6. $f(x) = \dfrac{\ln(1+x)}{x}, \quad x > 0$

ΝΔΟ $f(x) > 2^{f(x)} - 1 \quad \forall x > 0$

$f(x) + 1 > 2^{f(x)}$

$\ln(f(x)+1) > \ln 2^{f(x)}$

$\ln(f(x)+1) > f(x) \ln 2$

$$\frac{\ln(f(x)+1)}{f(x)} > \ln 2$$

$f(f(x)) > f(1)$

$f\downarrow$

$f(x) < 1 \quad \forall x > 0$

• $\ln x \leq x-1 \Rightarrow \ln(x+1) \leq x+1-1$

$\ln(x+1) \leq x$

$\dfrac{\ln(x+1)}{x} \leq 1 \quad \xrightarrow{f(x) \leq 1}$ ✓

$$f(x) = \frac{\ln(1+x)}{x}, \quad x > 0$$

$x > 0 \Rightarrow x+1 > 1$

$\ln(x+1) > \ln 1$

$\ln(x+1) > 0$

$f(x) > 0$

$-$

$$f'(x) = \frac{\frac{1}{x+1}x - \ln(x+1)}{x^2} = \frac{\frac{x}{x+1} - \ln(x+1)}{x^2}$$

$f \downarrow$

$$\boxed{\varphi(x) = \frac{x}{x+1} - \ln(x+1)}$$

$-$

$$\varphi'(x) = \frac{x+1-x}{(x+1)^2} - \frac{1}{x+1} = \frac{1}{(x+1)^2} - \frac{x+1}{(x+1)^2} = \frac{-x}{(x+1)^2}$$

$\varphi \downarrow$

Πρόσοχη Αν $x > 0$

$\varphi \downarrow$

$\varphi(x) < \varphi(0)$

$\varphi(x) < 0$

16. $f(x) = 2x\ln x - 2x + 1 \qquad , x > 0$

(a) $f'(x) = 2\ln x + 2x\,\frac{1}{x} - 2$

$f'(x) = 2\ln x + 2 - 2$

$f'(x) = 2\ln x$

$f'(x) = 0 \Rightarrow 2\ln x = 0 \Rightarrow \underline{\underline{x = 1}}$

x	0	1	$+\infty$
f'	$-$	0	$+$
f	$1 \searrow$	-1	$\nearrow +\infty$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(2\cancel{x\ln x} - 2\cancel{x} + 1\right) = 1$$

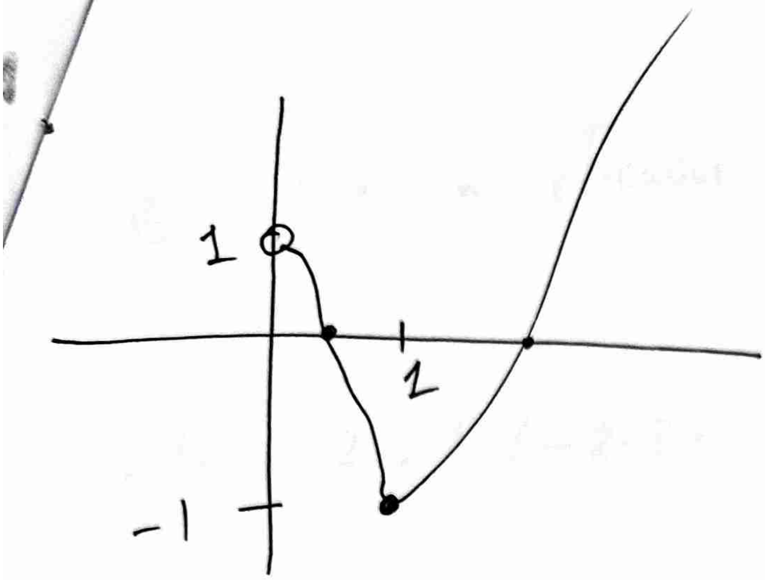
$$\rightarrow \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} =$$

$$= \lim_{x\to 0^+} -\frac{x^{\cancel{2}}}{\cancel{x}} = 0$$

$f(1) = -1$

$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty}\left(2x\ln x - 2x + 1\right) =$$

$$= \lim_{x\to +\infty} 2x\left(\ln x - 1 + \frac{1}{2x}\right) = +\infty$$



$\Sigma T_f = [-1, +\infty)$.

(B)

$\underline{x < 1}$

- $f \downarrow$
- f συνεχής.
- $\Sigma T_f = [-1, 1)$

Το $0 \in \Sigma T_f$

άρα $\exists!\ \xi_1$ τ.ω

$f(\xi_1) = 0$

$\underline{x \geq 1}$

- $f \nearrow$
- f συνεχής
- $\Sigma T_f = [-1, +\infty)$

Το $0 \in \Sigma T_f$

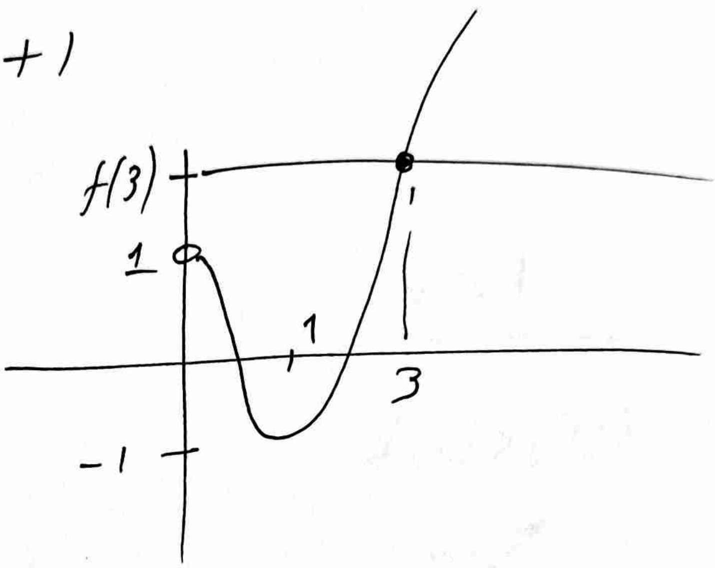
άρα $\exists!\ \xi_2$

τ.ω $f(\xi_2) = 0$.

(γ) Νδο η εξισωση $f(x)=f(3)$ εχει ριζα μοναδικη

$f(3) = 2\cdot 3 \ln 3 - 2\cdot 3 + 1 = 6\ln 3 - 6 + 1 = 6(\ln 3 - 1) + 1$

$f(3) = \underbrace{6(\ln 3 - 1)}_{+} + 1$



$3 > e$

$\ln 3 > \ln e$

$\ln 3 > 1$

$\ln 3 - 1 > 0$

$f(3) > 1$

$x < 1$

• $\Sigma T_f = [-1, 1)$

το $f(3) \notin \Sigma T_f$

αρα η $f(x) = f(3)$

Αδυνατη

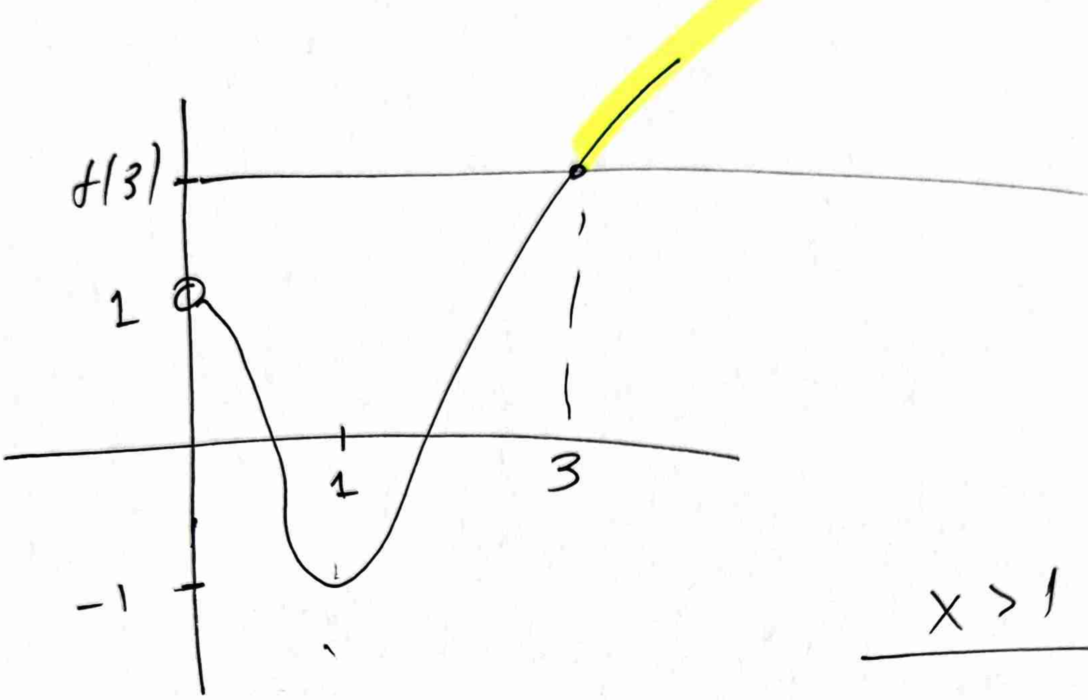
$x \geq 1$

$f(x) = f(3)$

f 1-1

$x = 3$

(δ) $f(x) > f(3)$



$0 < x < 1$

ΣΤ$_f = [-1, 1)$

άρα η ανίσωση

$f(x) > f(3)$

αδύνατη

$x > 1$

$f(x) > f(3)$

$f \nearrow$

$x > 3$

$x \in (3, +\infty)$

15. $f(x) = x\ln x + x^2 - 3x + 2$

$x > 0$

(01) $f(x) = f(e)$

$f'(x) = \ln x + x\,\frac{1}{x} + 2x - 3$

$f'(x) = \ln x + 2x - 2$ $\qquad f'(1) = 0$

$f''(x) = \frac{1}{x} + 2 > 0$

x	0	1	$+\infty$
f''	$+$	$+$	
f'	$\nearrow$ $-$	0	$+$ $\nearrow$
f	$2 \searrow$	0	$\nearrow +\infty$

$x < 1 \Rightarrow f'(x) < f'(1) \Rightarrow f'(x) < 0$

$x > 1 \Rightarrow f'(x) > f'(1) \Rightarrow f'(x) > 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (\cancel{x\ln x} + x^2 - 3x + 2) = 2$

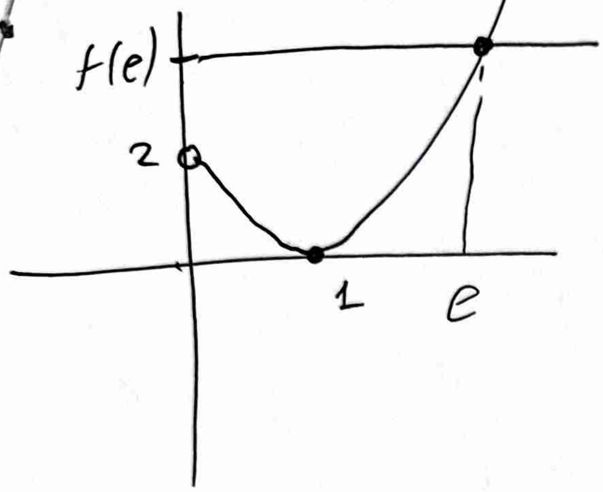
$\to \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} -\frac{x^{\cancel{2}}}{\cancel{x}}$

$= 0$

$f(1) = 0$

$\lim_{x\to+\infty} (\underbrace{x\ln x} + \underbrace{x^2 - 3x + 2}) = +\infty + \infty = +\infty$

$f(x)=f(e)$



$x>1$

- f συνεχής
- $f \nearrow \Rightarrow f$ 1-1

$f(x)=f(e)$

f 1-1

$x=e$

$f(e)= e\ln e+e^2-3e+2$

$f(e)=e^2-2e+2$

$f(e)=e(e-2)+2$

$f(e)>2$

$0<x\leq$

- $\Sigma T_f=[0,2)$

Το $f(e)>2$

άρα $f(e)\notin \Sigma T_f$

Αδύνατη

(B) ν.δ.ο. $\nexists\, x_0\in(0,1)$ τ.ω $f(x_0)=1+e^{1-x_0}$

$0<x_0<1$

$0>-x_0>-1$

$1>1-x_0>0$

$e^1>e^{1-x_0}>e^0$

$1+e>\underbrace{1+e^{1-x_0}}>2$

$1+e^{1-x_0}>2$

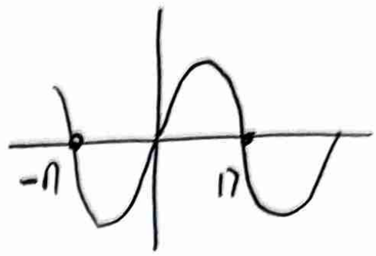
$\forall x\in(0,1)$

$0<f(x)<2$

$f(x)\neq 1+e^{1-x_0}$

11. (B) $x^2 = \cos x \quad x \in (-\pi, \pi)$.

$$\underbrace{x^2 - \cos x}_{f(x)} = 0$$



$f'(x) = 2x + \sin x \qquad f'(0) = 0$

$f''(x) = 2 + \cos x > 0 \qquad -1 \le \cos x \le 1$

$1 \le 2 + \cos x \le 3$

x	$-\pi$	0	π
f''	$+$		$+$
f'	$\nearrow$ $-$	0	$+$ $\nearrow$
f	$\searrow$		$\nearrow$

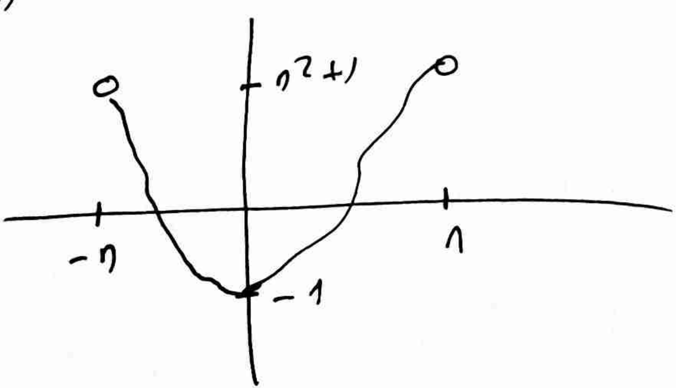
$x < 0 \Rightarrow f'(x) < f'(0) \Rightarrow f'(x) < 0$

$x > 0 \Rightarrow f'(x) > f'(0) \Rightarrow f'(x) > 0$

$$\lim_{x \to -\pi^+} f(x) = (-\pi)^2 - \cos(-\pi) = \pi^2 - \cos\pi = \pi^2 + 1$$

$f(0) = -1$

$$\lim_{x \to \pi^-} f(x) = \pi^2 - \cos\pi = \pi^2 + 1$$



2 [illegible]

① $x^2 = 2\ln x + 2$ $\qquad]0,+\infty[$

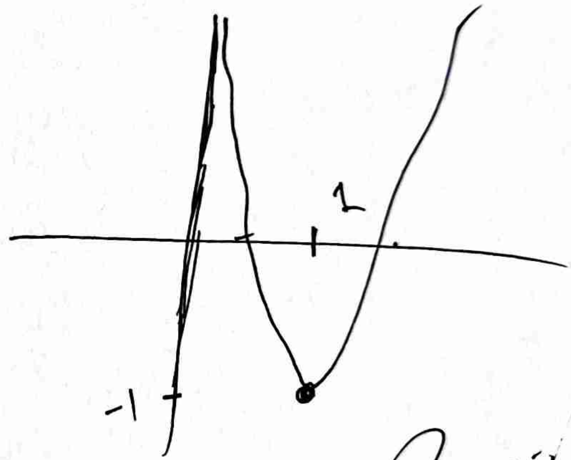
$x^2 - 2\ln x - 2 = 0$

$\boxed{f(x) = x^2 - 2\ln x - 2}$

$f'(x) = 2x - \frac{2}{x} = \frac{2x^2-2}{x} = \frac{2(x^2-1)}{x}$

$\to f'(x) = 0 \Rightarrow x^2 - 1 = 0 \qquad x = 1 \qquad \cancel{x = -1}$

x	0		1		$+\infty$
f'		$-$	0	$+$	
f	$+\infty$	$\searrow$	-1	$\nearrow$	$+\infty$



2 pi

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 - 2\ln x - 2 = +\infty$

$f(1) = -1$

$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} (x^2 - 2\ln x - 2) = \lim_{x\to+\infty} x^2\left(1 - 2\cancel{\frac{\ln x}{x^2}} - \cancel{\frac{2}{x^2}}\right) = +\infty$

$\to \lim_{x\to+\infty} \frac{\ln x}{x^2} = \lim_{x\to+\infty} \frac{\frac{1}{x}}{2x} = \lim_{x\to+\infty} \frac{1}{2x^2} = 0$

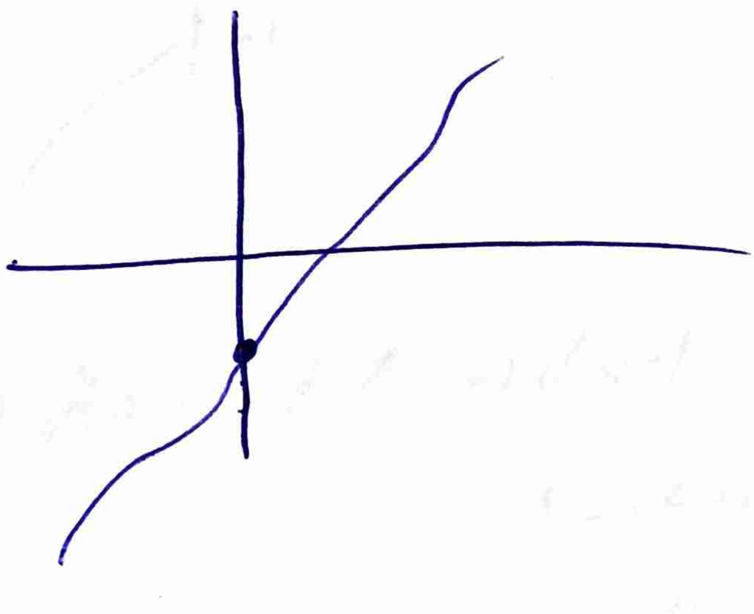
Ενότητα 27

2. $f(x) = x^3 - x^2 + 2x - 1$, $x \in \mathbb{R}$.

Δ1 $f'(x) = 3x^2 - 2x + 2 > 0$ $\quad f \nearrow$

$\Delta = 4 - 24 < 0$

$$\left.\begin{array}{l} \bullet \lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} x^3 = -\infty \\ \bullet \lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} x^3 = +\infty \end{array}\right\} \Sigma T_f = \mathbb{R}.$$



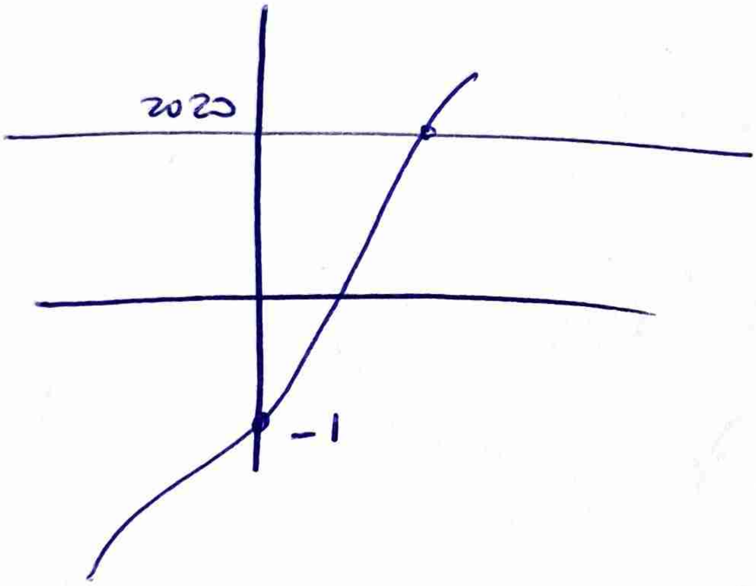
β) ΝΔΟ $f(f(x)-2019)=1$ έχει μοναδική ρίζα

$$f(f(x)-2019) = f(1)$$

f 1-1

$$f(x)-2019 = 1$$

$$f(x)=2020$$



- f συνεχής
- $f \nearrow$
- $\Sigma T_f = \mathbb{R}$.

Το $2020 \in \Sigma T_f$

άρα $\exists!\, \xi$ τ.ω $f(\xi)=2020$.

γ) Αφού $f \nearrow \Rightarrow f$ 1-1

$f(x)=x \Rightarrow x^3 - x^2 + 2x - 1 = x$

$$x^3 - x^2 + x - 1 = 0$$

1	-1	1	-1	1
↓	1	0	1	
1	0	1	0	

$(x-1)(x^2+1)=0$

$x-1=0$

$x=1$ $A(1,1)$.

(5) $f^{-1}(f(x)-6) > 1$

$f \nearrow$

$f\left(f^{-1}(f(x)-6) \right) > f(1)$

$f(x)-6 > 1$

$f(x) > 7$

$f(x) > f(2)$

$f \nearrow$

$x > 2$

Ασκησεις για Τριτη 20/1

26	27
9	1 α γ
12	7 α β
13	8
15	11 α
16	9
17 α γ	10 α
19 α β	12 α
23	13 α.
24	

Την Πρωτη Μιση ωρα θα πω κυρτοτητα και ασυμπτωτες.