

43. (a) No $\frac{a^3+b^3}{a+b} \geq ab$, $a, b \in \mathbb{R}$

$$\frac{(a+b)(a^2-ab+b^2)}{a+b} \geq ab$$

$$a^2-ab+b^2 \geq ab$$

$$a^2-2ab+b^2 \geq 0$$

$$(a-b)^2 \geq 0$$

(B) $\frac{a^3+b^3}{(a+b)^3} \geq \frac{1}{4}$

$$\frac{(a+b)(a^2-ab+b^2)}{(a+b)^3} \geq \frac{1}{4}$$

$$\frac{a^2-ab+b^2}{(a+b)^2} \geq \frac{1}{4}$$

$$4a^2-4ab+4b^2 \geq a^2+2ab+b^2$$

$$3a^2-6ab+3b^2 \geq 0$$

$$3(a^2-2ab+b^2) \geq 0$$

$$3(a-b)^2 \geq 0$$

$$\Rightarrow 4a^2-4ab+4b^2 \geq (a+b)^2$$

$$36. \quad \textcircled{a} \quad \begin{aligned} 3 \leq x \leq 5 &\Rightarrow -3 \geq -x \geq -5 \\ -2 \leq y \leq -1 &\Rightarrow -1 \geq y \geq -2 \end{aligned} \quad \left. \vphantom{\begin{aligned} 3 \leq x \leq 5 \\ -2 \leq y \leq -1 \end{aligned}} \right\} \textcircled{+}$$

$$-4 \geq y - x \geq -7$$

$$\textcircled{b} . \quad \begin{aligned} 3 \leq x \leq 5 &\Rightarrow 9 \leq x^2 \leq 25 \quad \rightarrow \textcircled{+} \\ -2 \leq y \leq -1 &\Rightarrow 4 \geq y^2 \geq 1 \Rightarrow 1 \leq y^2 \leq 4 \end{aligned}$$

$$\underline{\underline{10 \leq x^2 + y^2 \leq 29}}$$

$$\textcircled{1} \quad \left. \begin{aligned} 3 \leq x \leq 5 \\ -2 \leq y \leq -1 \end{aligned} \right\} \Rightarrow -\frac{1}{2} \geq \frac{1}{y} \geq -1 \quad \textcircled{-}$$

$$\frac{1}{2} \leq -\frac{1}{y} \leq 1$$

$$\frac{3}{2} \leq -\frac{x}{y} \leq 5$$

$$\underline{\underline{-\frac{3}{2} \geq \frac{x}{y} \geq -5}}$$

35.

$$\textcircled{a} \quad \left. \begin{array}{l} 2 \leq x \leq 3 \\ 1 \leq y \leq 2 \end{array} \right\} \textcircled{+} \quad \underline{\underline{3 \leq x+y \leq 5}}$$

$$\textcircled{b} \quad \begin{array}{lll} 2 \leq x \leq 3 & \Rightarrow 4 \leq 2x \leq 6 & \Rightarrow 6 \geq 2x \geq 4 \\ 1 \leq y \leq 2 & \Rightarrow -3 \geq -3y \geq -6 & \quad \quad \quad \textcircled{+} \end{array}$$

$$\underline{\underline{3 \geq 2x - 3y \geq -2}}$$

$$\textcircled{c} \quad \begin{array}{lll} 2 \leq x \leq 3 & \Rightarrow 3 \geq x \geq 2 & \\ 1 \leq y \leq 2 & \Rightarrow 1 \geq \frac{1}{y} \geq \frac{1}{2} & \end{array} \quad \left. \right\} \textcircled{+} \quad 3 \geq \frac{x}{y} \geq 1$$

$$\textcircled{d} \quad \begin{array}{lll} 2 \leq x \leq 3 & \Rightarrow 4 \leq x^2 \leq 9 & \Rightarrow 9 \geq x^2 \geq 4 \quad \textcircled{+} \\ 1 \leq y \leq 2 & \Rightarrow 1 \leq y^2 \leq 4 & \Rightarrow -1 \geq -y^2 \geq -4 \end{array}$$

$$\underline{\underline{8 \geq x^2 - y^2 \geq 0}}$$

38.

Minimálny počet

$$L = 2np \Rightarrow \underline{L = 2nx}$$

Minimálny počet

$$E = np^2$$

$$E = \underline{nx^2}$$

$$0,9 < x < 1,1$$

$$\Rightarrow 0,9 \cdot 2n < 2nx < 2n \cdot 1,1$$

$$\boxed{1,8n < L < 2,2n}$$

$$0,9 < x < 1,1$$

$$0,81 < x^2 < 1,21$$

$$0,81n < nx^2 < 1,21n$$

$$0,81n < E < 1,21n$$

39.

$$1,9 < x < 2,1$$

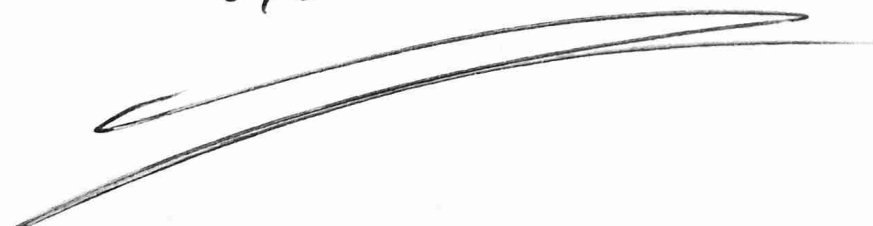
$$0,7 < y < 0,8$$

$$\Pi = 3y + x - y + 2y + x$$

$$\Pi = 4y + 2x$$

$$\begin{aligned} \bullet 1,9 < x < 2,1 &\Rightarrow 3,8 < 2x < 4,2 \\ \bullet 0,7 < y < 0,8 &\Rightarrow 2,8 < 4y < 3,2 \end{aligned} \quad \left. \vphantom{\begin{aligned} \bullet 1,9 < x < 2,1 \\ \bullet 0,7 < y < 0,8 \end{aligned}} \right\} (+)$$

$$6,6 < 2x + 4y < 7,4$$

$$6,6 < \Pi < 7,4$$


Exercícios 8

1. (a) $|+5| = 5$

(b) $|-3| = 3$

(c) $|0| = 0$

(d) $|-7| + |+3| - |1-3| =$

$$= 7 + 3 - |-2| =$$

$$= 10 - 2 = 8$$

(e) $|1+|-2|| - |3-|-5||$

$$= |1+2| - |3-5| =$$

$$= |3| - |-2| =$$

$$= 3 - 2$$

$$5. \textcircled{\beta} \left| 6x - 9 - x^2 \right| - \left| x^4 + 1 \right|^{\textcircled{+}}$$

$$= \left| - (x^2 - 6x + 9) \right| - (x^4 + 1)$$

$$= \left| - \textcircled{-} (x - 3)^2 \right| - x^4 - 1$$

$$= (x - 3)^2 - x^4 - 1$$

$$4. \quad \textcircled{a} \quad |x^2+1|^{\textcircled{+}} = x^2+1$$

$$\textcircled{b} \quad |{-x^2}|^{\textcircled{-}} = x^2$$

$$\textcircled{c} \quad |x^2-2x+1| = |(x-1)^2|^{\textcircled{+}} = (x-1)^2$$

$$\textcircled{d} \quad |-x^2+10x-25| = |-(x^2-10x+25)|$$

$$= |-(x-5)^2|^{\textcircled{-}} = (x-5)^2$$

$$\textcircled{e} \quad |(|x|+1)|^{\textcircled{+}} - |x| = \cancel{|x|+1} - \cancel{|x|} = 1$$

$$3. \textcircled{B} \quad |n-4| + |\sqrt{2}-2| - |\sqrt{2}-\sqrt{3}| =$$

$$\bullet \quad n < 4 \Rightarrow n-4 < 0$$

$$\bullet \quad \sqrt{2} < 2 \Rightarrow \sqrt{2}-2 < 0$$

$$\bullet \quad \sqrt{2} < \sqrt{3} \Rightarrow \sqrt{2}-\sqrt{3} < 0$$

$$= -n+4 -\sqrt{2}+2 -(-\sqrt{2}+\sqrt{3})$$

$$= -n+4 -\cancel{\sqrt{2}}+2 +\cancel{\sqrt{2}}-\sqrt{3}$$

$$= -n+6-\sqrt{3}$$

Επαναληπτική άσκηση

1. Να αποδειχθούν οι ανισότητες.

(α) Αν $a < -3$ και $b < 2$ νδo $ab - 6 > 2a - 3b$

(β) Νδo $b^2 \geq \frac{ab}{2} - \frac{(a-b)^2}{8}$

(γ) Νδo $x^2 + 6x + 2026 > 0$

(δ) Νδo αν $a+b \geq 0$ τότε $a^3 + b^3 \geq ab(a+b)$

(ε) Νδo $(a^2 + b^2)(x^2 + y^2) \geq (ax + by)^2$

(ς) Αν $x \geq -1$ νδo $x^3 - x^2 \geq x - 1$

(ζ) Αν $a < b < -4$ νδo $4b - a(4-b) < a^2$

(η) Αν a, b ορθοί και $a \leq b$

νδo $a - \frac{1}{a} \leq b - \frac{1}{b}$.

2. (α) $\forall x \quad x^2 + x + 1 > 0$

(β) $\forall x \neq 0 \quad x^2 + \frac{1}{x^2} \geq x + \frac{1}{x}$

3. $\forall a, b, \gamma \quad a^2 + b^2 + \gamma^2 \geq ab + b\gamma + \gamma a$.

4. (α) $\forall x, y > 0 \quad \frac{x}{y} + \frac{y}{x} \geq 2$

(β) $\forall a, b, \gamma > 0$

$$\frac{a+b}{\gamma} + \frac{b+\gamma}{a} + \frac{\gamma+a}{b} \geq 6$$

5. $\subseteq \sigma \tau \omega$ $0 < x < y$

Να συγκρίνουμε οι αριθμοί

$$A = (x-2y)^2 \quad B = y(4y-3x)$$

6. На брадов та $x, y \in \mathbb{R}$.

(a) $x^2 + y^2 + 10 = 2(x - 3y)$

(b) $(x+5)^2 + (y-2)^2 = 4(x+y+1)$.

7. $\subset \sigma \tau \omega$ $1 < x < 2$ ка $2 < y < 3$

(a) $2x - 4y$ (b) $x^2 - \frac{1}{x}$ (r) $y^2 - \frac{x}{y}$

(s) $(x+y)^2 - 2x$

(ε) $\frac{x-y}{x+y}$.