Ενότητα 23

2

ⓐ $f(x) = e^x + x^3 - 1$, $x \in R$

$f'(x) = e^x + 3x^2 > 0$ $f \nearrow$

ⓨ $f(x) = -x^3 + x^2 - x + 1$, $x \in R$.

$f'(x) = -3x^2 + 2x - 1 < 0 \Rightarrow f \downarrow$

$\Delta = 4 - 4(-3)\cdot(-1)$

$\Delta = 4 - 12$

$\Delta = -8$

ⓔ $f(x) = 1 + (x-2)^3$

$f'(x) = 3(x-2)^2 \geq 0$ $f \nearrow$

3.

(α) $f(x) = x^2 - 3x + 1$

$f'(x) = 2x - 3$

$\rightarrow f'(x) = 0 \Rightarrow 2x - 3 = 0 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$

x		$\frac{3}{2}$	
f'	$-$	0	$+$
f	$\searrow$		$\nearrow$

Η $f \downarrow (-\infty, \frac{3}{2}]$ και

Η $f \nearrow [\frac{3}{2}, +\infty)$

$f(x) \geq f(\frac{3}{2})$ Το $A(\frac{3}{2}, f(\frac{3}{2}))$ Ο.Ε.

$f(x) \geq -\frac{5}{4}$

(γ) $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1$

$f'(x) = \frac{1}{\cancel{3}} \cancel{3} x^2 - 2 \cdot 2x + 3$

$$\boxed{f'(x) = x^2 - 4x + 3}$$

x		1		3	
f'	$+$	0	$-$	0	$+$
f	$\nearrow$		$\searrow$		$\nearrow$

$\rightarrow f'(x) = 0 \Rightarrow x^2 - 4x + 3 = 0$

$\Delta = 16 - 12 = 4$

$x = \frac{4 \pm 2}{2}$ 3, 1

ε) $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

x		-1	1	
f'	$+$		$-$	$+$
f	$\nearrow$		$\searrow$	$\nearrow$

$\rightarrow f'(x) = 0 \Rightarrow 3x^2 - 3 = 0 \Rightarrow 3x^2 = 3 \Rightarrow x^2 = 1$

$x = 1$ $x = -1$

4. ⓐ $f(x) = \frac{x^4}{4} - x + 1$

$f'(x) = \frac{1}{4} \cdot 4x^3 - 1 = x^3 - 1$

x		1	
f'	$-$	0	$+$
f	$\searrow$		$\nearrow$

$\rightarrow f'(x) = 0 \Rightarrow x^3 - 1 = 0 \Rightarrow x^3 = 1$

$x = 1$

$f(x) \geq f(1)$

$f(x) \geq \frac{1}{4}$

γ) $f(x) = \frac{1}{4} x^4 - 2x$

$f'(x) = \frac{1}{\cancel{4}} \cancel{4} x^3 - 2 = x^3 - 2$

$f'(x) = 0 \Rightarrow x^3 - 2 = 0 \Rightarrow x^3 = 2 \Rightarrow x = \sqrt[3]{2}$

x		$\sqrt[3]{2}$	
f'	$-$		$+$
f	$\searrow$		$\nearrow$

$f(x) \geq f(\sqrt[3]{2})$

(ε) $f(x) = -\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 - 2x$

$$f'(x) = -\frac{1}{\cancel{4}}\cancel{4}x^3 + \frac{2}{\cancel{3}}\cancel{3}x^2 + \frac{1}{\cancel{2}}\cancel{2}x - 2$$

$$\boxed{f'(x) = -x^3 + 2x^2 + x - 2}$$

-1	2	1	-2	2
$\downarrow$	-2	0	2	
-1	0	1	0	

$$\boxed{f'(x) = (x-2)(-x^2+1)}$$

$f'(x) = 0 \Rightarrow x - 2 = 0$ ή $1 - x^2 = 0$

$x = 2$ $\quad$ $x = 1$ $\quad$ $x = -1$

x		-1		1		2	
$x-2$	$-$		$-$		$-$	0	$+$
$1-x^2$	$-$	0	$+$	0	$-$		$-$
f'	$+$		$-$		$+$		$-$
f	↗		↘		↗		↘

5. ⓐ $f(x) = x^4 - 2x^2 - 1$.

$$f'(x) = 4x^3 - 4x = 4x(x^2-1)$$

$\rightarrow f'(x) = 0 \Rightarrow 4x = 0$ ή $x^2 - 1 = 0$

$x=0$ $\quad x=1$ $\quad x=-1$

x		-1		0		1	
$4x$	$-$		$-$	0	$+$		$+$
x^2-1	$+$	0	$-$		$-$	0	$+$
$f'(x)$	$-$		$+$		$-$		$+$
$f(x)$	↘		↗		↘		↗

ⓑ $f(x) = 5x - x^5$

$$f'(x) = 5 - 5x^4 = 5(1-x^4) = 5(1-x^2)(1+x^2)$$

$\rightarrow f'(x) = 0 \Rightarrow 1 - x^2 = 0$

$x=1$ $\quad x=-1$

x		-1		1	
$1-x^2$	$-$		$+$		$-$
x^2+1	$+$		$+$		$+$
f'	$-$		$+$		$-$
f	↘		↗		↘

(δ) $f(x) = \frac{1}{5}x^5 - x^4 - \frac{1}{2}x^2 + 4x$

$$f'(x) = \frac{1}{5}5x^4 - 4x^3 - \frac{1}{2}2x + 4$$

$$f'(x) = x^4 - 4x^3 - x + 4 \quad = (x-1)(x^3 - 3x^2 - 3x - 4)$$

$$\boxed{f'(x) = (x-1)(x-4)(x^2+x+1)}$$

1	-4	0	-1	4		1
↓	1	-3	-3	-4		
1	-3	-3	-4	0		

1	-3	-3	-4	4
↓	4	4	4	
1	1	1	0	

$\longrightarrow f'(x) = 0 \Rightarrow x - 1 = 0$ ή $x - 4 = 0$ ή $x^2 + x + 1 = 0$

$\Delta < 0$

$x = 1$ $\qquad$ $x = 4$

x		1		4	
$x-1$	$-$	0	$+$		$+$
$x-4$	$-$		$-$	0	$+$
x^2+x+1	$+$		$+$		$+$
$f'(x)$	$+$		$-$		$+$
$f(x)$	↗		↘		↗

6. (α) $f(x) = \frac{1}{3}x^3 - x^2 + x - 1$

$f'(x) = \frac{1}{3}3x^2 - 2x + 1$

$f'(x) = x^2 - 2x + 1$

$f'(x) = (x-1)^2 \geq 0$ $\quad f \nearrow$

B' τρόπος

$f'(x) = x^2 - 2x + 1$

$\Delta = 0$

$x = 1$

x		1	
f'	+		+
f	$\nearrow$		$\nearrow$

(γ) $f(x) = -3x^4 + 4x^3 + 2$

$f'(x) = -12x^3 + 12x^2 = -12x^2(x-1)$

$\rightarrow f'(x) = 0 \Rightarrow -12x^2 = 0$ ή $x - 1 = 0$

$x = 0$ $\qquad$ $x = 1$

x		0		1	
$-12x^2$	−		−		−
$x-1$	−		−	0	+
f'	+		+		−

6. (ε) $f(x) = (x-1)^3 \cdot (x-3)^3$

$$f'(x) = 3(x-1)^2 (x-3) + (x-1)^3\, 3(x-3)^2$$

$$f'(x) = 3\,(x-1)^2 (x-3)\,(1 + x - 1)$$

$$\boxed{f'(x) = 3(x-1)^2 (x-3)\, x}$$

$\rightarrow f'(x) = 0 \Rightarrow (x-1)^2 = 0$ ή $x-3=0$ ή $x=0$

$x=1$ $\quad$ $x=3$

x		0	1	3	
$(x-1)^2$	+	+	0 +	+	
$(x-3)$	−	−	−	0 +	
x	−	0 +	+	+	
f'	+	−	−	+	
f	↗	↘	↘	↗	

7. (α) $f(x) = \dfrac{x}{x+1}$, $x \neq -1$

$$f'(x) = \frac{\cancel{x}+1-\cancel{x}}{(x+1)^2} = \frac{1}{(x+1)^2} > 0$$

$f \nearrow$ $(-\infty,-1)$ και $(-1,+\infty)$.

(β) $f(x) = x + \dfrac{1}{x}$, $x \neq 0$.

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$$

$\longrightarrow f'(x)=0 \rightarrow \dfrac{x^2-1}{x^2} = 0 \Rightarrow x^2-1=0$

$x=1$ $x=-1$

x		-1		0		1	
x^2-1	$+$	0	$-$	‖	$-$	0	$+$
x^2	$+$		$+$	‖	$+$		$+$
f'	$+$		$-$	‖	$-$		$+$
f	$\nearrow$		$\searrow$	‖	$\searrow$		$\nearrow$

7. ε) $f(x) = x^2 + \frac{2}{x}$, $x \neq 0$,

$$f'(x) = 2x + \frac{-2}{x^2} = \frac{2x^3 - 2}{x^2} = 2\,\frac{x^3-1}{x^2}$$

$$f'(x) = 0 \Rightarrow 2\,\frac{x^3-1}{x^2} = 0 \Rightarrow x^3 - 1 = 0$$

$$x^3 = 1$$

$$x = 1$$

x		0		1	
x^3-1	$-$		$-$	0	$+$
x^2	$+$		$+$		$+$
f'	$-$		$-$		$+$
f	↘		↘		↗

8.

(B) $f(x) = \ln|x| + \frac{1}{2x^2}$ $\qquad D_f = \mathbb{R}^*$

$$f'(x) = \frac{1}{x} + \frac{-4x}{4x^4} = \frac{1}{x} - \frac{1}{x^3} = \frac{x^2-1}{x^3}$$

$\longrightarrow f'(x) = 0 \Rightarrow \frac{x^2-1}{x^3} = 0 \Rightarrow x^2 - 1 = 0$

$x = 1$ $\quad x = -1$

x		-1		0		1	
x^2-1	$+$	0	$-$	‖	$-$	0	$+$
x^3	$-$		$-$	0	$+$		$+$
f'	$-$		$+$	‖	$-$		$+$
f	$\searrow$		$\nearrow$	‖	$\searrow$		$\nearrow$

(δ) $f(x) = \frac{x^3 - 2x}{x^2 - 1}$, $x \in \mathbb{R} - \{1, -1\}$.

$$f'(x) = \frac{(3x^2-2)(x^2-1) - (x^3-2x)2x}{(x^2-1)^2}$$

$$f'(x) = \frac{3x^4 - 3x^2 - 2x^2 + 2 - 2x^4 + 4x^2}{(x^2-1)^2}$$

$$f'(x) = \frac{x^4 - x^2 + 2}{(x^2-1)^2}$$

$$f'(x)=0 \Rightarrow \frac{x^4-x^2+2}{(x^2-1)^2}=0 \rightarrow x^4-x^2+2=0$$

$$\underline{\underline{x^2=t}}$$

$$t^2-t+2=0.$$

$$\underline{\underline{\Delta<0}}$$

$$\Rightarrow \boxed{x^4-x^2+2\neq 0}$$

x		-1		1	
x^4-x^2+2	$+$	‖	$+$	‖	$+$
$(x^2-1)^2$	$+$	‖	$+$	‖	$+$
f'	$+$	‖	$+$	‖	$+$
f	$\nearrow$	‖	$\nearrow$	‖	$\nearrow$

9. (B) $f(x) = \sqrt{4-x^2}$

При $4-x^2 \geq 0$

x		-2		2	
$4-x^2$	$-$		$+$		$-$

$D_f = [-2, 2]$

$$f'(x) = \frac{1}{2\sqrt{4-x^2}} (4-x^2)'$$

$$f'(x) = \frac{-\cancel{2}x}{\cancel{2}\sqrt{4-x^2}} = \frac{-x}{\sqrt{4-x^2}}$$

$$f'(x) = 0 \Rightarrow \frac{-x}{\sqrt{4-x^2}} = 0 \Rightarrow -x = 0$$

$x = 0$.

x	-2	0	2
$-x$	$+$	$-$	
$\sqrt{4-x^2}$	$+$	$+$	
f'	$+$	$-$	
f	$\nearrow$	$\searrow$	

$f(x) \leq f(0)$

$f(x) \leq 2$

9. δ) $f(x) = x - 4\sqrt{x}$, $D_f = [0, +\infty)$

$$f'(x) = 1 - 4 \cdot \frac{1}{2\sqrt{x}} = 1 - \frac{2}{\sqrt{x}} = \frac{\sqrt{x} - 2}{\sqrt{x}}$$

$$f'(x) = 0 \Rightarrow \frac{\sqrt{x} - 2}{\sqrt{x}} = 0 \Rightarrow \sqrt{x} - 2 = 0$$

$$\sqrt{x} = 2$$

$$x = 4$$

x	0		4	
$\sqrt{x} - 2$	$-$		0	$+$
$\sqrt{x}$	$+$			$+$
f'	$-$			$+$
f	$\searrow$			$\nearrow$

$$f(x) \geq f(4)$$

$$f(x) \geq -4$$

9.

02 $f(x) = x - 2\sqrt{x-2}\ ,\ x \geq 2$

$$f'(x) = 1 - \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{x-2}} = \frac{\sqrt{x-2} - 1}{\sqrt{x-2}}$$

$f'(x) = 0 \Rightarrow \sqrt{x-2} - 1 = 0$

$\sqrt{x-2} = 1$

$x - 2 = 1$

$x = 3$.

x	2	3	
$\sqrt{x-2} - 1$	$-$	0	$+$
$\sqrt{x-2}$	$+$		$+$
f'	$-$		$+$
f	$\searrow$		$\nearrow$

$f(x) \geq f(3)$

$f(x) \geq \underline{1}$

10. β) $f(x) = e^x - ex \quad , x \in \mathbb{R}.$

$$f'(x) = e^x - e$$

$\rightarrow f'(x) = 0 \Rightarrow e^x - e = 0$

$$e^x = e$$

$$x = 1.$$

x		1	
f'	$-$	0	$+$
f	$\searrow$		$\nearrow$

$f(x) \geq f(1)$

$f(x) \geq 0$

δ) $f(x) = (x^2+x+1)\, e^x$

$$f'(x) = (2x+1)\, e^x + (x^2+x+1)\, e^x$$

$$\boxed{f'(x) = e^x (x^2+3x+2)}$$

$f'(x) = 0 \Rightarrow x^2+3x+2 = 0$

$x = -2 \quad x = -1$

x		-2		-1	
e^x	$+$		$+$		$+$
x^2+3x+2	$+$	0	$-$	0	$+$
f'	$+$		$-$		$+$
f	$\nearrow$		$\searrow$		$\nearrow$

11. δ) $f(x) = e^{x} - e^{-x} - 2x$

$$f'(x) = e^{x} + e^{-x} - 2$$

$$f'(x) = e^{x} + \frac{1}{e^{x}} - 2$$

$$f'(x) = \frac{e^{2x} + 1 - 2e^{x}}{e^{x}} = \frac{(e^{x}-1)^2}{e^{x}}$$

$$f'(x) = \frac{(e^{x}-1)^2}{e^{x}} \geq 0$$

$f \nearrow$.

10. 52 $f(x) = \dfrac{e^x}{x^2+1}$ $\qquad D_f = \mathbb{R}$.

$$f'(x) = \frac{e^x(x^2+1) - e^x \cdot 2x}{(x^2+1)^2}$$

$$f'(x) = \frac{e^x(x^2-2x+1)}{(x^2+1)^2} = \frac{e^x(x-1)^2}{(x^2+1)^2} \geq 0$$

$f \nearrow$

11. B $f(x) = x e^{\frac{1}{x}}$, $\quad x \neq 0$

$$f'(x) = e^{\frac{1}{x}} + x \cdot e^{\frac{1}{x}} \left(-\frac{1}{x^2}\right)$$

$$f'(x) = e^{\frac{1}{x}} \left(1 - \frac{1}{x}\right) = e^{\frac{1}{x}} \frac{x-1}{x}$$

$\rightarrow f'(x) = 0 \quad \Rightarrow e^{\frac{1}{x}} \cdot \frac{x-1}{x} = 0 \quad \Rightarrow \frac{x-1}{x} = 0$

$x - 1 = 0$

$x = 1$

x		0		1	
$e^{\frac{1}{x}}$	+	‖	+		+
$x-1$	−	‖	−	0	+
x	−	‖	+		+
f'	+	‖	−		+
f	$\nearrow$	‖	$\searrow$		$\nearrow$

12. 8

$f(x) = \dfrac{\ln x}{\sqrt{x}}$, $D_f = (0, +\infty)$

$$f'(x) = \frac{\frac{1}{x}\sqrt{x} - \ln x \frac{1}{2\sqrt{x}}}{x}$$

$$f'(x) = \frac{2\cancel{x}\sqrt{x}\,\frac{1}{\cancel{x}}\sqrt{x} - \cancel{2}x\cancel{\sqrt{x}}\ln x \frac{1}{\cancel{2\sqrt{x}}}}{2x\sqrt{x}\;x}$$

$$f'(x) = \frac{2x - x\ln x}{2x^2\sqrt{x}} = \frac{2-\ln x}{2x\sqrt{x}}$$

$\rightarrow f'(x) = 0 \Rightarrow \dfrac{2-\ln x}{2x\sqrt{x}} = 0 \Rightarrow 2 - \ln x = 0$

$2 = \ln x$

$e^2 = e^{\ln x}$

$x = e^2$

x	0	e^2	$+\infty$
$2-\ln x$	$+$	0	$-$
$2x\sqrt{x}$	$+$		$+$
f'	$+$		$-$
f	$\nearrow$		$\searrow$

$f(x) \leq f(e^2)$

$f(x) \leq \frac{2}{e}$,

12. (B) $f(x) = x^2 \ln x \quad , x>0$

$$f'(x) = 2x \ln x + x^2 \cdot \frac{1}{x}$$

$$f'(x) = 2x \ln x + x$$

$$f'(x) = x(2\ln x + 1)$$

$\rightarrow f'(x) = 0 \Rightarrow x(2\ln x+1) = 0$

$\cancel{x=0}$

$$2\ln x + 1 = 0$$

$$\ln x = -\frac{1}{2}$$

$$e^{\ln x} = e^{-\frac{1}{2}}$$

$$x = \frac{1}{e^{1/2}} = \frac{1}{\sqrt{e}}$$

$$x = \frac{\sqrt{e}}{e}$$

x	0	$\frac{\sqrt{e}}{e}$	$+\infty$
x	$+$		$+$
$2\ln x+1$	$-$	0	$+$
f'	$-$		$+$
f	$\searrow$		$\nearrow$

$$f(x) \geq f(e^{-\frac{1}{2}})$$

$$f(x) \geq (e^{-\frac{1}{2}})^2 \ln e^{-\frac{1}{2}}$$

$$f(x) \geq e^{-1}\left(-\frac{1}{2}\right)$$

$$f(x) \geq -\frac{1}{2e}.$$

12. (a) $f(x) = x + \ln(x^2+1)$, $x \in \mathbb{R}$.

$$f'(x) = 1 + \frac{2x}{x^2+1} = \frac{x^2+1+2x}{x^2+1} = \frac{(x+1)^2}{x^2+1} \geq 0$$

$f \nearrow$

(b) $f(x) = x^{\ln x}$, $\underline{\underline{x>0}}$

$$f(x) = e^{\ln x^{\ln x}} = e^{\ln x \ln x} = e^{\ln^2 x}$$

$$f'(x) = e^{\ln^2 x}\, 2\ln x\, (\ln x)'$$

$$f'(x) = x^{\ln x}\, 2\ln x\, \frac{1}{x}$$

$$\boxed{f'(x) = 2\, x^{\ln x}\, \frac{\ln x}{x}}$$

x	0	1	$+\infty$
$x^{\ln x}$	$+$		$+$
$\ln x$	$-$		$+$
x	$+$		$+$
f'	$-$		$+$
f	$\searrow$		$\nearrow$

$\to f'(x) = 0 \Rightarrow 2\, x^{\ln x}\, \frac{\ln x}{x} = 0$

$\ln x = 0 \Rightarrow \underline{\underline{x = 1}}$

$f(x) \geq f(1)$

$\underline{\underline{f(x) \geq 1}}$

13. (B) $f(x) = 2xe^{-x} + (x-1)^2$

$f'(x) = 2e^{-x} - 2x\,e^{-x} + 2(x-1)$

$f'(x) = 2\left(e^{-x} - xe^{-x} + x - 1\right)$

$f'(x) = 2\left[e^{-x}(1-x) - (1-x)\right]$

$$\boxed{f'(x) = 2(1-x)\left(e^{-x} - 1\right)}$$

$\rightarrow f'(x) = 0 \Rightarrow 1 - x = 0$ ή $e^{-x} - 1 = 0$

$x = 1$

$e^{-x} = 1$

$\ln e^{-x} = \ln 1$

$-x = 0$

$x = 0$

x		0		1	
$1-x$	$+$		$+$	0	$-$
$e^{-x}-1$	$+$	0	$-$		$-$
f'	$+$		$-$		$+$
f	$\nearrow$		$\searrow$		$\nearrow$

13. (δ) $f(x) = (x^2 - x)\ln x + \frac{x^2}{2}$, $x>0$

$$f'(x) = (2x-1)\ln x + (x^2-x)\frac{1}{x} + \frac{1}{2}\cdot 2x$$

$$f'(x) = (2x-1)\ln x + x - 1 + x$$

$$f'(x) = (2x-1)\ln x + 2x - 1$$

$$\boxed{f'(x) = (2x-1)(\ln x + 1)}$$

$f'(x) = 0 \Rightarrow 2x-1=0$ ή $\ln x + 1 = 0$

$x = \frac{1}{2}$

$\ln x = -1$

$x = \frac{1}{e}$

x		$\frac{1}{e}$		$\frac{1}{2}$	
$2x-1$	$-$		$-$	0	$+$
$\ln x+1$	$-$	0	$+$		$+$
f'	$+$		$-$		$+$
f	↗		↘		↗

14. (B) $f(x) = \frac{x^2}{2} - x\,\sigma\upsilon\nu x + \eta\mu x$

$$f'(x) = \frac{1}{2}2x - (\sigma\upsilon\nu x - x\,\eta\mu x) + \sigma\upsilon\nu x$$

$$f'(x) = x - \cancel{\sigma\upsilon\nu x} + x\,\eta\mu x + \cancel{\sigma\upsilon\nu x}$$

$$\boxed{f'(x) = x(\underset{+}{1 + \eta\mu x})}$$

$$-1 \le \eta\mu x \le 1$$

$$0 \le 1 + \eta\mu x \le 2$$

$f'(x) = 0 \Rightarrow x = 0$

x		0	
x	$-$		$+$
$1+\eta\mu x$	$+$		$+$
f'	$-$		$+$
f	$\searrow$		$\nearrow$

$f(x) \ge f(0)$

$f(x) \ge 0$

13. 02

$$f(x) = \frac{x}{\ln^2 x}$$

$x > 0$

$\ln x \neq 0$

$x \neq 1$

$D_f = (0,1) \cup (1,+\infty)$.

$$f'(x) = \frac{\ln^2 x - x \cdot 2\ln x \cdot \frac{1}{x}}{\ln^4 x} = \frac{\ln x - 2}{\ln^3 x}$$

$$f'(x) = 0 \Rightarrow \frac{\ln x - 2}{\ln^3 x} = 0$$

$\ln x - 2 = 0$

$\ln x = 2$

$e^{\ln x} = e^2$

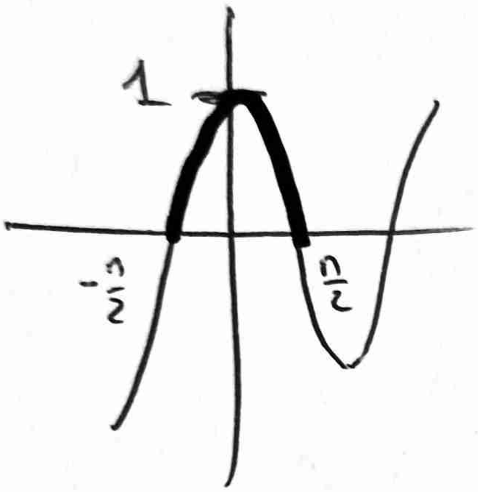
$x = e^2$

x	0	1	e^2
$\ln x - 2$	$-$	$-$ 0	$+$
$\ln^3 x$	$-$	0 $+$	$+$
f'	$+$	$-$	$+$
f	↗	↘	↗

15. Β) $f(x) = \text{ημ}x - \text{εφ}x \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$f'(x) = \text{συν}x - \frac{1}{\text{συν}^2 x} = \frac{\text{συν}^3 x - 1}{\text{συν}^2 x}$$

$$\rightarrow f'(x) = 0 \Rightarrow \frac{\text{συν}^3 x - 1}{\text{συν}^2 x} = 0$$



$\text{συν}^3 x - 1 = 0$

$\text{συν}^3 x = 1$

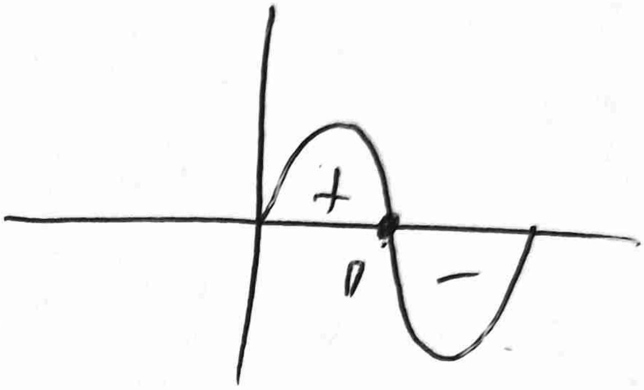
$\text{συν}x = 1$

$\underline{\underline{x = 0}}$

x	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$
$\text{συν}^3 x - 1$	$-$	$-$	
$\text{συν}^2 x$	$+$	$+$	
f'	$-$	$-$	
f	↘	↘	

14. ⑧ $f(x) = \dfrac{1}{1-\sigma\upsilon\nu x}$, $x \in (0, 2\pi)$,

$$f'(x) = \frac{-(\eta\mu x)}{(1-\sigma\upsilon\nu x)^2} = -\frac{\eta\mu x}{(1-\sigma\upsilon\nu x)^2}$$



x	0		π		2π
$-\eta\mu x$		$-$	0	$+$	
$(1-\sigma\upsilon\nu x)^2$		$+$		$+$	
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

$$f(x) \geq f(\pi)$$

$$f(x) \geq \frac{1}{2}.$$

16. (B) $f(x) = \begin{cases} x^3-3x, & x<0 \\ x^2-4x, & x \geq 0 \end{cases}$

$$\left.\begin{aligned} &\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2-4x = 0 \\ &\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^3-3x = 0 \end{aligned}\right\} \lim_{x\to 0} f(x) = 0$$

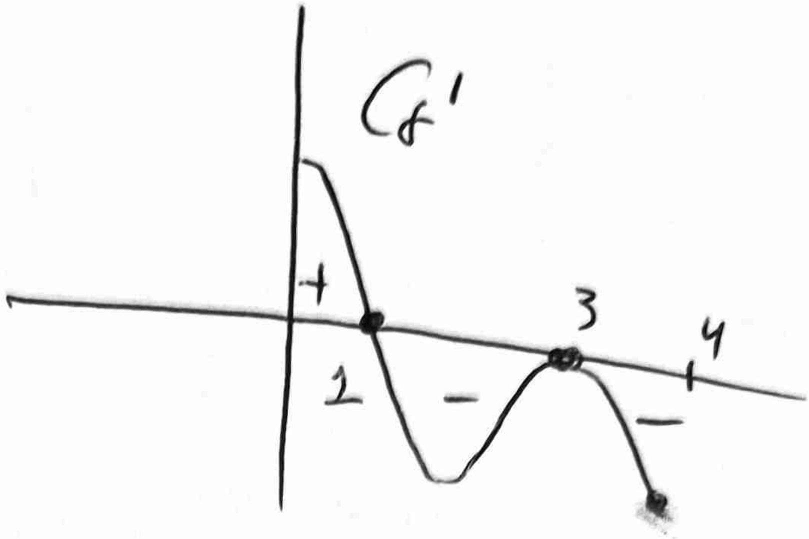
Συνεχής στο 0!

$f(0)=0$

$x<0$	$x>0$
$f_1(x) = x^3-3x$	$f_2(x) = x^2-4x$
$f_1'(x) = 3x^2-3$	$f_2'(x) = 2x-4$
$f_1'(x) = 3(x^2-1)$	$\rightarrow f_2'(x) = 0$
$\rightarrow f_1'(x) = 0 \Rightarrow x^2-1=0$	$2x-4=0$
$x=1$ $\quad$ $x=-1$	$x=2$

x		-1		0		1		2	
f_1'	$+$	0	$-$		$-$	0	$+$		$+$
f_2'	$-$		$-$		$-$		$-$	0	$+$
f'	$+$		$-$		$-$		$-$		$+$
f	$\nearrow$		$\searrow$		$\searrow$		$\searrow$		$\nearrow$

17.



x		1		3	
f'	+		–		–
f	↗		↘		↘

Επομενο Μαθημα

23

8 αγ.

9 αγς.

10 αγς