Βασίλης Γιαννακόπουλος

Μαθηματικά. 6933035551

Βοηθήματα Μαθηματικών: Μπαρλάς.

Ανδριάνα Αλεξανδρος Γ.

Θανος.

Συνάρτηση

Είναι μια διαδικασία αντιστοίχισης τιμών.

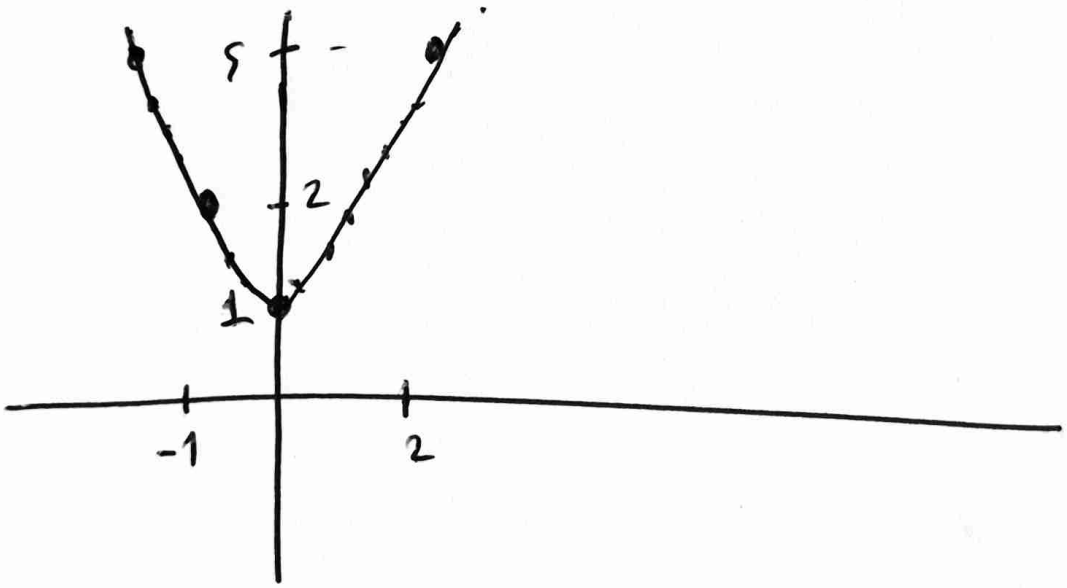
Π.Χ $f(x) = x^2 + 1$

x	0	-1	2
y	1	2	5

$f(0) = 0^2 + 1 = 1$

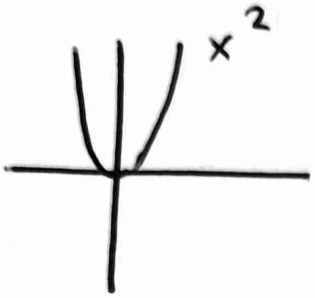
$f(-1) = (-1)^2 + 1 = 1 + 1 = 2$

$f(2) = 2^2 + 1 = 5$



Βασικές Συναρτήσεις

1.



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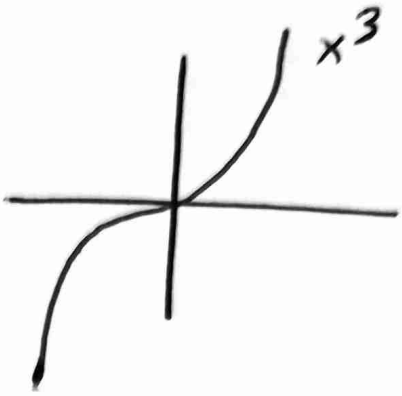

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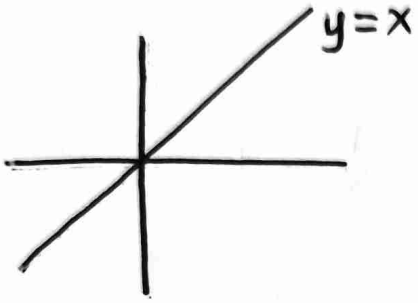

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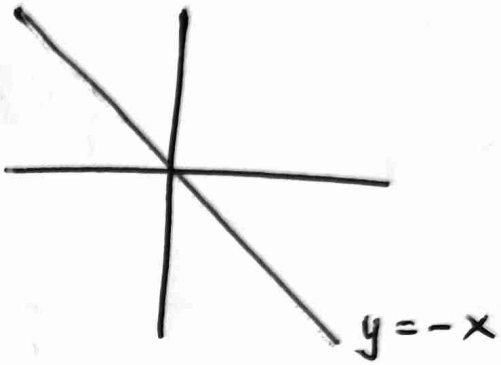

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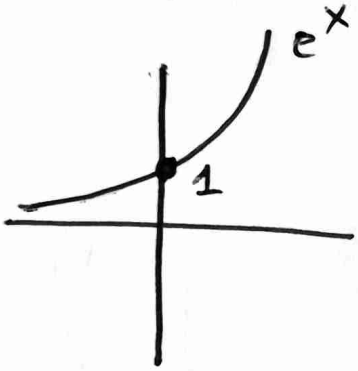

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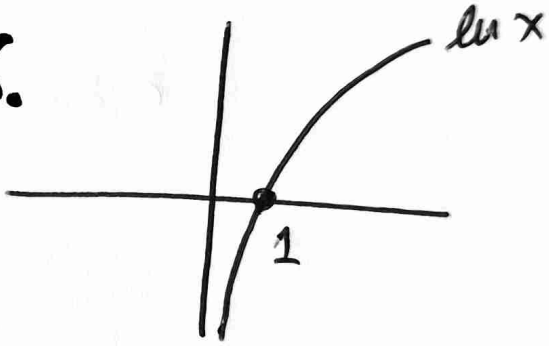

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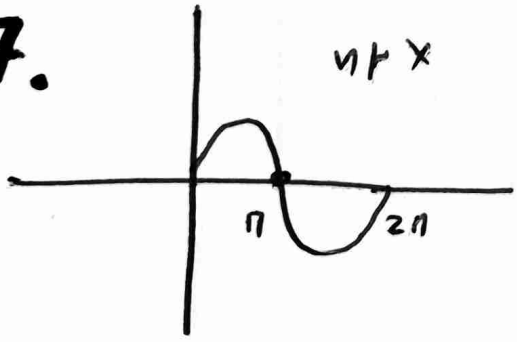

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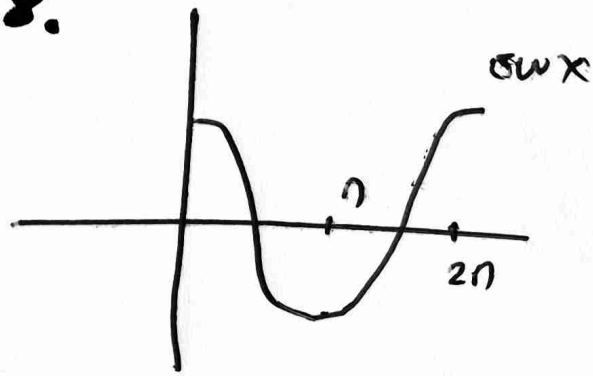

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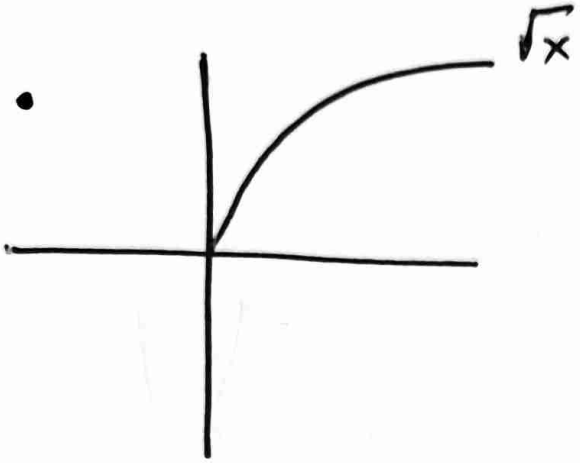

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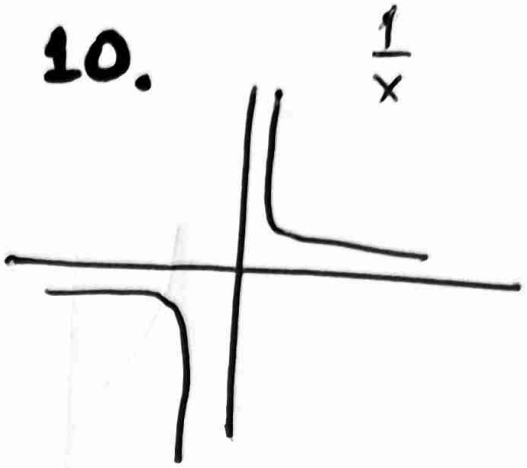

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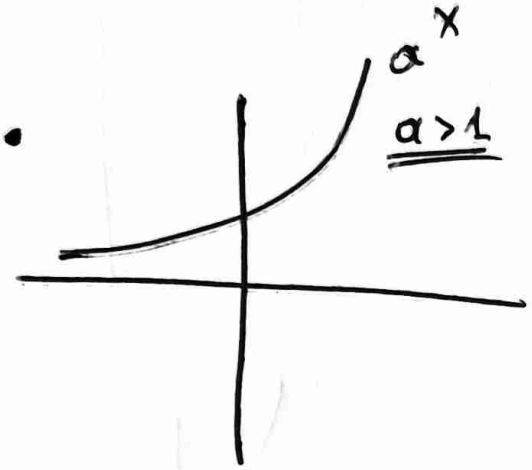

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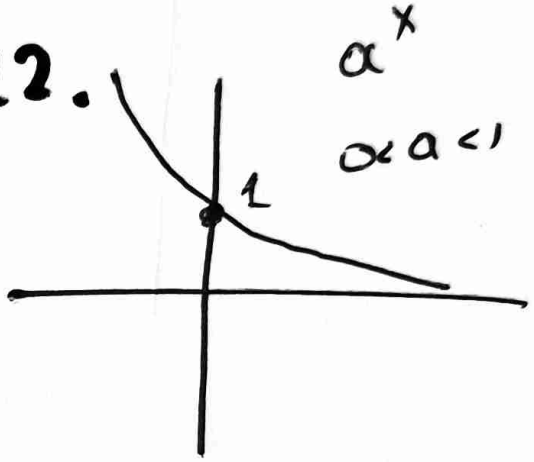


Πεδίο ορισμού

$f(x) = \sqrt{x-1}$

Πρέπει $x - 1 \geq 0 \Rightarrow x \geq 1$

$A_f = D_f = [1, +\infty)$

→ $\sqrt{x}$ πρέπει $x \geq 0$

→ $\frac{1}{x}$ πρέπει $x \neq 0$

→ $\ln x$ πρέπει $x > 0$.

Θετικά πράγματα του νου

$e^x > 0$

$\ln x > 0$ όταν $\underline{\underline{x > 1}}$

$\sqrt{x} \geq 0$

$a^x > 0$

$e^x - 1 > 0$ αν $x > 0$

$x^2 \geq 0$

$|x| \geq 0$.

Εξίσωση 2ου Βαθμού

$$ax^2+\beta x+\gamma=0 \quad , \quad a\neq 0.$$

$$\Delta=\beta^2-4a\gamma \qquad (\text{Διακρίνουσα})$$

1. Αν $\Delta<0$ τότε η εξίσωση είναι αδύνατη.

2. Αν $\Delta=0$ τότε η εξίσωση έχει μία διπλή ρίζα $x=-\frac{\beta}{2a}$.

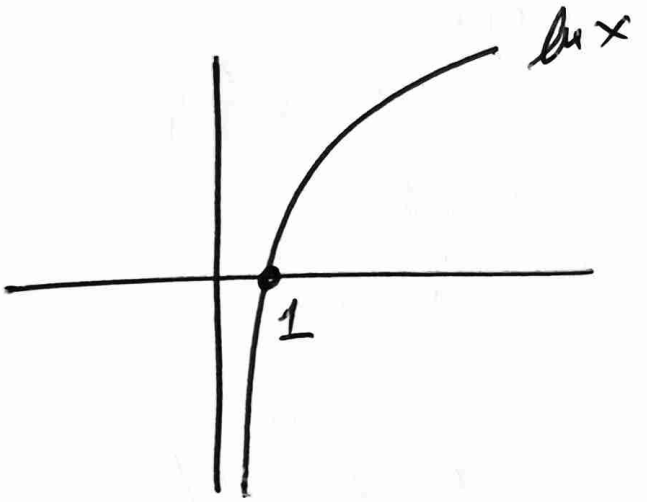
3. Αν $\Delta>0$ τότε η εξίσωση έχει δύο πραγματικές και άνισες ρίζες.

Свойства $\ln x$

$$\ln \frac{a}{b} = \ln a - \ln b$$

$$\ln a \cdot b = \ln a + \ln b$$

$$\ln a^{k} = k \ln a .$$



$$\ln 1 = 0$$

$$\ln e = 1$$

$$\ln e^{x} = x$$

$$e^{\ln x} = x .$$

Ιδιοτητ απολυτ τιμ

$$|x|^2 = x^2$$

$$|\oplus x| = x$$

$$|\ominus x| = -x$$

$$|x| \cdot |y| = |x \cdot y|$$

$$\frac{|x|}{|y|} = \left|\frac{x}{y}\right|$$

$$|x| + |y| \neq |x+y| .$$

Ενοτητα 3

2. (B) $f(x) = \frac{x+1}{x-1}$

Πρπ $x-1 \neq 0$

$x \neq 1$

$A_f = R - \{1\}$.

$g(x) = \frac{x^2-1}{x^2-2x+1}$

Πρπ $x^2-2x+1 \neq 0$

$A_g = R - \{1\}$.

$\longrightarrow x^2-2x+1 = 0$

$\Delta = (-2)^2 - 4 \cdot 1$

$\Delta - 4 - 4 = 0$.

$x = -\frac{\beta}{2\alpha} = -\frac{-2}{2 \cdot 1} = 1$

$x = 1$

$g(x) = \frac{x^2-1}{x^2-2x+1} = \frac{(x-1)(x+1)}{(x-1)^2} = \frac{x+1}{x-1} = f(x)$

Εχουν ιδιο πεδιο ορισμου, εχουν ιδιο τυπο αρα

είναι ίσες!

2. (β) $f(x) = x - \ln(1+e^x)$

Πρεπει $1+e^x > 0$ που ειναι!

$D_f = \mathbb{R}$.

$$g(x) = \ln \frac{e^x}{1+e^x}$$

πρεπει $1+e^x \neq 0$ που ειναι!

και $\frac{e^x}{1+e^x} > 0$ που ειναι!.

$D_g = \mathbb{R}$.

$g(x) = \ln e^x - \ln(1+e^x) = x - \ln(1+e^x) = H(x)$

Εχουν ιδιο πεδιο ορισμου

Εχουν ιδιο τυπο

Ειναι ισες!.

2. (ε) $f(x) = \frac{|x| - 3}{|x|}$ προ $|x| \neq 0$

$x \neq 0$

$A_f = R^* = R - \{0\}$.

$g(x) = \frac{x^2 - 3|x|}{x^2}$

προπι $x^2 \neq 0$

$x \neq 0$

$D_g = R^*$

$$g(x) = \frac{x^2 - 3|x|}{x^2} = \frac{|x|^2 - 3|x|}{|x|^2} = \frac{|x|(|x| - 3)}{|x|^2} =$$

$$= \frac{|x| - 3}{|x|} = f(x)$$

ΙδΕΣ!

3. ① $f(x) = \dfrac{x-4}{\sqrt{x}+2}$ προφανώς $x \geq 0$ και $\sqrt{x}+2 \neq 0$ πάντα.

$D_f = [0, +\infty)$

$g(x) = \sqrt{x} - 2$

προφανώς $x \geq 0$ δηλαδή $D_g = [0, +\infty)$

$$f(x) = \frac{x-4}{\sqrt{x}+2} = \frac{(x-4)(\sqrt{x}-2)}{(\sqrt{x}+2)(\sqrt{x}-2)} = \frac{(x-4)(\sqrt{x}-2)}{\sqrt{x}^2 - 2^2}$$

$$= \frac{(x-4)(\sqrt{x}-2)}{\sqrt{x}^2 - 4} = \frac{(x-4)(\sqrt{x}-2)}{x-4} = \sqrt{x} - 2 = g(x)$$

ισχύει.

Βασικές ρίζες

$\sqrt{x} \geq 0$ και $x \geq 0$

$\left(\begin{array}{ll} x^2 = 9 & \text{τότε} \quad x = \pm\sqrt{9} \\ 6^2 = 36 & \text{αρα} \quad \sqrt{36} = 6. \end{array} \right)$

$$\sqrt{x}^2 = x$$

$$\sqrt{x^2} = |x|$$

$$\sqrt{x}\sqrt{y} = \sqrt{xy}$$

$$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}.$$

Ασκηση

Πράξεις συναρτήσεων.

$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$, $D_f = (0, +\infty)$

$g(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$, $D_g = (0, +\infty)$

1. $(f+g)(x) = f(x) + g(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + \sqrt{x} - \frac{1}{\sqrt{x}}$

$= 2\sqrt{x}$

$D_{f+g} = D_f \cap D_g = (0, +\infty)$.

2. $(f-g)(x) = f(x) - g(x) = \sqrt{x} + \frac{1}{\sqrt{x}} - \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$

$= \sqrt{x} + \frac{1}{\sqrt{x}} - \sqrt{x} + \frac{1}{\sqrt{x}} = \frac{2}{\sqrt{x}}$

$D_{f-g} = D_f \cap D_g = (0, +\infty)$.

3. $(f \cdot g)(x) = f(x) \cdot g(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$

$$= \sqrt{x}^2 - \left(\frac{1}{\sqrt{x}}\right)^2 = x - \frac{1}{x}$$

$D_{f \cdot g} = D_f \cap D_g = (0, +\infty)$.

4. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x} + \frac{1}{\sqrt{x}}}{\sqrt{x} - \frac{1}{\sqrt{x}}} = \frac{\sqrt{x}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)}{\sqrt{x}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)}$

$$= \frac{\sqrt{x}^x + \sqrt{x}\frac{1}{\sqrt{x}}}{\sqrt{x}^2 - \sqrt{x}\frac{1}{\sqrt{x}}} = \frac{x+1}{x-1}$$

pri pogoju $g(x) \neq 0 \Rightarrow \sqrt{x} - \frac{1}{\sqrt{x}} \neq 0$

$\sqrt{x} \neq \frac{1}{\sqrt{x}}$

$\sqrt{x}^2 \neq 1$

$x \neq 1$

$D_{f/g} = (0, 1) \cup (1, +\infty)$.

Πράξεις συναρτήσεων

$(f+g)(x) = f(x) + g(x)$ $\qquad D_{f+g} = D_f \cap D_g$

$(f-g)(x) = f(x) - g(x)$ $\qquad D_{f-g} = D_f \cap D_g$

$(f \cdot g)(x) = f(x) \cdot g(x)$ $\qquad D_{f \cdot g} = D_f \cap D_g$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ $\qquad D_{f/g} = D_f \cap D_g$

για $g(x) \neq 0$.

Ασκηση

$f:(1,+\infty) \longrightarrow \mathbb{R}$ $\quad f(x) = \dfrac{x}{x-1}$

$g:[0,+\infty) \longrightarrow \mathbb{R}$ $\quad g(x) = \dfrac{x-1}{x+1}$

$$\longrightarrow (f+g)(x) = f(x)+g(x) = \frac{x}{x-1} + \frac{x-1}{x+1} =$$

$$= \frac{x(x+1)}{(x-1)(x+1)} + \frac{(x-1)^2}{(x-1)(x+1)} = \frac{x^2+x+x^2-2x+1}{(x-1)(x+1)} =$$

$$= \frac{2x^2-x+1}{(x-1)(x+1)}$$

$$D_{f+g} = (1,+\infty) = D_f \cap D_g$$

$\longrightarrow (f-g)(x) = f(x) - g(x) = \frac{x}{x-1} - \frac{x-1}{x+1} =$

$= \frac{x(x+1)}{(x-1)(x+1)} - \frac{(x-1)^2}{(x-1)(x+1)} = \frac{x^2+x-(x^2-2x+1)}{(x-1)(x+1)}$

$= \frac{x^2+x-x^2+2x-1}{(x-1)(x+1)} = \frac{3x-1}{(x-1)(x+1)}$

$D_{f-g} = D_f \cap D_g = (1, +\infty)$.

$\longrightarrow (f \cdot g)(x) = f(x) \cdot g(x) = \frac{x}{x-1} \cdot \frac{x-1}{x+1} = \frac{x}{x+1}$

$D_{f \cdot g} = D_f \cap D_g = (1, +\infty)$.

$\longrightarrow \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{x}{x-1}}{\frac{x-1}{x+1}} = \frac{x(x+1)}{(x-1)^2}$

$D_{f/g} = (1, +\infty)$.

προσοχή $g(x) \neq 0 \Rightarrow \frac{x-1}{x+1} \neq 0 \Rightarrow x-1 \neq 0 \Rightarrow x \neq 1$.

Μεγάλη προσοχή

$f: \mathbb{R} \to \mathbb{R}$ σημαίνει ότι $D_f = \mathbb{R}$

$f: \mathbb{R} \to (0, +\infty)$ σημαίνει ότι $D_f = \mathbb{R}$

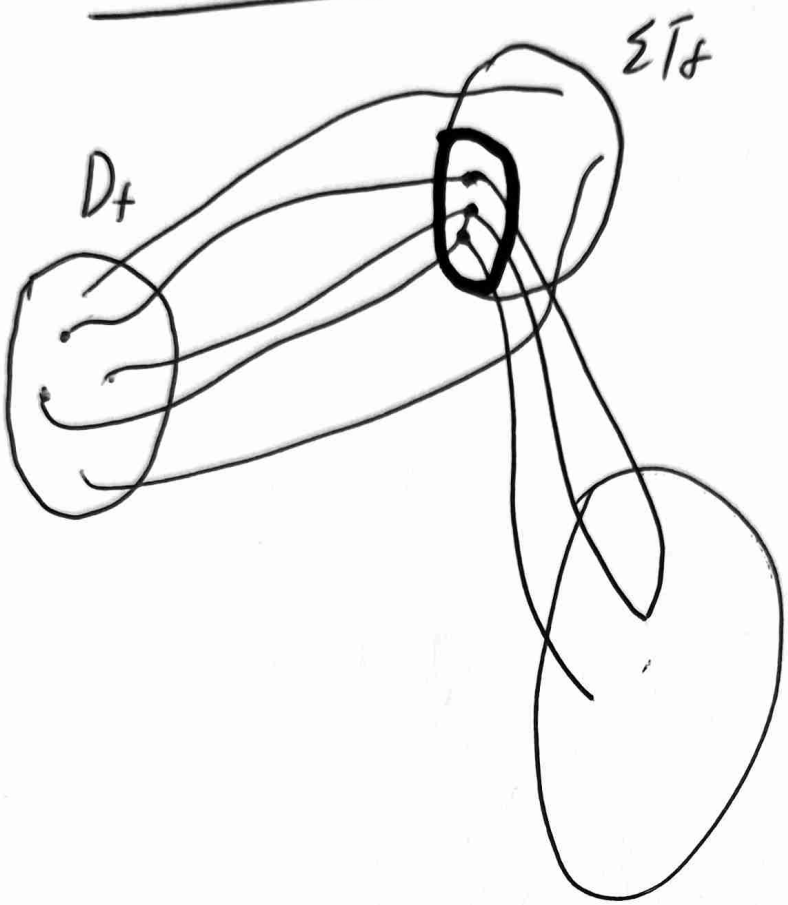
$\Sigma T_f = (0, +\infty)$

$f: [0,1] \to [3,4]$ σημαίνει ότι $D_f = [0,1]$

$\Sigma T_f = [3,4]$

$f(\mathbb{R}) = \mathbb{R}$ σημαίνει ότι $D_f = \mathbb{R}$ $\Sigma T_f = \mathbb{R}$.

Σύνθεση Συναρτήσεων



$$(g \circ f)(x) = g(f(x))$$

$x \in D_f$ και $f(x) \in D_g$

Ασκηση

$f: [1, +\infty) \longrightarrow \mathbb{R}$ $\quad f(x) = \frac{1}{x}$

$g: (0, +\infty) \longrightarrow \mathbb{R}$ $\quad g(x) = \ln(x+1)$

1. $(f \circ g)(x) = f(g(x)) = \frac{1}{\ln(x+1)}$

$x \in D_g$ και $g(x) \in D_f$

$x > 0$

$\ln(x+1) \geq 1$

$e^{\ln(x+1)} \geq e^1$

$x+1 \geq e$

$x \geq e-1$

$(f \circ g)(x) = \frac{1}{\ln(x+1)}$

$D_{f \circ g} = [e-1, +\infty)$.

2. $(g \circ f)(x) = g(f(x)) = \ln\left(\frac{1}{x} + 1\right)$

$x \in D_f$ και $f(x) \in D_g$

$x \geq 1$ $\quad \frac{1}{x} > 0$

$\Rightarrow \underline{\underline{x > 0}}$

$$(g \circ f)(x) = \ln\left(\frac{1}{x} + 1\right)$$

$$D_{g \circ f} = [1, +\infty)$$

3. $(f \circ f)(x) = f(f(x)) = \dfrac{1}{\frac{1}{x}} = \dfrac{\frac{1}{1}}{\frac{1}{x}} = \dfrac{x}{1} = x$

$x \in D_f$ και $f(x) \in D_f$

$x \geq 1$ $\quad \frac{1}{x} \geq 1$

$\frac{1}{x} - 1 \geq 0$

$\frac{1 - x}{x} \geq 0$

$x \in (0, 1]$

$$(f \circ f)(x) = x$$

$$D_{f \circ f} = \{1\}$$

x	0		1
$1-x$	$+$	$+$	$-$
x	$-$	$+$	$+$
$\frac{1-x}{x}$	$-$	$+$	$-$

4. $(g \circ g)(x) = g(g(x)) = \ln(\ln(x+1)+1)$

$x \in D_g$ και $g(x) \in D_g$

$x > 0$ $\quad \ln(x+1) > 0$

$e^{\ln(x+1)} > e^0$

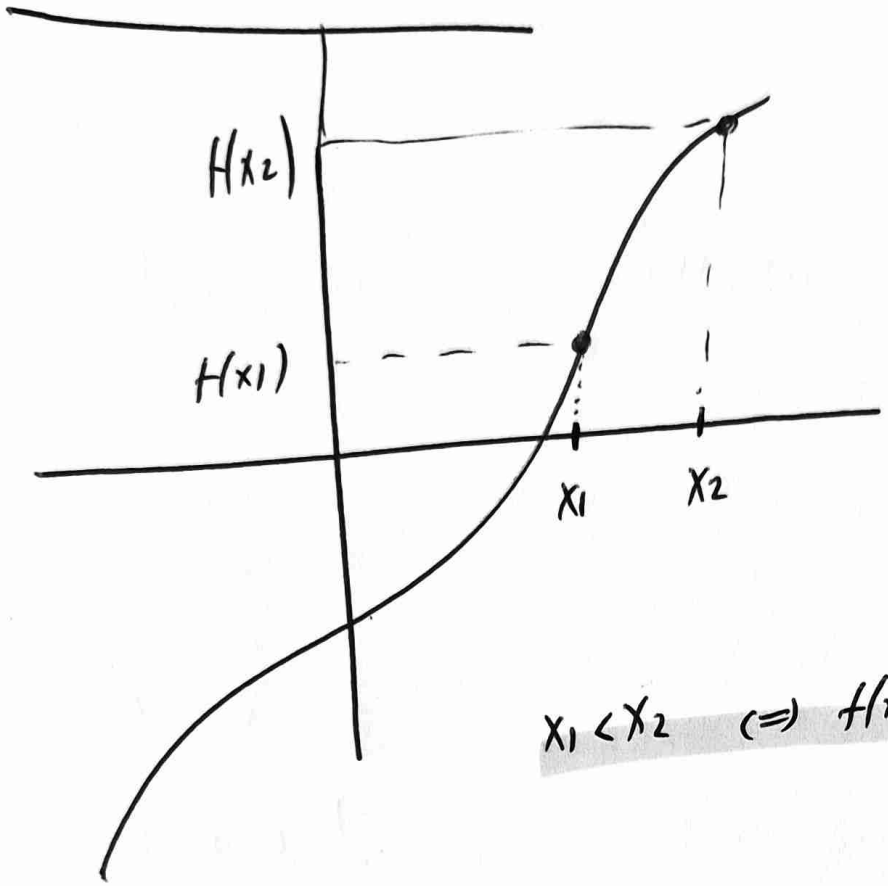
$x+1 > 1$

$x > 0$

$(g \circ g)(x) = \ln(\ln(x+1)+1)$

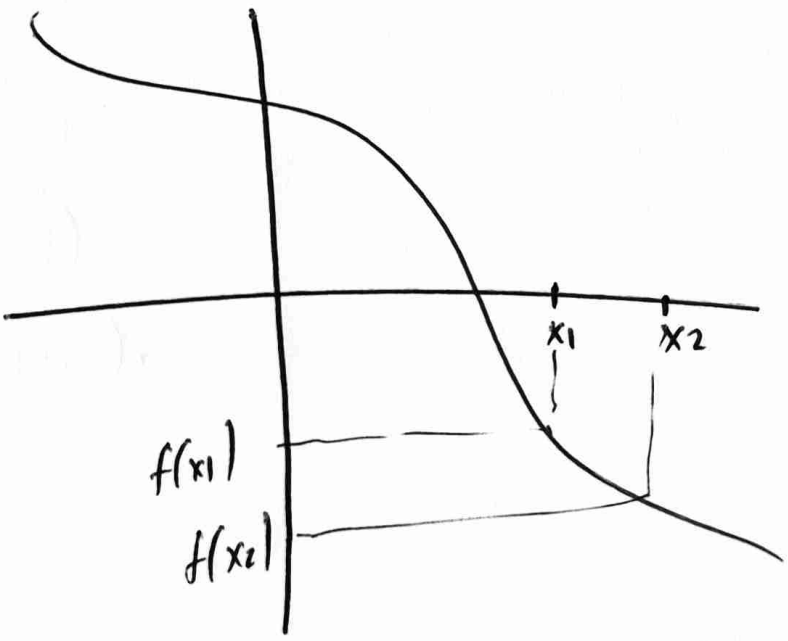
$D_{g \circ g} = (0, +\infty)$

Γνησίως αύξουσα



$x_1 < x_2 \Leftrightarrow f(x_1) < f(x_2)$

Γνησίως φθίνουσα



$x_1 < x_2 \Leftrightarrow f(x_1) > f(x_2)$

Παρατηρήσεις

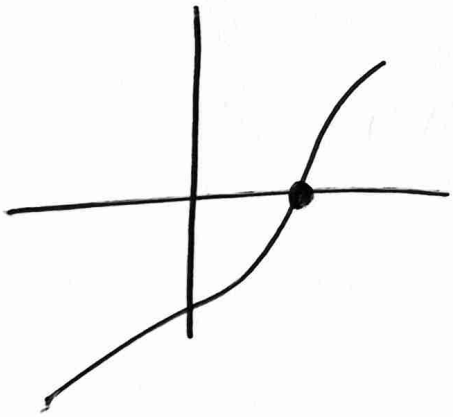
1. Αν η $f \nearrow$ τότε η f γν. μονότονη
2. Αν η $f \searrow$ τότε η f γν. μονότονη
3. Αν η f γν. μονότονη τότε η $f \nearrow$ ή $\searrow$

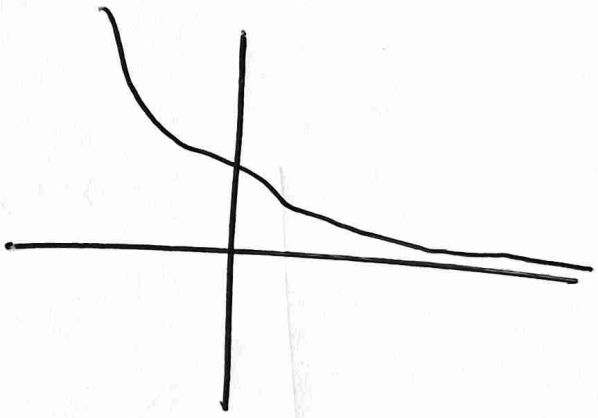
   δεν γνωρίζω τι από τα δύο ισχύει.

4. Αν μια συνάρτηση f είναι γν. μονότονη

   τότε τέμνει τον $x'x$ το πολύ

   μια φορά.

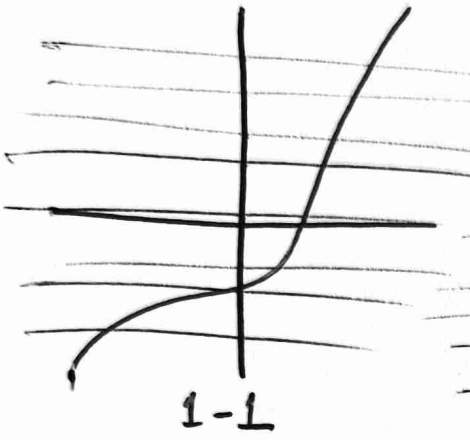




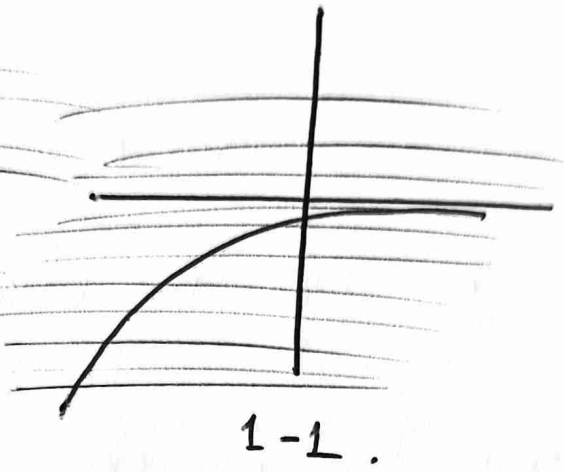
Το πολύ σημαίνει "έως"

Το τουλάχιστον σημαίνει "από εκεί και πάνω"

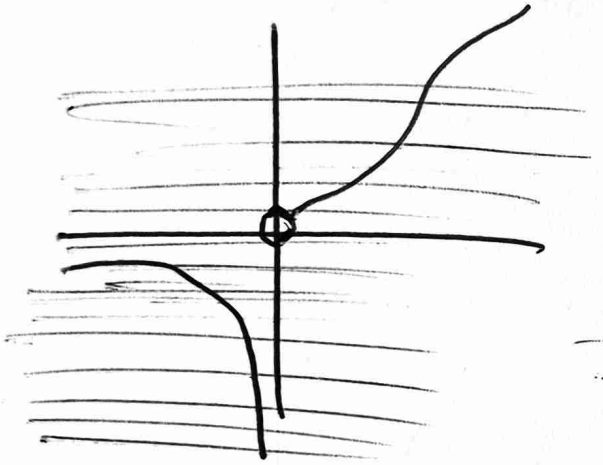
5. Μια συνάρτηση λέγεται "1-1" αν κάθε οριζόντια ευθεία τέμνει την $x'x$ το πολύ μια φορά.



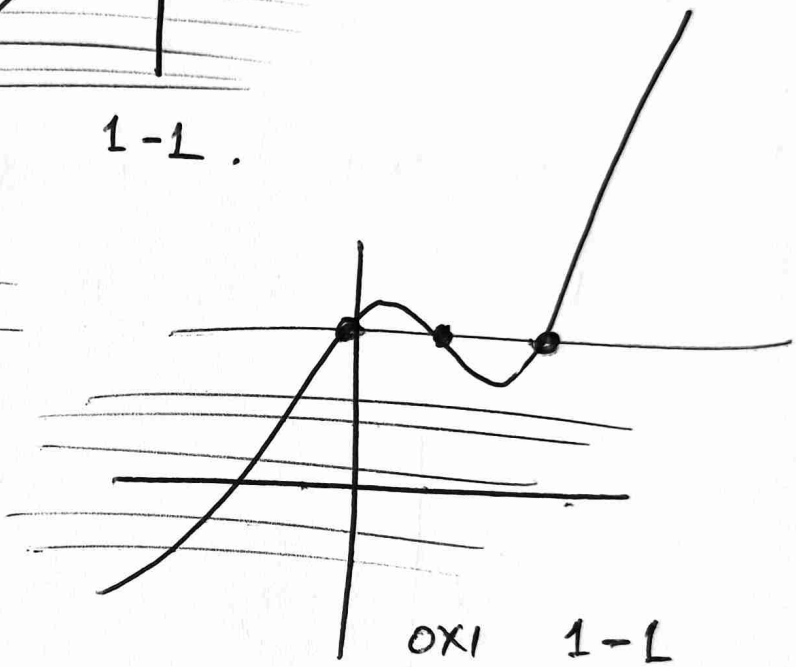
1-1



1-1.



1-1



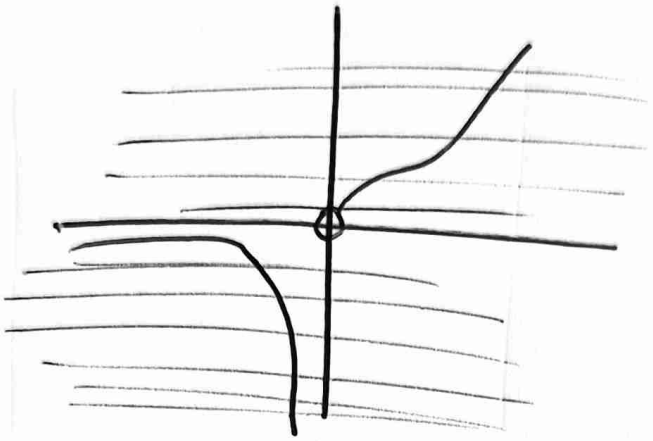
ΟΧΙ 1-1

$$f(x_1) = f(x_2) \iff x_1 = x_2$$

f 1-1

6. Αν η f γν. μονότονη $\Rightarrow$ f 1-1

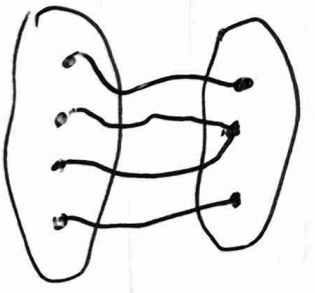
7. Αν η f 1-1 $\not\Rightarrow$ f γν. μονότονη.



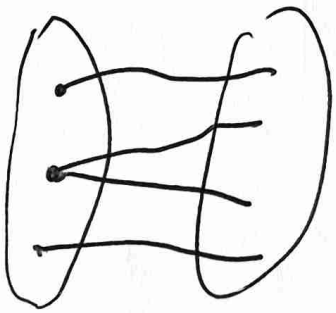
Είναι 1-1 αλλά όχι γν. μονότονη!

8. Αν η f συνάρτηση. Τότε

Αν $x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$



Συνάρτηση



όχι συνάρτηση.

9. Συμπεράσματα

$x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$ f συνάρτηση

$x_1 = x_2 \Leftrightarrow f(x_1) = f(x_2)$ $f: 1-1$

$x_1 < x_2 \Leftrightarrow f(x_1) < f(x_2)$ $f \nearrow$

$x_1 < x_2 \Leftrightarrow f(x_1) > f(x_2)$ $f \searrow$.

Επορεω Μαθημα

Τεταρτη 9 - 11:30

3

(2) αγ

(3) α β δ.

(5) α β δ

(6) ολη

(8)

(9)

(10).