

Exercise

14

5. $x^2 - 3x + 1 = 0$

$$\Delta = 5 > 0$$

2 roots x_1, x_2

$$x_1 + x_2 = 3$$

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 =$$

$$x_1 \cdot x_2 = 1$$

$$= 3^2 - 2 \cdot 1 = 9 - 2 = 7$$

$$x_1^2 + x_2^2 = 7$$

$$\textcircled{a} \sqrt{\frac{x_1}{x_2}} + \sqrt{\frac{x_2}{x_1}} = 3$$

$$\rightarrow \left(\sqrt{\frac{x_1}{x_2}} + \sqrt{\frac{x_2}{x_1}} \right)^2 = \sqrt{\frac{x_1}{x_2}}^2 + 2\sqrt{\frac{x_1}{x_2}}\sqrt{\frac{x_2}{x_1}} + \sqrt{\frac{x_2}{x_1}}^2$$

$$= \frac{x_1}{x_2} + 2\sqrt{\frac{\cancel{x_1}}{\cancel{x_2}} \cdot \frac{\cancel{x_2}}{\cancel{x_1}}} + \frac{x_2}{x_1}$$

$$= \frac{x_1}{x_2} + 2 + \frac{x_2}{x_1} = \frac{x_1}{x_2} + \frac{x_2}{x_1} + 2$$

$$= \frac{x_1^2 + x_2^2}{x_1 x_2} + 2 = \frac{7}{1} + 2 = 9$$

$$\begin{aligned}
 \textcircled{B} \quad \frac{1}{x_1-2} + \frac{1}{x_2-2} &= \frac{x_2-2+x_1-2}{(x_1-2)(x_2-2)} = \\
 &= \frac{x_1+x_2-4}{x_1x_2-2x_1-2x_2+4} = \frac{3-4}{1-2(x_1+x_2)+4} = \\
 &= \frac{-1}{1-2 \cdot 3+4} = \frac{-1}{-1} = 1.
 \end{aligned}$$

$$\textcircled{D} \quad \frac{2}{x_1^2} + \frac{2}{x_2^2} = \frac{2x_2^2+2x_1^2}{x_1^2x_2^2} = \frac{2(x_2^2+x_1^2)}{(x_1x_2)^2} =$$

$$= \frac{2 \cdot 7}{1^2} = \frac{14}{1} = 14$$

$$\textcircled{E} \quad (3x_1-x_2)(3x_2-x_1) =$$

$$= 9x_1x_2 - 3x_1^2 - 3x_2^2 + x_1x_2 =$$

$$= 10x_1x_2 - 3(x_1^2+x_2^2) = 10 \cdot 1 - 3 \cdot 7 = -11$$

$$18. \quad x^2 - (\lambda - 1)x + \lambda - 2 = 0$$

$$\begin{aligned} \textcircled{a} \quad \Delta &= [-(\lambda - 1)]^2 - 4 \cdot (\lambda - 2) = (\lambda - 1)^2 - 4\lambda + 8 = \\ &= \lambda^2 - 2\lambda + 1 - 4\lambda + 8 = \\ &= \lambda^2 - 6\lambda + 9 = (\lambda - 3)^2 \geq 0 \end{aligned}$$

$$\textcircled{b} \quad \frac{x_1}{x_2} + \frac{x_2}{x_1} = -4$$

$$\underline{x_1 + x_2 = \lambda - 1}$$

$$\underline{x_1 \cdot x_2 = \lambda - 2}$$

$$\frac{x_1^2 + x_2^2}{x_1 x_2} = -4$$

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2$$

$$= (\lambda - 1)^2 - 2(\lambda - 2)$$

$$= \lambda^2 - 2\lambda + 1 - 2\lambda + 4$$

$$= \lambda^2 - 4\lambda + 5$$

$$\underline{\lambda^2 - 4\lambda + 5} = -4$$

$$\lambda - 2$$

$$\lambda^2 - 4\lambda + 5 = -4(\lambda - 2)$$

$$\lambda^2 - 4\lambda + 5 = -4\lambda + 8$$

$$\lambda^2 = 3$$

$$\lambda = \pm \sqrt{3}$$

$$\lambda \neq 2$$

$$\textcircled{1} \quad \frac{x_1}{x_2} = 2$$

$$\Rightarrow \underline{x_1 = 2x_2}$$

↓

$$\underline{x_1 + x_2 = 2-1}$$

$$2x_2 + x_2 = 2-1$$

$$\underline{3x_2 = 2-1}$$

$$x_2 = \frac{2-1}{3}$$

$$\underline{x_1 x_2 = 2-2}$$

⇓

$$\left(2-1 - \frac{2-1}{3}\right) \cdot \left(\frac{2-1}{3}\right) = 2-2$$

$$\checkmark \quad x_1 + \frac{2-1}{3} = 2-1$$

$$x_1 = 2-1 - \frac{2-1}{3}$$

$$\frac{3 \cdot 2 - 3 - 2 + 1}{3} \cdot \frac{2-1}{3} = 2-2$$

$$\frac{2 \cdot 2 - 2}{3} \cdot \frac{2-1}{3} = 2-2$$

$$\frac{(2 \cdot 2 - 2)(2-1)}{9} = 2-2$$

$$(2\lambda - 2)(\lambda - 1) = 9\lambda - 18$$

$$2\lambda^2 - 2\lambda - 2\lambda + 2 = 9\lambda - 18$$

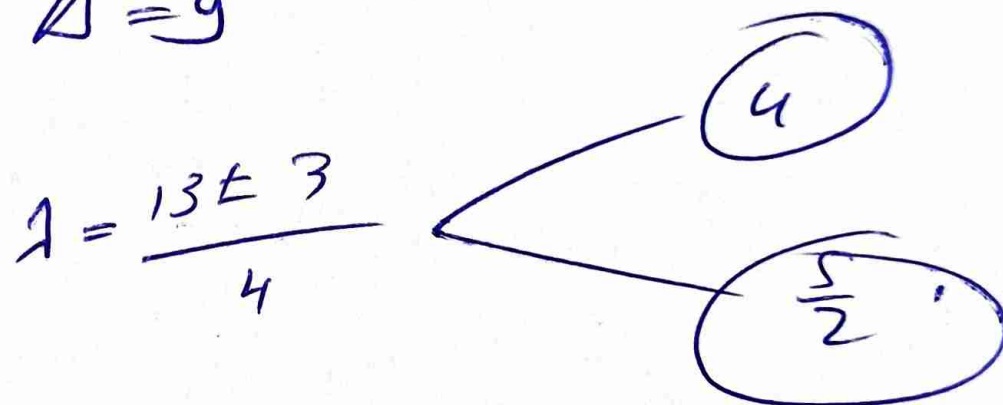
$$2\lambda^2 - 4\lambda - 9\lambda + 2 + 18 = 0$$

$$2\lambda^2 - 13\lambda + 20 = 0$$

$$\Delta = 169 - 160$$

$$\Delta = 9$$

$$\lambda = \frac{13 \pm 3}{4}$$



22. $x^2 - 2x - 5 = 0$

Να κατασκευάσεται εἰσφορά του.

με πωλ $2x_1 - 3x_2$ και $2x_2 - 3x_1$,

$\Delta = 24 > 0$

x_1, x_2

$x_1 + x_2 = 2$

$x_1 \cdot x_2 = -5$

$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 =$

$= 2^2 - 2(-5)$

$= 4 + 10$

$= 14$

$x^2 + 2x - 149 = 0$

$x^2 - Sx + P = 0.$

$S = (2x_1 - 3x_2) + 2x_2 - 3x_1$

$S = -x_1 - x_2 = -(x_1 + x_2)$

$S = -2$

$P = (2x_1 - 3x_2)(2x_2 - 3x_1).$

$P = 4x_1x_2 - 6x_1^2 - 6x_2^2 + 9x_2x_1$

$P = 4(-5) - 6(x_1^2 + x_2^2) + 9(-5)$

$P = -20 - 6 \cdot 14 - 45$

$P = -20 - 45 - 84 = -149$
 $P = -149$

$$17. \quad x^2 - 2x + 1 = 0$$

Ⓐ Врл то 1 wccc ca cww p1✓
програмаки .

$$\text{Анаитw } \Delta \geq 0$$

$$B^2 - 4ax \geq 0$$

$$(-2)^2 - 4 \cdot 1 \cdot 1 \geq 0$$

$$4 - 4 \geq 0$$

$$-4 \geq -4$$

$$\underline{\underline{1 \leq 1}}$$

$$\textcircled{B} \quad x_1 = 3x_2$$

↓

$$x_1 + x_2 = 2$$

$$\Rightarrow 3x_2 + x_2 = 2$$

$$4x_2 = 2$$

$$x_2 = \frac{1}{2}$$

$$x_1 x_2 = 1$$

↓

$$\frac{1}{2} \cdot \frac{3}{2} = 1 \quad \left(2 = \frac{3}{2} \right)$$

$$x_1 = \frac{3}{2}$$

Евотута 14

16. $x^2 - 32x - 27 = 0$

а) $\Delta = b^2 - 4ac = (-32)^2 - 4 \cdot (-27) =$
 $= 92^2 + 108 > 0$ ✓

б) Ав $x_1 = x_2^2$ Врд $x_1, x_2, 1$

$$x_1 + x_2 = - \frac{-32}{1}$$

$$\underline{x_1 + x_2 = 32}$$

$$x_1 x_2 = \frac{-27}{1}$$

$$\underline{x_1 x_2 = -27}$$

$$x_2^2 x_2 = -27$$

$$x_2^3 = (-3)^3$$

$$x_2 = -3$$

$$x_1 = 9$$

$$9 + (-3) = 32$$

$$6 = 32$$

$$2 = 2$$

6. (γ) Η εξίσωση που φαχνω αωι.

$$x^2 - Sx + P = 0$$

2

-2

$$S = 2 + (-2) = 0$$

$$P = 2 \cdot (-2) = -4$$

$$x^2 - 0 \cdot x + (-4) = 0$$

$$x^2 - 4 = 0$$

(δ) Η εξίσωση που φαχνω

$$x^2 - Sx + P = 0$$

1

$$x^2 - (1 - \sqrt{2}) \cdot x - \sqrt{2} = 0$$

$-\sqrt{2}$

$$S = 1 + (-\sqrt{2}) = 1 - \sqrt{2}$$

$$P = 1 \cdot (-\sqrt{2}) = -\sqrt{2}$$

Εποραιο Μαθημα

Τεταρτη 21/1/26.

Παλιω

14

①

②

⑥ α β

⑩

⑪

⑫

⑭

NEES

14

③

②0

②1 α β