{-x^2+2x+1}{x^2-1}.$$

$D_{f-g} = D_f \cap D_g = \mathbb{R} - \{1,-1\}$.

ⓑ $(f \cdot g)(x) = f(x) \cdot g(x) = \dfrac{1}{x-1} \cdot \dfrac{x}{x+1} = \dfrac{x}{x^2-1}$

$D_{f \cdot g} = D_f \cap D_g = \mathbb{R} - \{1,-1\}$.

(γ) $\left(\frac{1}{g}\right)(x) = \frac{1}{g(x)} = \frac{1}{\frac{x}{x+1}} = \frac{x+1}{x}$

$D_{1/g} = R - \{-1, 0\}$.

$x \in D_g$ και $g(x) \neq 0$

$x \neq -1$ $\quad \frac{x}{x+1} \neq 0$

$x \neq 0$

(δ) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{1}{x-1}}{\frac{x}{x+1}} = \frac{x+1}{x(x-1)} = \frac{x+1}{x^2 - x}$

$D_{f/g} = D_f \cap D_g$ και $g(x) \neq 0$.

$D_{f/g} = R - \{1, -1, 0\}$.

8. ⓐ $f(x) = \ln(x-1)$ $D_f = (1, +\infty)$.

πρέπει $x-1>0 \Rightarrow x>1$

$g(x) = \frac{1}{x}$ $D_g = \mathbb{R}^*$

$(g \circ f)(x) = g(f(x)) = \frac{1}{\ln(x-1)}$

$x \in D_f$ και $f(x) \in D_g$

$x>1$ και $\ln(x-1) \neq 0$

$e^{\ln(x-1)} \neq e^0$

$x-1 \neq 1$

$x \neq 2$.

$D_{g \circ f} = (1, 2) \cup (2, +\infty)$.

(β) $f(x) = \ln x$ $\qquad\qquad$ $g(x) = \sqrt{1-x}$

$D_f = (0,+\infty)$ $\qquad\qquad$ $D_g = (-\infty, 1]$

$\rightarrow (g \circ f)(x) = g(f(x)) = \sqrt{1 - \ln x}$

$x \in D_f$ και $f(x) \in D_g$

$\underline{\underline{x > 0}}$ $\qquad\qquad$ $\ln x \leq 1$

$e^{\ln x} \leq e^1$

$x \leq e.$

$D_{g\circ f} = (0, e].$

(γ) $f(x) = \dfrac{x}{x-1}$ $\qquad\qquad$ $g(x) = \ln x$

$D_f = \mathbb{R} - \{1\}$ $\qquad\qquad$ $D_g = (0,+\infty).$

$\rightarrow (g \circ f)(x) = g(f(x)) = \ln\left(\dfrac{x}{x-1}\right)$

$x \in D_f$ και $f(x) \in D_g$

$\underline{\underline{x \neq 1}}$ $\qquad\qquad$ $\dfrac{x}{x-1} > 0$

x		0		1	
x	$-$	0	$+$		$+$
$x-1$	$-$		$-$		$+$
$\dfrac{x}{x-1}$	$+$		$-$		$+$

$x \in (-\infty, 0) \cup (1, +\infty).$

(8) $f(x) = \dfrac{2x-1}{x+1}$ $\quad D_f = \mathbb{R} - \{-1\}.$

$g(x) = \sqrt{x-1}$ $\quad D_g = [1, +\infty).$

$\longrightarrow (g \circ f)(x) = g\big(f(x)\big) = \sqrt{\dfrac{2x-1}{x+1} - 1}$

$= \sqrt{\dfrac{2x-1}{x+1} - \dfrac{x+1}{x+1}} = \sqrt{\dfrac{x-2}{x+1}}$

$x \in D_f$ και $f(x) \in D_g.$

$\underline{\underline{x \neq -1}}$

$\dfrac{2x-1}{x+1} \geq 1.$

$\dfrac{2x-1}{x+1} - 1 \geq 0$

$\dfrac{2x-1}{x+1} - \dfrac{x+1}{x+1} \geq 0$

$\dfrac{x-2}{x+1} \geq 0$

x		-1		2	
$x-2$	$-$		$-$	0	$+$
$x+1$	$-$	0	$+$		$+$
$\frac{x-2}{x+1}$	$+$		$-$		$+$

$\underline{\underline{x \in (-\infty, -1) \cup [2, +\infty)}}$

9. $f(x)=\frac{1}{x-1}$, $D_f=\mathbb{R}-\{1\}$.

$g(x)=\sqrt{x}$, $D_g=[0,+\infty)$

α) $(g\circ f)(x)=g(f(x))=\sqrt{\frac{1}{x-1}}$

$x\in D_f$ και $f(x)\in D_g$

$\underline{\underline{x\neq 1}}$ $\qquad \frac{1}{x-1}\geq 0$

$x-1>0$

$\underline{\underline{x>1}}$

$D_{g\circ f}=(1,+\infty)$.

β) $(f\circ g)(x)=f(g(x))=\frac{1}{\sqrt{x}-1}$

$x\in D_g$ και $g(x)\in D_f$

$\underline{\underline{x\geq 0}}$ $\qquad \sqrt{x}\neq 1$

$\underline{\underline{x\neq 1}}$

$D_{f\circ g}=[0,1)\cup(1,+\infty)$

10. $f(x) = \dfrac{x-1}{x+1}$ $\quad D_f = \mathbb{R} - \{-1\}$

$g(x) = \dfrac{1}{x}$ $\quad D_g = \mathbb{R} - \{0\}$.

ⓐ $(f \circ g)(x) = f(g(x)) = \dfrac{\frac{1}{x} - 1}{\frac{1}{x} + 1} =$

$= \dfrac{1-x}{1+x}$

$x \in D_g$ και $g(x) \in D_f$

$x \neq 0$ $\qquad \dfrac{1}{x} \neq -1$

$x \neq -1$

$D_{f \circ g} = \mathbb{R} - \{0, -1\}$.

ⓑ $(f \circ f)(x) = f(f(x)) = \dfrac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} =$

$= \dfrac{x-1-x-1}{x-1+x+1} = \dfrac{-2}{2x} = -\dfrac{1}{x}$.

$x \in D_f$ και $f(x) \in D_f$

$x \neq -1$ $\qquad \dfrac{x-1}{x+1} \neq -1$

$x-1 \neq -x-1$

$2x \neq 0 \qquad x \neq 0$.

$$D_{f\circ f} = \mathbb{R} - \{-1, 0\}.$$

(i) $\left(g \circ \frac{1}{f}\right)(x) = g\left(\frac{1}{f}(x)\right) = \dfrac{1}{\frac{x+1}{x-1}} = \dfrac{x-1}{x+1}$.

$x \in D_{1/f}$ και $\left(\frac{1}{f}\right)(x) \in D_g$

$x \neq -1 \qquad \dfrac{x+1}{x-1} \neq 0$

$x \neq 1 \qquad x \neq -1$

$$D_{g\circ\frac{1}{f}} = \mathbb{R} - \{-1, 1\}.$$

$\rightarrow \left(\frac{1}{f}\right)(x) = \dfrac{1}{f(x)} = \dfrac{1}{\frac{x-1}{x+1}} = \dfrac{x+1}{x-1}$

$D_{\frac{1}{f}} = \mathbb{R} - \{-1, 1\}$

$x \in D_f$ και $f(x) \neq 0$.

$\underline{x \neq -1} \qquad \dfrac{x-1}{x+1} \neq 0 \Rightarrow x-1 \neq 0 \Rightarrow \underline{x \neq 1}$

Ενότητα 4

4. $f(x)=\frac{1}{x}-\sqrt{x}+1.$

$D_f=(0,+\infty).$

(Β1) $x_1<x_2 \Rightarrow \frac{1}{x_1}>\frac{1}{x_2}$ $\quad$ (+)

$\cdot\ x_1<x_2 \Rightarrow \sqrt{x_1}<\sqrt{x_2} \Rightarrow -\sqrt{x_1}>-\sqrt{x_2}$

$-\sqrt{x_1}+1>-\sqrt{x_2}+1$

$$\underbrace{\frac{1}{x_1}-\sqrt{x_1}+1}_{f(x_1)} > \underbrace{\frac{1}{x_2}-\sqrt{x_2}+1}_{f(x_2)}$$

$f(x_1)>f(x_2)$

$f\downarrow$

(Β) $e<\pi \xRightarrow{f\downarrow} f(e)>f(\pi)$

Tips

$x_1 < x_2 \Rightarrow -x_1 > -x_2$

$x_1 < x_2 \Rightarrow \frac{1}{x_1} > \frac{1}{x_2}$ x_1, x_2 ομοσημοι

$x_1 < x_2 \Rightarrow x_1^2 > x_2^2$ x_1, x_2 αρνητικοι

$x_1 < x_2 \Rightarrow \alpha^{x_1} > \alpha^{x_2}$ $0 < \alpha < 1$

$x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2}$

$x_1 < x_2 \Rightarrow \sqrt{x_1} < \sqrt{x_2}$

$x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2$

$x_1 < x_2 \Rightarrow x_1^3 < x_2^3$

$x_1 < x_2 \Rightarrow x_1^2 < x_2^2$ x_1, x_2 θετικα.

5. $f(x)=e^{x-1}+\ln x-1$

$D_f=(0,+\infty)$.

ⓐ

- $x_1<x_2 \Rightarrow x_1-1<x_2-1 \Rightarrow e^{x_1-1}<e^{x_2-1}$ ⊕
- $x_1<x_2 \Rightarrow \ln x_1<\ln x_2 \Rightarrow \ln x_1-1<\ln x_2-1$

$$\underbrace{e^{x_1-1}+\ln x_1-1}_{f(x_1)} < \underbrace{e^{x_2-1}+\ln x_2-1}_{f(x_2)}$$

$f \nearrow$

Ⓑ i) Αν $x>1$ τοτε νδο $f(x)>0$

$\rightarrow \quad x>1 \xRightarrow{f\nearrow} f(x)>f(1) \Rightarrow f(x)>0$

ii) Αν $\alpha<\beta$ νδο $\ln\frac{\alpha}{\beta}<e^{\beta-1}-e^{\alpha-1}$

$\rightarrow \quad \alpha<\beta \Rightarrow f(\alpha)<f(\beta) \Rightarrow e^{\alpha-1}+\ln\alpha-1<e^{\beta-1}+\ln\beta-1$

$\ln\alpha-\ln\beta<e^{\beta-1}-e^{\alpha-1}$

$\ln\frac{\alpha}{\beta}<e^{\beta-1}-e^{\alpha-1}$

iii) Αν $x>0$ νδο $f(x)-f(x+1)<0$

$\rightarrow$ $x<x+1 \Rightarrow f(x)<f(x+1) \Rightarrow f(x)-f(x+1)<0$

iv). Νδο αν $x>0$ το $f(2x)<f(5x)$

Προφανώς $2x<5x \Rightarrow f(2x)<f(5x)$

v). $\forall x>1$ νδο $f(x)<f(x^2)$.

Ισχυει οτι $x<x^2 \Rightarrow f(x)<f(x^2)$.

Προσοχη

Η περιοχη $(0,1)$ ειναι υπαλη!
Θελει μεγαλη προσοχη.

vi) Νδο $f(a) - f(a+3) < f(a+1) - f(a+2)$.

$a < a+1 \Rightarrow f(a) < f(a+1)$

$+$

$a+2 < a+3 \Rightarrow f(a+2) < f(a+3)$

$$f(a) + f(a+2) < f(a+1) + f(a+3)$$

$$f(a) - f(a+3) < f(a+1) - f(a+2)$$

10. $f(x) = x^5 + x - 2$

(a) $x_1 < x_2 \Rightarrow x_1^5 < x_2^5$

(+)

$x_1 < x_2 \Rightarrow x_1 - 2 < x_2 - 2$

$x_1^5 + x_1 - 2 < x_2^5 + x_2 - 2$

$f(x_1) < f(x_2)$

$f \nearrow \Rightarrow f$ ȝ $1-1$

(B). P.T.U $f(x)$

$f(x) = 0$

$f(x) = f(1)$

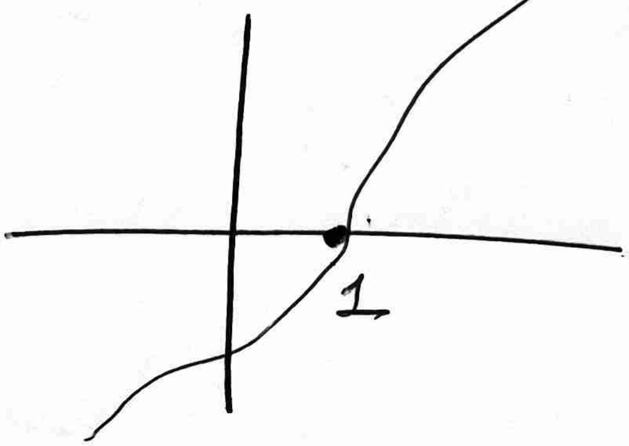
f ȝ $1-1$

$\underline{\underline{x = 1}}$

x		1	
$f(x)$	$-$	0	$+$

$x < 1 \Rightarrow f(x) < f(1) \Rightarrow f(x) < 0$

$x > 1 \Rightarrow f(x) > f(1) \Rightarrow f(x) > 0$



(6) i) $f(x) > 0$

$f(x) > f(1)$

$f \nearrow$

$x > 1$

ii). $x^5 + x + 2 < 0$

$x^5 + x < -2$

$x^5 + x - 2 < -2 - 2$

$f(x) < -4$

$f(x) < f(-1)$

$f \nearrow$

$x < -1$

iii) $f(x^5 + x) \geq 32$.

$f(x^5 + x) \geq f(2)$

$f \nearrow$

$x^5 + x \geq 2$

$x^5 + x - 2 \geq 0$

$f(x) \geq 0$

$f(x) \geq f(1)$

$f \nearrow$

$x \geq 1$

11. $f(x) = e^x + \ln(x+1) - 1$

$D_f = (-1, +\infty)$.

Ανίσωση

$f(x) > 0$

$f(x) > f(1)$

$f \nearrow$

$x > 1$

Μονοτονία $f(x)$

- $x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2}$

$+$

- $x_1 < x_2 \Rightarrow x_1 + 1 < x_2 + 1 \Rightarrow$

$\ln(x_1+1) < \ln(x_2+1)$

$\ln(x_1+1) - 1 < \ln(x_2+1) - 1$

$f(x_1) < f(x_2) \quad f \nearrow$

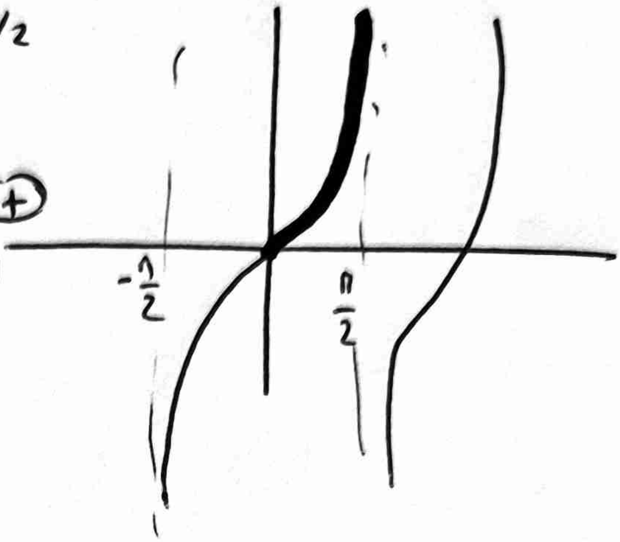
13. $f(x)=εφx+e^x-1$, $x\in[0,\frac{\pi}{2}]$.

(α) • $x_1<x_2 \Rightarrow εφx_1<εφx_2$

• $x_1<x_2 \Rightarrow e^{x_1}<e^{x_2}$

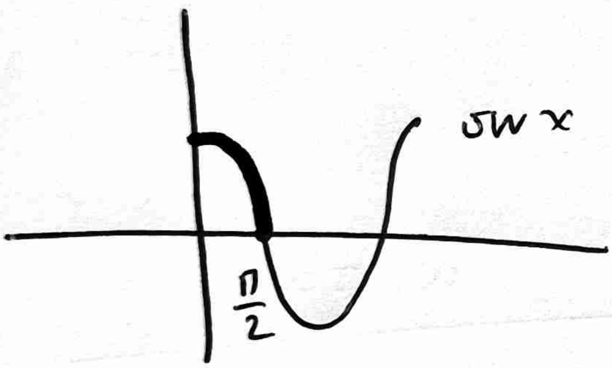
$\oplus$

$e^{x_1}-1<e^{x_2}-1$



$f\nearrow$

(β) Ν.δ.ο. $ημx+e^x συνx > συνx \quad \forall x\in(0,\frac{\pi}{2})$



$\frac{ημx}{συνx}+e^x>1$

$εφx+e^x>1$.

$\underbrace{εφx+e^x-1}>0$

$f(x)>0$

$f(x)>f(0)$

$f\nearrow$

$x>0$.

16. $f(x) = x + \ln(x+3)$

$D_f = (-3, +\infty)$.

(a) $x_1 < x_2$ (+)

$x_1 < x_2 \Rightarrow x_1 + 3 < x_2 + 3$

$\Rightarrow \ln(x_1+3) < \ln(x_2+3)$

$f \nearrow$

(B) i) $\underbrace{x^2 + \ln(x^2+3)}_{} > \ln 3$.

$f(x^2) > f(0)$

$f \nearrow$

$x^2 > 0$

$x \in \mathbb{R}^*$

11) $x^4 - x^2 < \ln \dfrac{x^2+3}{x^4+3}$

$$x^4 - x^2 < \ln(x^2+3) - \ln(x^4+3)$$

$$\underbrace{\ln(x^4+3) + x^4}_{f(x^4)} < \underbrace{\ln(x^2+3) + x^2}_{f(x^2)}$$

$$f \nearrow$$

$$x^4 < x^2$$

$$x^4 - x^2 < 0$$

$$x^2(x^2-1) < 0$$

$+$

$$x^2 - 1 < 0$$

x	-1		1	
x^2-1	$+$	$-$		$+$

$$x \in (-1, 1)$$

Λυση: $x \in (-1, 0) \cup (0, 1)$

iii) $\ln \dfrac{x+6}{x^2+4} < x^2 - x - 2$

$$\ln(x+6) - \ln(x^2+4) < x^2 - x - 2$$

$$\ln(x+6) < \ln(x^2+4) + x^2 - x - 2$$

$$\ln(x+3+3) < \ln(x^2+1+3) + x^2 - x - 2$$

$$\ln(x+3+3) + x+3 < \ln(x^2+1+3) + x^2+1.$$

$$f(x+3) < f(x^2+1)$$

$$f \nearrow$$

$$x+3 < x^2+1$$

$$0 < x^2 - x - 2$$

x		-1		2	
x^2-x-2	$+$	|	$-$	|	$+$

$$x \in (-\infty, -1) \cup (2, +\infty).$$

17. $f(x) = \ln x \qquad D_f = (0, +\infty)$

$g(x) = \sqrt{2-x} - 1$

$D_g = (-\infty, 2]$

$\rightarrow f(x) < g(x) \Rightarrow \ln x < \sqrt{2-x} - 1$

$$\underbrace{\ln x - \sqrt{2-x} + 1}_{\varphi(x)} < 0$$

$\varphi(x) < 0$

$\varphi(x) < \varphi(1)$

$\varphi \nearrow$

$x < 1$

Μονοτονια $\varphi(x)$

$\varphi \nearrow.$

- $x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2.$
- $x_1 < x_2 \rightarrow -x_1 > -x_2 \Rightarrow 2 - x_1 > 2 - x_2$

$\sqrt{2-x_1} > \sqrt{2-x_2}$ +

$-\sqrt{2-x_1} < -\sqrt{2-x_2}$

Όταν $x<1$ η $f(x)<g(x)$.

Όταν $x>1$ η $f(x)>g(x)$

Το $A(1,0)$ είναι κοινό τους σημείο.

(B) <u>Ανίσωση</u> στο $\left(0,\frac{\pi}{2}\right)$.

$$\ln\frac{1-\eta\mu x}{1-\sigma\upsilon\nu x}<\sqrt{1+\eta\mu x}-\sqrt{1+\sigma\upsilon\nu x}$$

$$\ln(1-\eta\mu x)-\ln(1-\sigma\upsilon\nu x)<\sqrt{1+\eta\mu x}-\sqrt{1+\sigma\upsilon\nu x}$$

$$\ln(1-\eta\mu x)-\sqrt{1+\eta\mu x}<\ln(1-\sigma\upsilon\nu x)-\sqrt{1+\sigma\upsilon\nu x}$$

$$\underbrace{\ln(1-\eta\mu x)-\sqrt{2-(1-\eta\mu x)}+1}<1+\ln(1-\sigma\upsilon\nu x)-\sqrt{2-(1-\sigma\upsilon\nu x)}$$

$$\varphi(1-\eta\mu x)<\varphi(1-\sigma\upsilon\nu x)$$

$$\varphi\nearrow$$

$$1-\eta\mu x<1-\sigma\upsilon\nu x.$$

$$-\eta\mu x < -\sigma\upsilon\nu x$$

$$\eta\mu x > \sigma\upsilon\nu x . \qquad \left(0, \frac{\pi}{2}\right).$$

$$\frac{\eta\mu x}{\sigma\upsilon\nu x} > 1$$

$$\varepsilon\varphi x > 1$$

$$\varepsilon\varphi x > \varepsilon\varphi \frac{\pi}{4}$$

$$\varepsilon\varphi \nearrow$$

$$\frac{\pi}{2} > x > \frac{\pi}{4}$$

20. γ. $x < \dfrac{2}{x^4+1}$

$$x(x^4+1) < 2$$

$$\underbrace{x^5+x-2}_{f(x)} < 0$$

$$f(x) < 0$$

$$f(x) < f(1)$$

$$f \nearrow$$

$$x < 1$$

Μονοτονια $f(x)$

$x_1 < x_2 \Rightarrow x_1^5 < x_2^5$

$+$

$x_1 < x_2 \Rightarrow x_1 - 2 < x_2 - 2$

$f(x_1) < f(x_2)$ $f \nearrow$

11. $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$

$$\left.\begin{array}{l} \text{(a)}\quad x_1 < x_2 \Rightarrow \left(\frac{3}{5}\right)^{x_1} > \left(\frac{3}{5}\right)^{x_2} \\[2ex] x_1 < x_2 \Rightarrow \left(\frac{4}{5}\right)^{x_1} > \left(\frac{4}{5}\right)^{x_2} \end{array}\right\} (+)$$

$$f(x_1) > f(x_2)$$

$$f \downarrow$$

(b) $3^x + 4^x > 5^x$

$$\frac{3^x}{5^x} + \frac{4^x}{5^x} > \frac{5^x}{5^x}$$

$$\left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x > 1$$

$$\left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1 > 0$$

$$f(x) > 0$$

$$f(x) > f(2)$$

$$f \downarrow$$

$$x < 2$$

23. (B) $\ln(x^2+1) - \frac{1}{x^2+1} < \ln 5 - \frac{1}{5}$

$\boxed{f(x) = \ln x - \frac{1}{x}}$

$x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2$

$x_1 < x_2 \Rightarrow \frac{1}{x_1} > \frac{1}{x_2}$ (+)

$-\frac{1}{x_1} < -\frac{1}{x_2}$

$f \nearrow$

$f(x^2+1) < f(5)$

$f \nearrow$

$x^2+1 < 5$

$x^2 - 4 < 0$

$x^2 < 4$

$x^2 < 2^2$

$|x| < |2|$

$|x| < 2$

$-2 < x < 2$

23. (δ) $e^x+x+\ln(e^x+x+3) > \ln 4\cdot e$

$[-3,+\infty)$

$e^x+x+\ln(e^x+x+3) > \ln 4+\ln e$

$e^x+x+\ln(e^x+x+3) > \ln 4+1$

$e^x+x+\ln(e^x+x+3) > \ln(1+3)+1$

$$\boxed{f(x)=x+\ln(x+3)}$$

$f(e^x+x) > f(1)$

$f\nearrow$

$e^x+x>1$

$\underbrace{e^x+x-1}_{\varphi(x)}>0$

$\varphi(x)>0$

$\varphi(x)>\varphi(0)$

$\varphi\nearrow$

$x>0$

Μονοτονία $f(x)$

- $x_1<x_2 \Rightarrow \ln(x_1+3)<\ln(x_2+3)$
- $x_1<x_2$

$\oplus$

$f\nearrow$

Μονοτονία $\varphi(x)$

- $x_1<x_2 \Rightarrow e^{x_1}<e^{x_2}$
- $x_1<x_2 \Rightarrow x_1-1<x_2-1$

$\oplus$ $\varphi\nearrow$

24. B III). $e^x - e^{x^3} > 2\ln x$, $x > 0$

$$e^x - e^{x^3} > \ln x^2$$

$$e^x - e^{x^3} > \ln \frac{x^3}{x}$$

$$e^x - e^{x^3} > \ln x^3 - \ln x$$

$$e^x + \ln x > \ln x^3 + e^{x^3}$$

$$\boxed{\begin{array}{c} f(x) = e^x + \ln x \\ f \nearrow \end{array}}$$

$$f(x) > f(x^3)$$

$$f \nearrow$$

$$x > x^3$$

$$1 > x^2$$

$$1^2 > x^2$$

$$|1| > |x|$$

$$1 > |x| \qquad x \in (-1, 1).$$

25. f gr. povozovu $\begin{cases} f \nearrow \\ \text{ili} \\ f \searrow \end{cases}$

ⓐ $A(1,5) \quad f(1)=5$

$B(5,-2) \quad f(5)=-2$.

$$\boxed{\begin{array}{l} 1 < 5 \quad \Rightarrow \quad \underset{\underset{5}{\|}}{f(1)} > \underset{\underset{-2}{\|}}{f(5)} \\ \qquad\qquad\qquad\qquad f \searrow \end{array}}$$

ⓑ $x_1 < x_2 \overset{f \searrow}{\Rightarrow} f(x_1) > f(x_2)$

$$f(f(x_1)) < f(f(x_2))$$

$$(f \circ f)(x_1) < (f \circ f)(x_2)$$

$$f \circ f \nearrow$$

⑧ $f(f(e^x)) < -2$

$f(f(e^x)) < f(5)$

$f \downarrow$

$f(e^x) > 5$

$f(e^x) > f(1)$

$f \downarrow$

$e^x < 1$

$x < \ln 1$

$\underline{\underline{x < 0}}$

⑥ $g(x) = x + \ln x$

$g \nearrow$

$\cdot\ \underbrace{x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2}_{\oplus}$

$f(g(x)) > 5$

$f(g(x)) > f(1)$

$g(x) < 1$

$g(x) < g(1)$

$g \nearrow$

$0 < x < 1$

Επομενο Μαθημα

Δευτερα 9-11:30.

4

3

9

12

14

15

18

20 α β

22

23 α γ

24 α β) i) ii).

28.