3. (a) $\int_1^3 2x\ln x\,dx = \int_1^3 (x^2)'\ln x\,dx$

$= \left(x^2\ln x\right)_1^3 - \int_1^3 x^2\cdot\frac{1}{x}\,dx =$

$= 9\ln 3 - \int_1^3 x\,dx =$

$= 9\ln 3 - \frac{1}{2}\left(x^2\right)_1^3 = 9\ln 3 - 4$

(β) $\int_1^2 \ln x\,dx = \int_1^2 1\cdot\ln x\,dx = \int_1^2 (x)'\ln x\,dx$

$= \left(x\ln x\right)_1^2 - \int_1^2 x\cdot\frac{1}{x}\,dx =$

$= 2\ln 2 - \int_1^2 1\,dx = 2\ln 2 - (x)_1^2 =$

$= 2\ln 2 - 1.$

(γ) $\int_{-1}^0 \ln(x+2)\,dx = \int_{-1}^0 1\cdot\ln(x+2)\,dx$

$= \int_{-1}^0 (x)'\ln(x+2)\,dx = \cancel{\left(x\ln(x+2)\right)_{-1}^0} - \int_{-1}^0 x\,\frac{1}{x+2}\,dx$

$= -\int_{-1}^0 \frac{x}{x+2}\,dx \quad \underline{(*)}$

$$\begin{array}{r|l} x & x+2 \\ \hline -(x+2) & 1 \\ \hline -2 & \end{array}$$

(*)

$$- \int_{-1}^{0} \frac{1\cdot(x+2)-2}{x+2}\, dx =$$

$$= - \int_{-1}^{0} \frac{x+2}{x+2} - \frac{2}{x+2}\, dx =$$

$$= - \int_{-1}^{0} 1 - \frac{2}{x+2}\, dx =$$

$$= - \int_{-1}^{0} 1\, dx + 2 \int_{-1}^{0} \frac{1}{x+2}\, dx$$

$$= - (x)_{-1}^{0} + 2 \left(\ln|x+2|\right)_{-1}^{0}$$

$$= -(0+1) + 2(\ln 2 - 0)$$

$$= -1 + 2\ln 2.$$

4. (a) $\int_1^2 \frac{\ln x}{x^2}\, dx = \int_1^2 \frac{1}{x^2} \cdot \ln x \, dx$

$= \int_1^2 \left(-\frac{1}{x}\right)' \ln x \, dx =$

$= \left(-\frac{1}{x}\ln x\right)_1^2 - \int_1^2 \left(-\frac{1}{x}\right)\frac{1}{x}\, dx$

$= -\left(\frac{\ln x}{x}\right)_1^2 + \int_1^2 \frac{1}{x^2}\, dx$

$= -\left(\frac{\ln 2}{2}\right) + \left(-\frac{1}{x}\right)_1^2 =$

$= -\frac{1}{2}\ln 2 - \left(\frac{1}{2} - 1\right) = -\frac{1}{2}\ln 2 + \frac{1}{2}.$

(B) $\int_1^4 \frac{\ln x}{\sqrt{x}}\, dx = \int_1^4 \frac{1}{\sqrt{x}} \ln x \, dx =$

$= \int_1^4 (2\sqrt{x})' \ln x \, dx = (2\sqrt{x}\ln x)_1^4 - \int_1^4 2\sqrt{x}\,\frac{1}{x}\, dx$

$= 4\ln 4 - 2\int_1^4 \frac{\sqrt{x}}{x}\, dx = 4\ln 4 - 2\int_1^2 \frac{\cancel{t}}{\cancel{t^2}}\, 2\cancel{t}\, dt$

$\boxed{\begin{array}{c}\sqrt{x} = t \\ x = t^2\end{array}}$ $dx = 2t\, dt$

$= 4\ln 4 - 4.$

γ $\displaystyle\int_1^a x^2 \ln x \, dx = \int_1^a \left(\frac{x^3}{3}\right)' \ln x \, dx$

$$= \left(\frac{x^3}{3}\ln x\right)_1^a - \int_1^a \frac{x^3}{3}\frac{1}{x}\,dx =$$

$$= \frac{a^3}{3}\ln a - \frac{1}{3}\int_1^a x^2\,dx$$

$$= \frac{a^3}{3}\ln a - \frac{1}{3}\frac{1}{3}(x^3)_1^a$$

$$= \frac{a^3}{3}\ln a - \frac{1}{9}(a^3-1).$$

δ. $\displaystyle\int_1^t \ln^2 x\,dx = \int_1^t (x)' \ln^2 x\,dx$

$$= (x\ln^2 x)_1^t - \int_1^t \cancel{x}\, 2\ln x \frac{1}{\cancel{x}}\,dx$$

$$= t\ln^2 t - 2\int_1^t \ln x\,dx.$$

$$= t\ln^2 t - 2\int_1^t (x)'\ln x\,dx =$$

$$= t\ln^2 t - 2\left[(x\ln x)_1^t - \int_1^t x\cdot\frac{1}{x}dx\right] =$$

$$= t\ln^2 t - 2\left[t\ln t - (x)_1^t\right]$$

$$= t\ln^2 t - 2t\ln t + 2(t-1)$$

ε) $$\int_1^2 \frac{1+\ln x}{x^2}\,dx = \int_1^2 \frac{1}{x^2}(1+\ln x)\,dx$$

$$= \int_1^2 \left(-\frac{1}{x}\right)'(1+\ln x)\,dx =$$

$$= \left(-\frac{1}{x}(1+\ln x)\right)_1^2 - \int_1^2 -\frac{1}{x}\frac{1}{x}\,dx$$

$$= -\left(\frac{1}{2}(1+\ln 2) - 1\right) + \int_1^2 \frac{1}{x^2}\,dx$$

$$= -\frac{1}{2}(1+\ln 2) + 1 + \left(-\frac{1}{x}\right)_1^2$$

$$= -\frac{1}{2}(1+\ln 2) + 1 - \left(\frac{1}{2} - 1\right)$$

$$= -\frac{1}{2} - \frac{1}{2}\ln 2 + 1 - \frac{1}{2} + 1 = 1 - \frac{1}{2}\ln 2.$$

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2. (α) $\int_{-1}^{0} x e^{x}\, dx =$

$= \int_{-1}^{0} x (e^{x})'\, dx =$

$= (x e^{x})_{-1}^{0} - \int_{-1}^{0} e^{x} \cdot 1\, dx =$

$= (0 e^{0} - (-1) e^{-1}) - \int_{-1}^{0} e^{x}\, dx =$

$= \frac{1}{e} - (e^{x})_{-1}^{0} = \frac{1}{e} - (1 - \frac{1}{e}) = \frac{2}{e} - 1.$

(β) $\int_{0}^{\pi} x\, ημx\, dx = \int_{0}^{\pi} x\, (-συνx)'\, dx =$

$= (-x\, συνx)_{0}^{\pi} - \int_{0}^{\pi} -συνx \cdot 1\, dx$

$= -(x\, συνx)_{0}^{\pi} + \int_{0}^{\pi} συνx\, dx =$

$= -(\pi\, συν\pi - 0\, συν0) + (ημx)_{0}^{\pi} =$

$= -(-\pi) + \cancel{ημ\pi - ημ0} = \pi.$

(γ) $\int_0^{-1} x^2 e^x \, dx = \int_0^{-1} x^2 (e^x)' \, dx =$

$= (x^2 e^x)_0^{-1} - \int_0^{-1} e^x \, 2x \, dx =$

$= \frac{1}{e} - 2 \int_0^{-1} e^x x \, dx =$

$= \frac{1}{e} - 2 \int_0^{-1} (e^x)' x \, dx =$

$= \frac{1}{e} - 2 \left[(e^x x)_0^{-1} - \int_0^{-1} e^x \, 1 \, dx \right]$

$= \frac{1}{e} - 2 \left[\left(-\frac{1}{e} - 0\right) - (e^x)_0^{-1} \right]$

$= \frac{1}{e} + \frac{2}{e} + 2 (e^{-1} - e^0) =$

$= \frac{3}{e} + \frac{2}{e} - 2 = \frac{5}{e} - 2 .$

8 $\int_0^{\pi} x^2 \, \sigma\upsilon\nu x \, dx = \int_0^{\pi} x^2 (\eta\mu x)' \, dx$

$= \cancel{\left(x^2 \eta\mu x\right)_0^{\pi}}^{\,0} - \int_0^{\pi} 2x \, \eta\mu x \, dx$

$= -2 \int_0^{\pi} x \, (-\sigma\upsilon\nu x)' \, dx =$

$= -2 \left[\left(-x \, \sigma\upsilon\nu x\right)_0^{\pi} - \int_0^{\pi} -\sigma\upsilon\nu x \, dx \right]$

$= -2 \left[-\left(x \, \sigma\upsilon\nu x\right)_0^{\pi} + \cancel{\left(\eta\mu x\right)_0^{\pi}} \right]$

$= 2\left(\pi \, \sigma\upsilon\nu \pi - 0 \cdot \sigma\upsilon\nu 0\right) = 2\pi(-1) = -2\pi.$

5. (a) $I=\int_0^\pi e^x \eta\mu x \, dx$

$$I=\int_0^\pi (e^x)' \eta\mu x \, dx$$

$$I=\cancel{(e^x \eta\mu x)_0^\pi} - \int_0^\pi e^x \sigma\upsilon\nu x \, dx$$

$$I=-\int_0^\pi (e^x)' \sigma\upsilon\nu x \, dx$$

$$I=-\left[(e^x \sigma\upsilon\nu x)_0^\pi - \int_0^\pi e^x(-\eta\mu x) \, dx \right]$$

$$I=-(-e^\pi-1)-I$$

$$2I=e^\pi+1$$

$$I=\frac{e^\pi+1}{2}$$

5. (B) $\int_0^{\pi} e^x \,\text{συν}2x \, dx$

$$I = \int_0^{\pi} (e^x)' \,\text{συν}2x \, dx$$

$$I = \left(e^x \,\text{συν}2x\right)_0^{\pi} - \int_0^{\pi} e^x \left(-2\,\text{ημ}2x\right) dx$$

$$I = e^{\pi} - 1 + 2\int_0^{\pi} e^x \,\text{ημ}2x \, dx$$

$$I = e^{\pi} - 1 + 2\int_0^{\pi} (e^x)' \,\text{ημ}2x \, dx$$

$$I = e^{\pi} - 1 + 2\left[\overset{0}{\cancel{\left(e^x \,\text{ημ}2x\right)_0^{\pi}}} - \int_0^{\pi} 2e^x \,\text{συν}2x \, dx\right]$$

$$I = e^{\pi} - 1 - 2 \cdot 2\int_0^{\pi} e^x \,\text{συν}2x \, dx$$

$$I = e^{\pi} - 1 - 4I$$

$$5I = e^{\pi} - 1$$

$$I = \frac{e^{\pi} - 1}{5}$$

(γ) $\displaystyle\int_0^{\pi} \frac{\sigma\upsilon\nu x}{e^x}\,dx = \int_0^{\pi} e^{-x}\,\sigma\upsilon\nu x\,dx$

$$I = \int_0^{\pi} e^{-x}\,\sigma\upsilon\nu x\,dx$$

$$I = \int_0^{\pi} (-e^{-x})'\,\sigma\upsilon\nu x\,dx$$

$$I = \left(-e^{-x}\,\sigma\upsilon\nu x\right)_0^{\pi} - \int_0^{\pi} -e^{-x}(-\eta\mu x)\,dx$$

$$I = -\left(-e^{-\pi} - 1\right) - \int_0^{\pi} e^{-x}\,\eta\mu x\,dx$$

$$I = e^{-\pi} + 1 - \int_0^{\pi} (-e^{-x})'\,\eta\mu x\,dx$$

$$I = e^{-\pi} + 1 - \left[\cancel{\left(-e^{-x}\,\eta\mu x\right)_0^{\pi}} - \int_0^{\pi} -e^{-x}\,\sigma\upsilon\nu x\,dx\right]$$

$$I = e^{-\pi} + 1 - I$$

$$2I = e^{-\pi} + 1$$

$$I = \frac{e^{-\pi}+1}{2}$$

6. ⓐ $\int_0^1 x e^{2x} \, dx = \int_0^1 x \left(\frac{1}{2} e^{2x}\right)' dx$

$= \left(\frac{1}{2} x e^{2x}\right)_0^1 - \int_0^1 \frac{1}{2} e^{2x} \, dx$

$= \frac{1}{2} \left(x e^{2x}\right)_0^1 - \frac{1}{2} \int_0^1 e^{2x} \, dx$

$= \frac{1}{2} \left(e^2 - 0\right) - \frac{1}{2} \left(\frac{1}{2} e^{2x}\right)_0^1$

$= \frac{1}{2} e^2 - \frac{1}{4} \left(e^2 - 1\right)$

Ⓑ $\int_0^\pi 4x \, \sigma\upsilon\nu 2x \, dx = \int_0^\pi 4x \left(\frac{1}{2} \eta\mu 2x\right)' dx$

$= \cancel{\left(\frac{1}{2} 4x \, \eta\mu 2x\right)_0^\pi} - \int_0^\pi \frac{1}{2} \eta\mu 2x \cdot 4 \, dx$

$= -2 \int_0^\pi \eta\mu 2x \, dx =$

$= -2 \left(-\frac{\sigma\upsilon\nu 2x}{2}\right)_0^\pi = \sigma\upsilon\nu 2\pi - \sigma\upsilon\nu 0$

$= 0$

10) $\int_0^1 x e^{-x}\, dx = \int_0^1 x\left(-e^{-x}\right)' dx$

$= \left(-x e^{-x}\right)_0^1 - \int_0^1 -e^{-x}\, dx$

$= -\left(x e^{-x}\right)_0^1 + \int_0^1 e^{-x}\, dx$

$= -\left(\frac{1}{e} - 0\right) - \left(e^{-x}\right)_0^1 =$

$= -\frac{1}{e} - \left(\frac{1}{e} - 1\right) = 1 - \frac{2}{e}.$

8 $\int_0^1 (x^2 - x - 1)\, e^{-x}\, dx =$

$= \int_0^1 (x^2 - x - 1)\left(-e^{-x}\right)' dx =$

$= \left(-(x^2 - x - 1) e^{-x}\right)_0^1 - \int_0^1 -e^{-x}(2x - 1)\, dx$

$= -\left(-\frac{1}{e} + 1\right) + \int_0^1 \left(-e^{-x}\right)'(2x - 1)\, dx$

$$= \frac{1}{e} - 1 + \left[\left[-e^{-x}(2x-1) \right]_0^1 - \int_0^1 -e^{-x} 2\,dx \right]$$

$$= \frac{1}{e} - 1 - \left(\frac{1}{e} + 1 \right) + 2 \int_0^1 e^{-x}\,dx$$

$$= \cancel{\frac{1}{e}} - 1 - \cancel{\frac{1}{e}} - 1 + 2 \left(-e^{-x} \right)_0^1$$

$$= -2 - 2 \left(\frac{1}{e} - 1 \right) =$$

$$= -2 - \frac{2}{e} + 2 = -\frac{2}{e},$$

(E) $\int_0^1 \frac{2x+1}{e^x}\,dx = \int_0^1 (2x+1)e^{-x}\,dx$

$= \int_0^1 (2x+1)\left(-e^{-x}\right)'\,dx =$

$= \left(-(2x+1)e^{-x}\right)_0^1 - \int_0^1 -e^{-x}\cdot 2\,dx$

$= -\left(3\frac{1}{e} - 1\right) + 2\int_0^1 e^{-x}\,dx$

$= -\frac{3}{e} + 1 + 2\left(-e^{-x}\right)_0^1 =$

$= -\frac{3}{e} + 1 - 2\left(\frac{1}{e} - 1\right)$

$= -\frac{3}{e} + 1 - \frac{2}{e} + 2 = -\frac{5}{e} + 3.$

15. (α) $\int_0^{\frac{\pi}{2}} \eta\mu\left(x-\frac{\pi}{6}\right)dx = \int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \eta\mu\, t\, dt$

$x-\frac{\pi}{6}=t$

$dx = dt$

$= \left(-\sigma\upsilon\nu\, t\right)_{-\frac{\pi}{6}}^{\pi/3} =$

$= -\left(\sigma\upsilon\nu\frac{\pi}{3} - \sigma\upsilon\nu\left(-\frac{\pi}{6}\right)\right) =$

$= -\frac{1}{2}+\frac{\sqrt{3}}{2}$

(β) $\int_0^1 \frac{2}{4x+1} = 2\int_0^1 \frac{1}{4x+1}\,dx$

$= 2\int_1^5 \frac{1}{u}\,\frac{1}{4}\,du$

$4x+1=u$

$4\,dx = du$

$dx = \frac{1}{4}\,du$

$= \frac{1}{2}\left(\ln u\right)_1^5 =$

$= \frac{1}{2}\ln 5.$

(1) $\int_0^1 \frac{1}{\sqrt{2x+1}}\,dx = \int_1^{\sqrt{3}} \frac{1}{\cancel{t}} \cancel{t}\,dt$

$\sqrt{2x+1} = t$

$2x+1 = t^2$

$2dx = 2t\,dt$

$dx = t\,dt$

$= \int_1^{\sqrt{3}} 1\,dt$

$= (t)_1^{\sqrt{3}} = \sqrt{3}-1.$

9. $f(0)=1$ $\quad f \nearrow$

$f(2)=2$

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ΝΔΟ $\quad 4 < \int_1^{e^4} \frac{f(\sqrt{\ln x})}{x}\, dx < 8$.

$\sqrt{\ln x} = t$

$\ln x = t^2$

$\frac{1}{x} dx = 2t\, dt$

$4 < \int_0^2 2f(t)\, t\, dt < 8$

Αρκεί νδο $\quad 2 < \int_0^2 f(t)\, t\, dt < 4$

$2 < \int_0^2 f(x)\, x\, dx < 4$.

$$0 < x < 2$$

$$f \nearrow$$

$$f(0) < f(x) < f(2)$$

$$1 < f(x) < 2$$

$$x < x f(x) < 2x$$

$$\int_0^2 x \, dx < \int_0^2 x f(x) \, dx < \int_0^2 2x \, dx$$

$$\frac{1}{2} \left(x^2\right)_0^2 < \int_0^2 x f(x) \, dx < \left(x^2\right)_0^2$$

$$2 < \int_0^2 x f(x) \, dx < 4$$

13. $f(x) = e^{-x^2}$

ⓐ Ν.δ.ο $1 - x^2 \le f(x) \le 1 \quad \forall x \in [0,1]$

$$\boxed{1 - x^2 \le e^{-x^2} \le 1}$$

- $e^x \geq x+1$

  $e^{-x^2} \geq -x^2+1$ ✓

- $x^2 \geq 0 \Rightarrow -x^2 \le 0 \Rightarrow e^{-x^2} \le e^0$

  $e^{-x^2} \le 1$ ✓

ⓑ. Ν.δ.ο $\frac{2}{3} < \int_0^1 e^{-x^2}\,dx < 1$

$$\boxed{\frac{2}{3} < \int_0^1 f(x)\,dx < 1}$$

$$1 - x^2 \le f(x) \le 1$$

$$\int_0^1 1 - x^2\,dx < \int_0^1 f(x)\,dx < \int_0^1 1\,dx$$

$$(x)\Big|_0^1 - \frac{1}{3}(x^3)\Big|_0^1 < \int_0^1 f(x)\,dx < (x)\Big|_0^1$$

$$1 - \frac{1}{3} < \int_0^1 f(x)\,dx < 1$$

$$\frac{2}{3} < \int_0^1 f(x)\,dx < 1$$

1 νδο $\frac{4}{5} < \int_0^1 f(x^2)\,dx < 1$

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$$1 - x^2 \le f(x) \le 1$$

$$1 - x^4 \le f(x^2) \le 1$$

$$\int_0^1 1 - x^4\,dx < \int_0^1 f(x^2)\,dx < \int_0^1 1\,dx$$

$$(x)_0^1 - \frac{1}{5}(x^5)_0^1 < \int_0^1 f(x^2)\,dx < (x)_0^1$$

$$1 - \frac{1}{5} < \int_0^1 f(x^2)\,dx < 1$$

$$\frac{4}{5} < \int_0^1 f(x^2)\,dx < 1$$

9. $f(0)=1$ $f \nearrow$ Ενότητα 37

$f(2)=2$

ν.δ.ο $4 < \int_1^{e^4} \frac{f(\sqrt{\ln x})}{x} dx < 8$.

$\sqrt{\ln x} = t$

$\ln x = t^2$

$\frac{1}{x} dx = 2t\,dt$

$4 < \int_0^2 2f(t)\,t\,dt < 8$

Αρκεί ν.δ.ο $2 < \int_0^2 f(t)\,t\,dt < 4$

$2 < \int_0^2 f(x)\,x\,dx < 4$.

$0 < x < 2$

$f \nearrow$

$f(0) < f(x) < f(2)$

$1 < f(x) < 2$

$x < x f(x) < 2x$

$$\int_0^2 x \, dx < \int_0^2 x f(x) \, dx < \int_0^2 2x \, dx$$

$$\frac{1}{2} \left(x^2\right)_0^2 < \int_0^2 x f(x) \, dx < \left(x^2\right)_0^2$$

$$2 < \int_0^2 x f(x) \, dx < 4$$

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