20. ① $f(x) = e^{x-1} + x\ln x - 2x \quad , x > 0$

$$f'(x) = e^{x-1} + \ln x + x\,\frac{1}{x} - 2$$

$$f'(x) = e^{x-1} + \ln x - 1 \,. \qquad \underline{\underline{f'(1) = 0}}$$

$$f''(x) = e^{x-1} + \frac{1}{x} > 0$$

x	0	1
f''	$+$	$+$
f'	$\nearrow$ $-$	0 $\nearrow$ $+$
f	$\searrow$	$\nearrow$

$x < 1 \Rightarrow f'(x) < f'(1) \Rightarrow f'(x) < 0$

$x > 1 \Rightarrow f'(x) > f'(1) \Rightarrow f'(x) > 0$

$$f(x) \geq f(1)$$

$$\underline{\underline{f(x) \geq -1}}$$

(ε) $f(x) = e^x + \frac{x^2}{2} - 2x + ημx \quad , x \in R.$

$f'(x) = e^x + \frac{1}{2}\, 2x - 2 + συνx$

$f'(x) = e^x + x - 2 + συνx$ $\qquad \underline{\underline{f'(0) = 0}}$

$f''(x) = e^x + \underbrace{1 - ημx}_{+} > 0$

- $-1 \leq ημx \leq 1$

$1 \geq -ημx \geq -1$

$2 \geq 1 - ημx \geq 0$

x		0	
f''	$+$		$+$
f'	$\nearrow$ $-$	0	$\nearrow$ $+$
f	$\searrow$		$\nearrow$

$x < 0 \Rightarrow f'(x) < f'(0) \Rightarrow f'(x) < 0$

$x > 0 \Rightarrow f'(x) > f'(0) \Rightarrow f'(x) > 0$

$f(x) \geq f(0)$

$f(x) \geq 1$

21. (δ) $f(x) = \begin{cases} \dfrac{1-\sigma\upsilon\nu x}{x}, & x \in (0, \frac{\pi}{2}] \\ 0, & x = 0 \end{cases}$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} \frac{1-\sigma\upsilon\nu x}{x} = \lim_{x\to 0} \frac{-(-\eta\mu x)}{1} = 0$$

$f(0) = 0$ Συνεχής στο 0!

$$f'(x) = \frac{\eta\mu x \cdot x - (1-\sigma\upsilon\nu x)}{x^2} = \frac{x\,\eta\mu x - 1 + \sigma\upsilon\nu x}{x^2}$$

$\varphi(x) = x\,\eta\mu x - 1 + \sigma\upsilon\nu x$ $\quad \underline{\underline{\varphi(0) = 0}}$

$\varphi'(x) = \cancel{\eta\mu x} + x\,\sigma\upsilon\nu x \cancel{- \eta\mu x}$

$\varphi'(x) = \overset{(+)}{x}\,\overset{(+)}{\sigma\upsilon\nu x} > 0$ $\quad x \in (0, \frac{\pi}{2})$

x	0		$\frac{\pi}{2}$
φ'		$+$	
φ	0	$+ \nearrow$	
x^2		$+$	
f'		$+$	
f		$\nearrow$	

$x > 0 \Rightarrow \varphi(x) > \varphi(0) \Rightarrow \varphi(x) > 0$

21. (B) $f(x) = \dfrac{x^3 - 2\ln x}{2x}$, $x > 0$

$$f'(x) = \frac{\left(3x^2 - 2\frac{1}{x}\right)2x - \left(x^3 - 2\ln x\right)2}{4x^2}$$

$$f'(x) = \frac{6x^3 - 4 - 2x^3 + 4\ln x}{4x^2}$$

$$f'(x) = \frac{4x^3 + 4\ln x - 4}{4x^2}$$

$$\boxed{f'(x) = \frac{x^3 + \ln x - 1}{x^2}}$$

$g(x) = x^3 + \ln x - 1$ $\qquad$ $g(1) = 0$,

$g'(x) = 3x^2 + \dfrac{1}{x} > 0$

x	0		1	
g'		$+$		$+$
g		$\nearrow$ $-$	0	$+$ $\nearrow$
x^2		$+$		$+$
f'		$-$		$+$
f		$\searrow$		$\nearrow$

$x<1 \Rightarrow g(x)<g(1) \Rightarrow g(x)<0$

$x>1 \Rightarrow g(x)>g(1) \Rightarrow g(x)>0$

$$f(x) \geq f(1)$$

$$f(x) \geq \frac{1}{2}.$$

22. (B) $f(x) = x\ln x - \frac{x^2}{2}$, $x>0$.

$f'(x) = \ln x + x\frac{1}{x} - \frac{1}{2}2x$

$f'(x) = \ln x + 1 - x \leq 0$. $\quad f \downarrow$

• $\ln x \leq x-1 \Rightarrow \ln x - x + 1 \leq 0$

(δ) $f(x) = x^2 \ln x + \frac{1}{2}x^2 - \frac{2}{3}x^3$, $x>0$.

$f'(x) = 2x\ln x + x^2\frac{1}{x} + \frac{1}{2}2x - \frac{2}{3}3x^2$

$f'(x) = 2x\ln x + x + x - 2x^2$

$f'(x) = 2x\ln x + 2x - 2x^2$

$f'(x) = \underset{+}{2x}\,\underset{-}{(\ln x + 1 - x)} \leq 0$ $\quad f \downarrow$.

• $\ln x \leq x-1 \Rightarrow \ln x - x + 1 \leq 0$

23. Ⓑ $f(x) = \frac{1}{2}x^2 + x - x\ln x \quad , x>0$

$f'(x) = \frac{1}{2}2x + 1 - \ln x - x\frac{1}{x}$

$f'(x) = x + \cancel{1} - \ln x - \cancel{1}$

$f'(x) = x - \ln x > 0 \quad f \nearrow$

- $\ln x \leq x - 1$

$1 \leq x - \ln x$

$0 < x - \ln x$

⑥ $f(x) = (x+1)\ln x + e^x \quad , x>0$

$f'(x) = \ln x + (x+1)\frac{1}{x} + e^x$

$f'(x) = \ln x + 1 + \frac{1}{x} + e^x > 0 \quad f \nearrow$

- $\ln x \leq x - 1 \Rightarrow \ln\frac{1}{x} \leq \frac{1}{x} - 1 \Rightarrow \ln 1 - \ln x \leq \frac{1}{x} - 1$

$0 \leq \ln x + \frac{1}{x} - 1$

$2 \leq \ln x + \frac{1}{x} + 1$

$2 \leq \ln x + \frac{1}{x} + 1 + e^x$

23. σε $f(x) = e^{1-x} + \ln x \quad , x > 0$

$$f'(x) = -e^{1-x} + \frac{1}{x}$$

Σεναριο Α

$f'(x) = -e^{1-x} + \frac{1}{x} \qquad f'(1) = 0$

$f''(x) = e^{1-x} - \frac{1}{x^2} \qquad f''(1) = 0$

Παιζω την κολοκυθια.

Σεναριο Β

$$f'(x) = \frac{-xe^{1-x} + 1}{x}$$

$g(x) = 1 - xe^{1-x} \qquad g(1) = 0$

$g'(x) = -e^{1-x} + xe^{1-x} = e^{1-x}(x-1)$

$\rightarrow g'(x) = 0 \Rightarrow x - 1 = 0 \Rightarrow \underline{\underline{x = 1}}$

x	0		1	
g'		$-$	0	$+$
g		$\searrow$ $+$		$+$ $\nearrow$
x		$+$		$+$
f'		$+$		$+$
f		$\nearrow$		$\nearrow$

$g(x) \geq g(1) \Rightarrow g(x) \geq 0$

24. (B) $f(x) = \dfrac{x}{e^x - x}$ $\qquad A_f = \mathbb{R}$

Πρέπει $e^x - x \neq 0$ που ισχύει!

- $e^x \geq x+1 \Rightarrow e^x - x \geq 1 \Rightarrow e^x - x > 0$

$$f'(x) = \frac{e^x - x - x(e^x - 1)}{(e^x - x)^2}$$

$$f'(x) = \frac{e^x - \cancel{x} - xe^x + \cancel{x}}{(e^x - x)^2}$$

$$f'(x) = \frac{e^x(1-x)}{(e^x - x)^2}$$

$$f'(x) = 0 \Rightarrow \frac{e^x(1-x)}{(e^x-x)^2} = 0 \Rightarrow e^x(1-x) = 0$$

$1 - x = 0$

$x = 1$

x		1	
$1-x$	$+$		$-$
$(e^x-x)^2$	$+$		$+$
f'	$+$		$-$
f	$\nearrow$		$\searrow$

$f(x) \leq f(1)$

$f(x) \leq \dfrac{1}{e-1}$.

(δ) $f(x) = \frac{1}{x} - \frac{1}{εφx} \qquad x \in \left(0, \frac{\pi}{2}\right)$

$$f'(x) = -\frac{1}{x^2} - \frac{-\frac{1}{συν^2x}}{εφ^2x} = -\frac{1}{x^2} + \frac{1}{συν^2x\, εφ^2x}$$

$$f'(x) = -\frac{1}{x^2} + \frac{1}{\cancel{συν^2x}\, \frac{ημ^2x}{\cancel{συν^2x}}} = -\frac{1}{x^2} + \frac{1}{ημ^2x}$$

$$f'(x) = \frac{1}{ημ^2x} - \frac{1}{x^2} = \frac{\overset{+}{x^2 - ημ^2x}}{x^2 ημ^2x} \geq 0$$

• $|ημx| \leq |x| \Rightarrow |ημx|^2 \leq |x|^2 \qquad f \nearrow$

$$ημ^2x \leq x^2 \Rightarrow x^2 - ημ^2x \geq 0$$

25. (B) $f(x) = \frac{1}{2}x^2 + \sigma\upsilon\nu x$

$$f'(x) = \frac{1}{2}2x - \eta\mu x$$

$$f'(x) = x - \eta\mu x \qquad f'(0)=0$$

$$f''(x) = 1 - \sigma\upsilon\nu x \geq 0$$

$$-1 \leq \sigma\upsilon\nu x \leq 1$$

$$1 \geq -\sigma\upsilon\nu x \geq -1$$

$$2 \geq 1 - \sigma\upsilon\nu x \geq 0$$

x		0	
f''	$+$		$+$
f'	$\nearrow$ $-$	0	$\nearrow$ $+$
f	$\searrow$		$\nearrow$

$x<0 \Rightarrow f'(x) < f'(0) \Rightarrow f'(x)<0$

$x>0 \Rightarrow f'(x) > f'(0)$

$f'(x)>0$

$$f(x) \geq f(0)$$

$$f(x) \geq 1$$

8. $f(x) = x\,\varepsilon\varphi x \quad , \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$

$$f'(x) = \varepsilon\varphi x + x \cdot \frac{1}{\sigma\upsilon\nu^2 x}$$

$$f'(x) = \frac{\eta\mu x}{\sigma\upsilon\nu x} + \frac{x}{\sigma\upsilon\nu^2 x}$$

$$f'(x) = \frac{\eta\mu x\,\sigma\upsilon\nu x + x}{\sigma\upsilon\nu^2 x}.$$

$g(x) = \eta\mu x\,\sigma\upsilon\nu x + x \qquad g(0) = 0$

$$g'(x) = \sigma\upsilon\nu^2 x - \eta\mu^2 x + 1$$

$$g'(x) = \sigma\upsilon\nu^2 x + \sigma\upsilon\nu^2 x > 0$$

x	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$
g'	$+$		$+$
g	$\nearrow$ $-$	0	$\nearrow$ $+$
$\sigma\upsilon\nu^2 x$	$+$		$+$
f'	$-$		$+$
f	$\searrow$		$\nearrow$

$x < 0 \Rightarrow g(x) < g(0) \Rightarrow g(x) < 0$

$x > 0 \Rightarrow g(x) > g(0) \Rightarrow g(x) > 0$

Βασικές ΓΝΩΣΤΕΣ

ανισότητες της ύλης

1. $e^x \geq x+1 \quad \forall x \in \mathbb{R}$

2. $\ln x \leq x-1 \quad \forall x > 0$

3. $|\eta\mu x| \leq |x| \quad \forall x \in \mathbb{R}$.

1. Νδο $e^x \geq x+1$

$\underbrace{e^x - x - 1}_{f(x)} \geq 0 \Rightarrow$ Αρκει νδο $f(x) \geq 0$

$f'(x) = e^x - 1$

$\rightarrow f'(x) = 0 \Rightarrow e^x - 1 = 0 \Rightarrow e^x = 1$

$x = 0$

x		0	
f'	$-$	0	$+$
f	$\searrow$		$\nearrow$

$f(x) \geq f(0)$

$e^x - x - 1 \geq 0$

$e^x \geq x + 1$.

2. Νδο $\ln x \le x - 1$

$\underbrace{\ln x - x + 1}_{f(x)} \le 0 \Rightarrow$ Αρκει νδο $f(x) \le 0$

$$f'(x) = \frac{1}{x} - 1 = \frac{1-x}{x}$$

$$f'(x) = 0 \Rightarrow \frac{1-x}{x} = 0 \Rightarrow 1 - x = 0 \Rightarrow x = 1$$

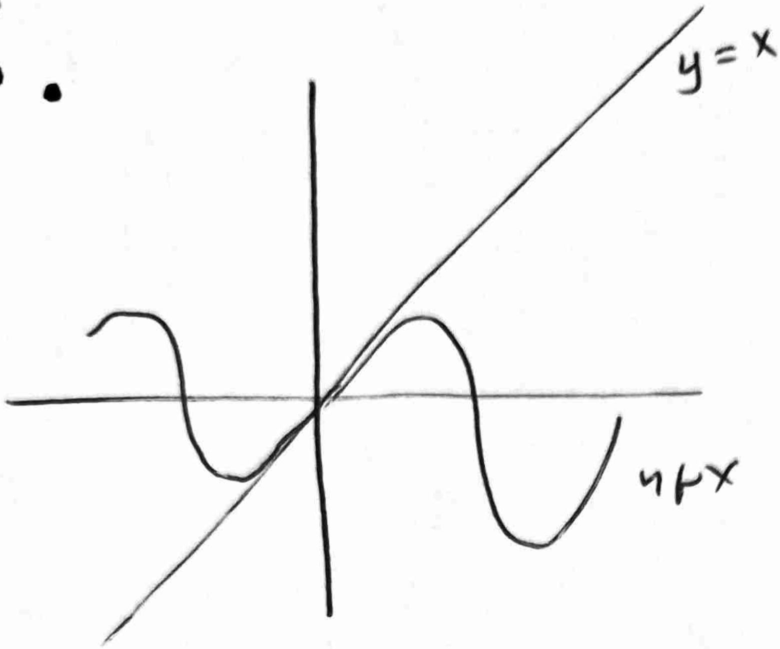
x	0	1	$+\infty$
$1-x$	$+$	$-$	
x	$+$	$+$	
f'	$+$	$-$	
f	$\nearrow$	$\searrow$	

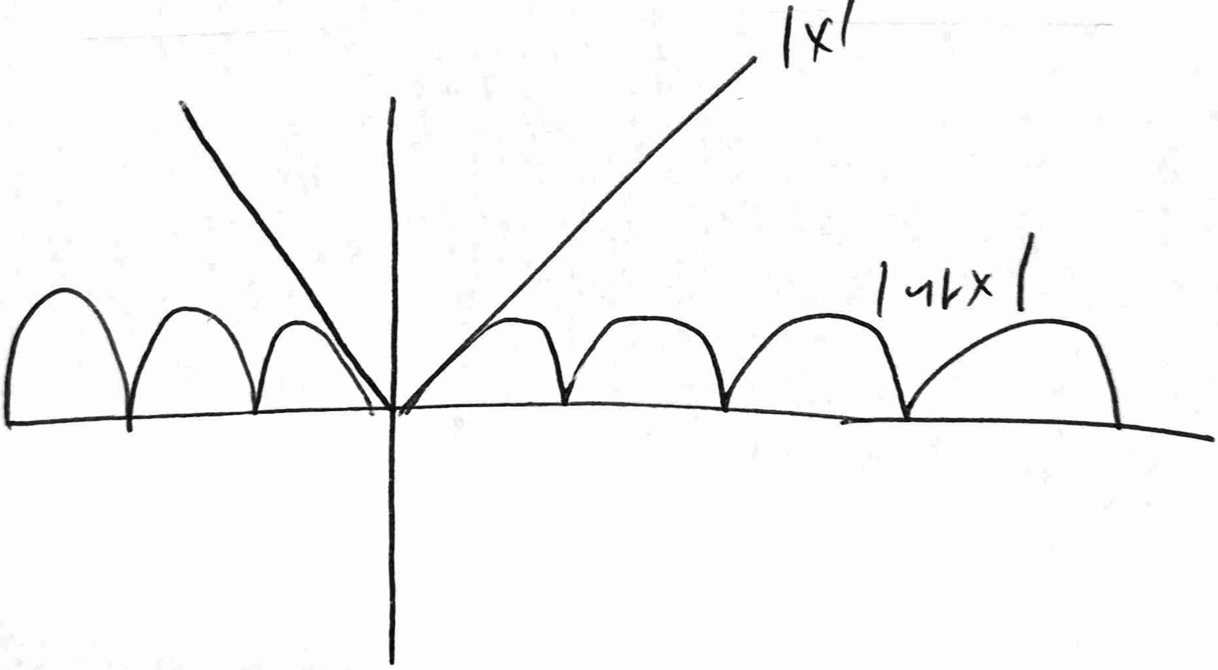
$$f(x) \le f(1)$$

$$\ln x - x + 1 \le 0$$

$$\ln x \le x - 1$$

3.





$|\sin x| \le |x|$.

Επομενο Μαθημα

23

8 α γ

9 α γ ε

10 α γ ε

11 α γ

12 α γ ε ζ

13 α γ ε

14 α γ

15 α.

16 α.

20 α δ

21 α γ.

22 α γ

23 α γ ε

24 α γ

25 α γ.